Compositional Semantics and Behavioral Equivalences for P Systems

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Outline of the talk

1. Introduction
    - P Systems
    - Behavioral Equivalences for P Systems
2. The P Algebra
    - Syntax
    - Semantics
    - Behavioral Preorders and Equivalences
3. The PP Algebra
    - Few Words
4. Conclusions and Future Work

Introduction

P Systems are distributed parallel computing devices inspired by the structure and the functioning of living cells

Recently, some operational semantics of P Systems have been defined (e.g. by Ciobanu et Al., Busi, and Freund et Al.)

The aim of this work is to introduce *behavioral equivalences* for P Systems

- We introduce a process algebraic representation of P Systems
- We define a compositional semantics as a *Labeled Transition System (LTS)*
- We study well–known behavioral equivalences over LTSs on our semantics

P Systems

A *P System* Π is given by

$$\Pi = (V, \mu, w_1, \ldots, w_n, (R_1, \rho_1), \ldots, (R_n, \rho_n))$$

where:

- V is an *alphabet* whose elements are called *objects*;
- $\mu \subset \mathbb{N} \times \mathbb{N}$ is a *membrane structure*, such that $(i, j) \in \mu$ denotes that the membrane labeled by j is contained in the membrane labeled by i;
- w_i with $1 \leq i \leq n$ are strings from V^* representing multisets over V associated with the membranes $1, 2, \ldots, n$ of μ;
- R_i with $1 \leq i \leq n$ are finite sets of *evolution rules* associated with the membranes $1, 2, \ldots, n$ of μ;
- ρ_i is a partial order relation over R_i, specifying a *priority* relation between rules: $(r_1, r_2) \in \rho_1$ iff $r_1 > r_2$ (i.e. r_1 has a higher priority than r_2).

Evolution Rules

The products of a rule are denoted with a multiset of *messages* of the following forms:

- $(v, here)$: objects v remain in the same membrane;
- (v, out): objects v are sent out;
- (v, in_l): objects v are sent into the child membrane l.

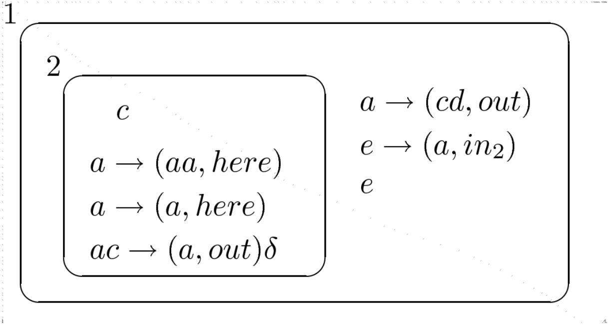
We can assume that all evolution rules have the following form, where $\{l_1, \ldots, l_n\}$ is a set of membrane labels in $\mathbb{N}$.

$$u \to (v_h, here)(v_o, out)(v_1, in_{l_1}) \ldots (v_n, in_{l_n})$$

A dissolving evolution rule is denoted by adding to the products the special message δ such that $\delta \notin V$:

$$u \to (v_h, here)(v_o, out)(v_1, in_{l_1}) \ldots (v_n, in_{l_n})\delta$$

An Example



A P System that may send out of the skin membrane (if the computation terminates) a multiset of objects $c^n d^n$.

Maximal Parallelism

Evolution rules are applied with *maximal parallelism*:

> *A* **multiset of instances** *of evolution rules is chosen non–deterministically such that* **no other rule can be applied** *to the system obtained by removing all the objects necessary to apply the chosen instances of rules.*

Priority relations between rules are such that:

> *A rule with a priority smaller than another cannot be chosen for application if the one with greater priority* **is applicable**.

