

Formal Methods for Interactive Systems

Part 10 — Reverse Engineering with Matrix Algebra

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Reverse Engineering

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⇒ formal specifications **must be reconstructed**

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- checking interaction results against the **real device**
- observing user-machine interaction in the **real operating environment**
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- checking documentation (e.g., **user manual**)

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- **case studies are more credible** \longleftarrow not tailored for the used modelling / verification approach

Matrix Algebra

to describe interaction human-machine

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[Thimbleby 04]

Example: Pushbutton Switch

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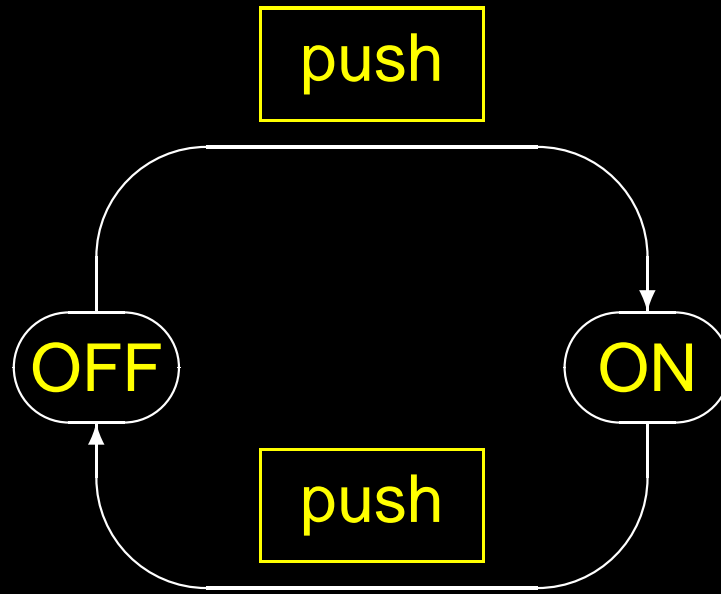
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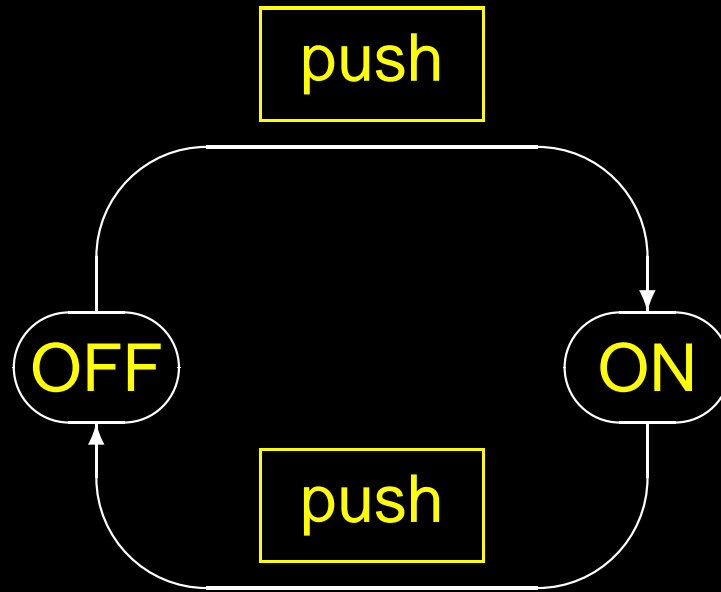
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[Thimbleby 04]

Model of Pushbutton Switch



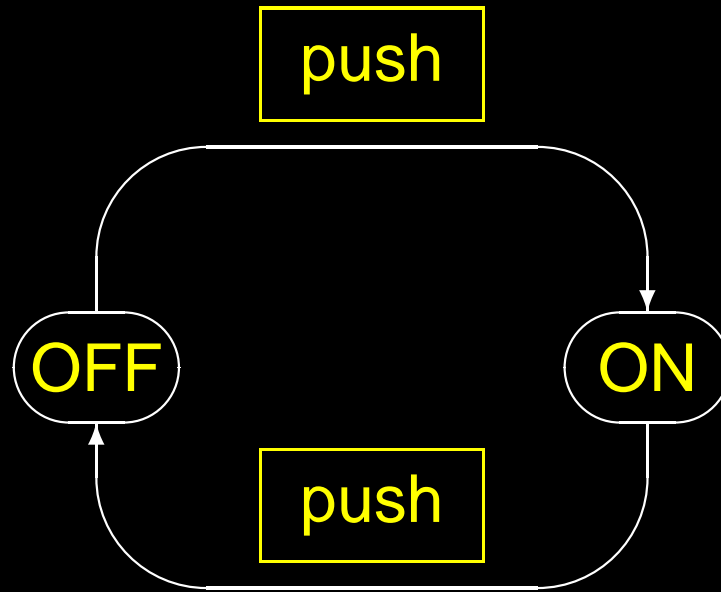
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Switch Properties

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$$\text{No because } \boxed{\text{push}}^2 = I$$

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Properties

- The system is **closed**
- any sequence of actions is equivalent to the last actions the user does

Remarks

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 - **mechanisable [Gow and Thimbleby 04]**

Calculator

- Model: **CASIO HL-820LC**

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- Operations: **binary 2×2 matrices**

Calculator Functions

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Remarks on MRC

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 $\boxed{\text{MRC}} (\boxed{\text{MRC}} \boxed{\text{MRC}}) = \boxed{\text{MRC}} \boxed{\text{MRC}}$

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- Do we really need to give such a special function to **MRC** **MRC**?

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[Thimbleby 00]

Safety-critical

- Is a calculator safety-critical?

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- Is a calculator safety-critical?
- May a calculator-like interface be safety-critical?

Safety-critical

- Is a calculator safety-critical?
- May a calculator-like interface be safety-critical?
- Yes!

Syringe pump have calculator-like interfaces

Examinations

Seminar 6 — Usable Calculator

Topic: Calculator Interfaces

Towards a truly usable calculator

- Harold Thimbleby
Computer Algebra for User Interface Design, 2004
- Harold Thimbleby
A Novel Pen-based Calculator and its Evaluation, 2004
- Will Thimbleby and Harold Thimbleby
A Novel Gesture-based Calculator and its Design Principles, 2005
- Websites on
 - Computer Algebra: <http://www.cs.swan.ac.uk/~csharold/CA/>
 - Calculator: <http://www.cs.swan.ac.uk/calculators/>

References

Reverse Engineering

- [Thimbleby 04]: Reverse Engineering and Matrix Algebra
- [Gow and Thimbleby 2004]: Matrix Algebra and Tools
- [Thimbleby 00]: Reverse Engineering

[Thimbleby 04]

Harold Thimbleby.

User interface design with matrix algebra.

ACM Transactions on Computer-Human

Interaction, Vol. 11, No. 2, 2004, pages 181–236.

About:

- Reverse Engineering
- Matrix Algebra

[Gow and Thimbleby 04]

Jeremy Gow and Harold Thimbleby.

MAUI: An Interface Design Tool Based on Matrix Algebra.

In *Proceedings of CADUI 2004*, Springer, 2004, pages 81–94.

About:

- Tools

[Thimbleby 00]

Harold Thimbleby.

Calculators are needlessly bad.

International Journal of Computer-Human Studies, Vol. 52, No. 6, 2000, pages 1031–1069.

About:

- Reverse Engineering of Calculatos