Observable Behavior

In order to define reasonable semantics and behavioral equivalences we have to characterize what is reasonable to observe of the behavior of a P System

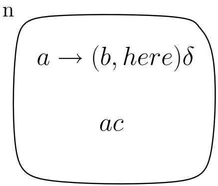
- There are many choices…

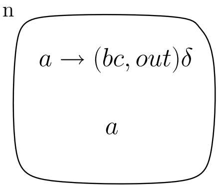
We choose to observe the input/output behavior of membranes:

> *Two membranes are equivalent if, at each step, they can:*
>
> - *receive the same objects from outer and inner membranes*
> - *send the same objects to the outer membrane (or to the external environment)*
> - *send the same objects to the same inner membranes*

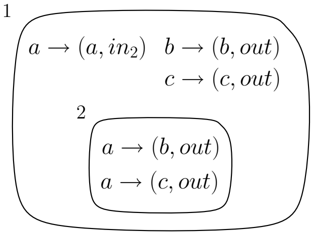
Examples of Equivalent Membranes

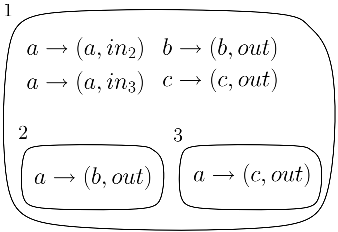
The following membranes could be considered as equivalent:





and also the following two:





Examples of Equivalent Membrane Contents

A *membrane content* is a pair $(\mathcal{R}, u)$ where

- $\mathcal{R}$ is a set of evolution rules
- u is a multiset of objects

that can be (a part of) the content of a membrane

These membrane contents should be considered pairwise equivalent:

$$(a \rightarrow (b, here)\delta\ ,\ \emptyset) \qquad \text{and} \qquad (a \rightarrow (b, out)\delta\ ,\ \emptyset)$$

$$(a \rightarrow (b, here)\delta\ ,\ abc) \qquad \text{and} \qquad (a \rightarrow (b, out)\delta\ ,\ abc)$$

$$(\mathcal{R}_1\ ,\ \emptyset) \qquad \text{and} \qquad (\mathcal{R}_2\ ,\ \emptyset)$$

where $\mathcal{R}_1 = \{\ \ a \rightarrow (b, here)\ \ ,\ \ a \rightarrow (c, here)\ \ \}$ and
$\mathcal{R}_2 = \mathcal{R}_1 \cup \{\ \ aa \rightarrow (bc, here)\ \ \}$.

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The P Algebra (for P Systems without Priorities)

Def. (P Algebra) The syntax of *membrane contents* c, *membranes* m, and *membrane systems* ms is given by the following grammar:

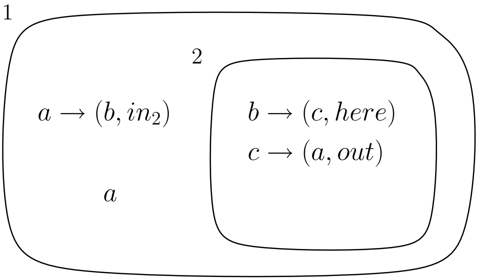
$$c ::= (\varnothing, \varnothing) \;\big|\; (u \rightarrow v_h v_o \{v_{l_i}\}, \varnothing) \;\big|\; (u \rightarrow v_h v_o \{v_{l_i}\}\delta, \varnothing) \;\big|\; (\varnothing, a) \;\big|\; c \cup c$$

$$m ::= [_l c]_l \qquad ms ::= m \;\big|\; ms \mid ms \;\big|\; \mu(m, ms) \;\big|\; \mathbf{v}$$

where l and l_i range over $\mathbb{N}$ and a ranges over V.

- $u \rightarrow v_h v_o \{v_{l_i}\}$ stands for
  $u \rightarrow (v_h, here)(h_o, out)(v_{l_1}, in_{l_1}) \ldots (v_{l_n}, in_{l_n})$
- $c_1 \cup c_2$ denotes the membrane content obtained by merging the rules and the objects of c_1 and c_2
- $[_l c]_l$ denotes a membrane whose content is c and whose label is l
- $ms|ms$ denotes *juxtaposition* of membranes
- $\mu(m, ms)$ denotes the containment of the membranes ms in m (hierarchical composition)
- $\mathbf{v}$ represents the *dissolved* membrane

Example of Term of the P Algebra



corresponds to the term:

$$\mu\big(\ [_1\ (a \to (b, in_2), \varnothing)\ \cup\ (\varnothing, a)\ ]_1\ ,\ [_2\ (b \to (c, here), \varnothing)\ \cup\ (c \to (a, out), \varnothing)\ ]_2\ \big)$$

that is (for short):

$$\mu\big(\ [_1\ a \to (b, in_2)\ ,\ a\ ]_1\ ,\ [_2\ b \to (c, here)\ ,\ c \to (a, out)\ ]_2\ \big)$$

Semantics of the P Algebra

The semantics of the P Algebra is a Labeled Transition System (LTS)

- states are terms
- transitions are labeled by information about the input/output behavior of the system (observation)

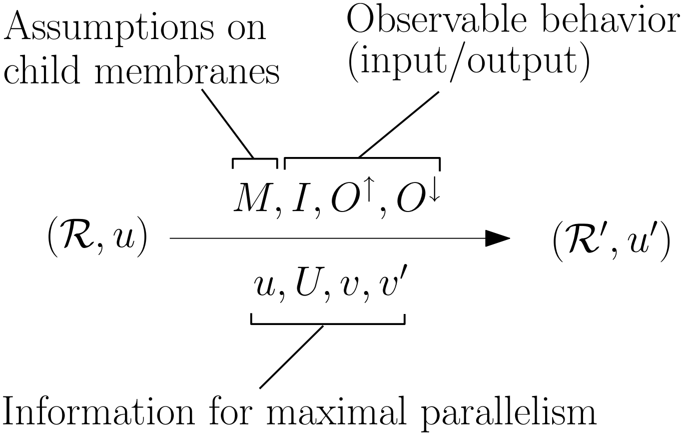
Let's start with membrane contents. We would like to

- define the behavior of individual evolution rules and objects
- infer the behavior of a membrane content from the behaviors of its rules and objects

Problem: it is hard to express the concept of maximal parallelism in a compositional way

Solution: we enrich transition labels with information concerning the (potential) application and non application of evolution rules

Transitions of Membrane Contents



Inference Rules for Membrane Contents

$$\frac{I \in V^*}{(\varnothing,\varnothing) \xrightarrow[\varnothing,\varnothing,\varnothing,\varnothing]{\varnothing,I,\varnothing,\varnothing} (\varnothing, I)} \quad (mc8)$$

$$\frac{I \in V^*}{(\varnothing,a) \xrightarrow[\varnothing,\varnothing,\varnothing,a]{\varnothing,I,\varnothing,\varnothing} (\varnothing, Ia)} \quad (mc7)$$

$$\frac{I \in V^*}{(\varnothing,a) \xrightarrow[\varnothing,\varnothing,a,\varnothing]{\varnothing,I,\varnothing,\varnothing} (\varnothing, I)} \quad (mc6)$$

$$\frac{I \in V^* \quad n \in \mathbb{N}}{(u \rightarrow v_h v_o\{v_{l_i}\},\varnothing) \xrightarrow[u^n,\{u\},\varnothing,\varnothing]{\varnothing,I,v_o^n,\{(l_i,v_{l_i}^n)\}} (u \rightarrow v_h v_o\{v_{l_i}\}, I v_h^n)} \quad (mc1_n)$$

Inference Rules for Membrane Contents (2)

$$\frac{I \in V^* \quad n \in \mathbb{N} \quad n > 0}{(u \to v_h v_o \{v_{l_i}\}\delta, \varnothing) \xrightarrow[u^n, \{u\}, \varnothing, \varnothing]{\varnothing, I, I v_o^n v_h^n \delta, \{(l_i, v_{l_i}^n)\}} \mathbf{v}} \quad (mc2_n)$$

$$\frac{I \in V^*}{(u \to v_h v_o \{v_{l_i}\}\delta, \varnothing) \xrightarrow[\varnothing, \{u\}, \varnothing, \varnothing]{\varnothing, I, \varnothing, \varnothing} (u \to v_h v_o \{v_{l_i}\}\delta, I)} \quad (mc3)$$

$$\frac{I \in V^* \quad M \subseteq \mathsf{Labels}(\{v_{l_i}\}) \quad M \neq \varnothing}{(u \to v_h v_o \{v_{l_i}\}, \varnothing) \xrightarrow[\varnothing, \varnothing, \varnothing, \varnothing]{M, I, \varnothing, \varnothing} (u \to v_h v_o \{v_{l_i}\}, I)} \quad (mc4)$$

$$\frac{I \in V^* \quad M \subseteq \mathsf{Labels}(\{v_{l_i}\}) \quad M \neq \varnothing}{(u \to v_h v_o \{v_{l_i}\}\delta, \varnothing) \xrightarrow[\varnothing, \varnothing, \varnothing, \varnothing]{M, I, \varnothing, \varnothing} (u \to v_h v_o \{v_{l_i}\}\delta, I)} \quad (mc5)$$

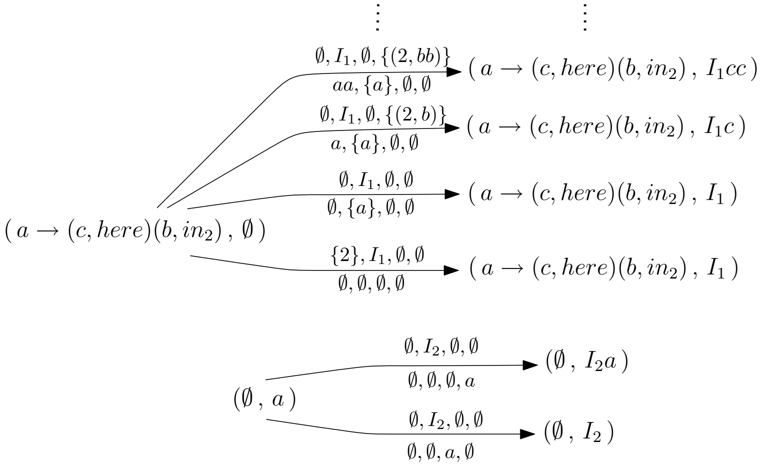
Inference Rules for Membrane Contents (3)

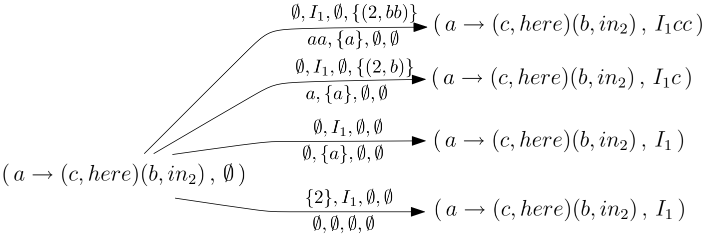
$$\frac{x_1 \xrightarrow[u_1, U_1, v_1, v_1']{M_1, I_1, O_1^{\uparrow}, O_1^{\downarrow}} y_1 \quad x_2 \xrightarrow[u_2, U_2, v_2, v_2']{M_2, I_2, O_2^{\uparrow}, O_2^{\downarrow}} y_2 \quad \begin{array}{c} M_1 M_2 \cap \mathsf{Labels}(O_1^{\downarrow} \cup_{\mathbb{N}} O_2^{\downarrow}) = \varnothing \\ v_1' v_2' \nvdash U_1 \oplus U_2 \quad \delta \notin O_1^{\uparrow} O_2^{\uparrow} \end{array}}{x_1 \cup x_2 \xrightarrow[u_1 u_2, U_1 \oplus U_2, v_1 v_2, v_1' v_2']{M_1 M_2, I_1 I_2, O_1^{\uparrow} O_2^{\uparrow}, O_1^{\downarrow} \cup_{\mathbb{N}} O_2^{\downarrow}} y_1 \cup y_2}$$

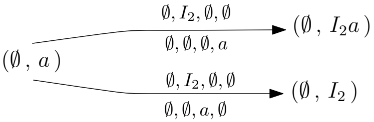
+ similar rules to handle dissolution of x_1 or/and x_2

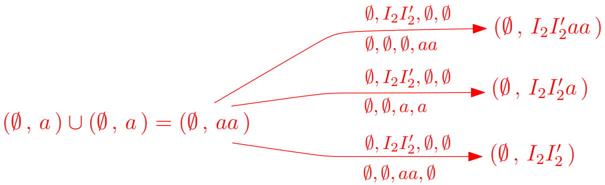
- $u_1 u_2$ is the union of multisets u_1 and u_2
- $v \vdash U$ means $\exists u.(u \subseteq v \wedge u \in U)$
- $\cup_{\mathbb{N}}$ groups objects sent to the same child membrane
- $\oplus$ merges sets of multisets by removing redundant ones

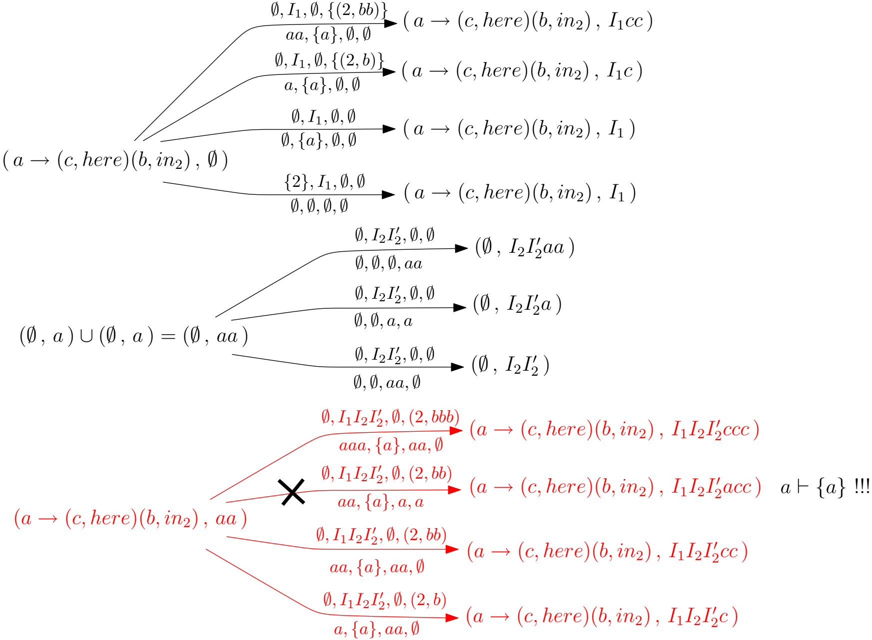
Example of Semantics of Membrane Contents

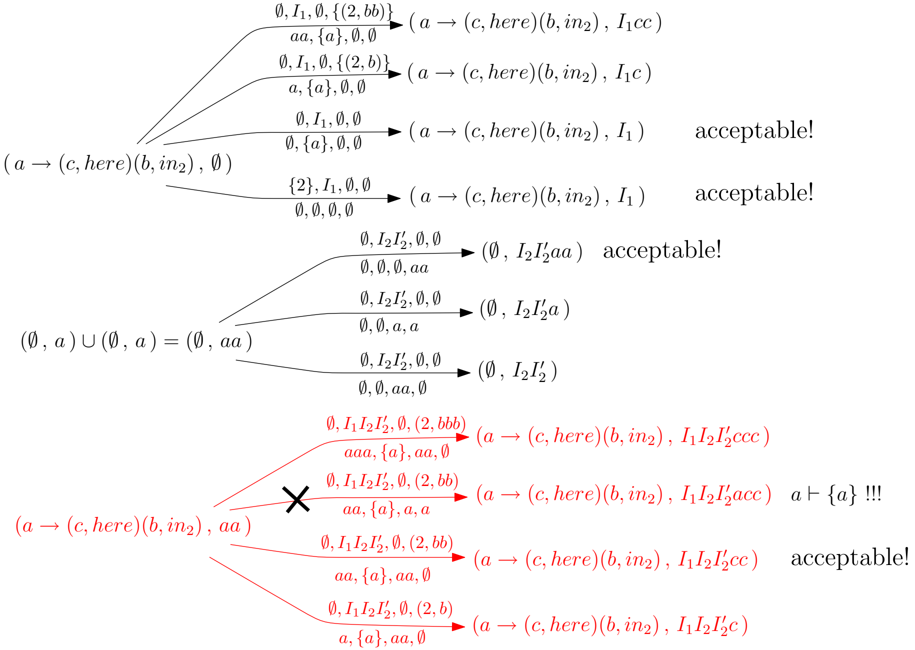



$(\, a \rightarrow (c, here)(b, in_2)\, ,\, \emptyset\,)$
$\emptyset, I_1, \emptyset, \{(2, bb)\}$
$aa, \{a\}, \emptyset, \emptyset$
$(\, a \rightarrow (c, here)(b, in_2)\, ,\, I_1cc\,)$
$\emptyset, I_1, \emptyset, \{(2, b)\}$
$a, \{a\}, \emptyset, \emptyset$
$(\, a \rightarrow (c, here)(b, in_2)\, ,\, I_1c\,)$
$\emptyset, I_1, \emptyset, \emptyset$
$\emptyset, \{a\}, \emptyset, \emptyset$
$(\, a \rightarrow (c, here)(b, in_2)\, ,\, I_1\,)$
$\{2\}, I_1, \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, \emptyset$
$(\, a \rightarrow (c, here)(b, in_2)\, ,\, I_1\,)$


$(\emptyset\, ,\, a\,)$
$\emptyset, I_2, \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, a$
$(\emptyset\, ,\, I_2a\,)$
$\emptyset, I_2, \emptyset, \emptyset$
$\emptyset, \emptyset, a, \emptyset$
$(\emptyset\, ,\, I_2\,)$


$(\emptyset\, ,\, a\,) \cup (\emptyset\, ,\, a\,) = (\emptyset\, ,\, aa\,)$
$\emptyset, I_2I_2', \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, aa$
$(\emptyset\, ,\, I_2I_2'aa\,)$
$\emptyset, I_2I_2', \emptyset, \emptyset$
$\emptyset, \emptyset, a, a$
$(\emptyset\, ,\, I_2I_2'a\,)$
$\emptyset, I_2I_2', \emptyset, \emptyset$
$\emptyset, \emptyset, aa, \emptyset$
$(\emptyset\, ,\, I_2I_2'\,)$


$(\, a \to (c, here)(b, in_2)\,,\ \emptyset\,)$
$\emptyset, I_1, \emptyset, \{(2, bb)\}$
$aa, \{a\}, \emptyset, \emptyset$
$(\, a \to (c, here)(b, in_2)\,,\ I_1cc\,)$
$\emptyset, I_1, \emptyset, \{(2, b)\}$
$a, \{a\}, \emptyset, \emptyset$
$(\, a \to (c, here)(b, in_2)\,,\ I_1c\,)$
$\emptyset, I_1, \emptyset, \emptyset$
$\emptyset, \{a\}, \emptyset, \emptyset$
$(\, a \to (c, here)(b, in_2)\,,\ I_1\,)$
$\{2\}, I_1, \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, \emptyset$
$(\, a \to (c, here)(b, in_2)\,,\ I_1\,)$
$(\emptyset\,,\ a\,) \cup (\emptyset\,,\ a\,) = (\emptyset\,,\ aa\,)$
$\emptyset, I_2I'_2, \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, aa$
$(\emptyset\,,\ I_2I'_2aa\,)$
$\emptyset, I_2I'_2, \emptyset, \emptyset$
$\emptyset, \emptyset, a, a$
$(\emptyset\,,\ I_2I'_2a\,)$
$\emptyset, I_2I'_2, \emptyset, \emptyset$
$\emptyset, \emptyset, aa, \emptyset$
$(\emptyset\,,\ I_2I'_2\,)$
$(a \to (c, here)(b, in_2)\,,\ aa\,)$
$\emptyset, I_1I_2I'_2, \emptyset, (2, bbb)$
$aaa, \{a\}, aa, \emptyset$
$(a \to (c, here)(b, in_2)\,,\ I_1I_2I'_2ccc\,)$
$\emptyset, I_1I_2I'_2, \emptyset, (2, bb)$
$aa, \{a\}, a, a$
$(a \to (c, here)(b, in_2)\,,\ I_1I_2I'_2acc\,)$
$a \vdash \{a\}$!!!
$\emptyset, I_1I_2I'_2, \emptyset, (2, bb)$
$aa, \{a\}, aa, \emptyset$
$(a \to (c, here)(b, in_2)\,,\ I_1I_2I'_2cc\,)$
$\emptyset, I_1I_2I'_2, \emptyset, (2, b)$
$a, \{a\}, aa, \emptyset$
$(a \to (c, here)(b, in_2)\,,\ I_1I_2I'_2c\,)$


$(a \to (c, here)(b, in_2), \emptyset)$
$\emptyset, I_1, \emptyset, \{(2, bb)\}$
$aa, \{a\}, \emptyset, \emptyset$
$(a \to (c, here)(b, in_2), I_1cc)$
$\emptyset, I_1, \emptyset, \{(2, b)\}$
$a, \{a\}, \emptyset, \emptyset$
$(a \to (c, here)(b, in_2), I_1c)$
$\emptyset, I_1, \emptyset, \emptyset$
$\emptyset, \{a\}, \emptyset, \emptyset$
$(a \to (c, here)(b, in_2), I_1)$
acceptable!
$\{2\}, I_1, \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, \emptyset$
$(a \to (c, here)(b, in_2), I_1)$
acceptable!
$(\emptyset, a) \cup (\emptyset, a) = (\emptyset, aa)$
$\emptyset, I_2I'_2, \emptyset, \emptyset$
$\emptyset, \emptyset, \emptyset, aa$
$(\emptyset, I_2I'_2aa)$
acceptable!
$\emptyset, I_2I'_2, \emptyset, \emptyset$
$\emptyset, \emptyset, a, a$
$(\emptyset, I_2I'_2a)$
$\emptyset, I_2I'_2, \emptyset, \emptyset$
$\emptyset, \emptyset, aa, \emptyset$
$(\emptyset, I_2I'_2)$
$(a \to (c, here)(b, in_2), aa)$
$\emptyset, I_1I_2I'_2, \emptyset, (2, bbb)$
$aaa, \{a\}, aa, \emptyset$
$(a \to (c, here)(b, in_2), I_1I_2I'_2ccc)$
$\emptyset, I_1I_2I'_2, \emptyset, (2, bb)$
$aa, \{a\}, a, a$
$(a \to (c, here)(b, in_2), I_1I_2I'_2acc)$
$a \vdash \{a\}$!!!
$\emptyset, I_1I_2I'_2, \emptyset, (2, bb)$
$aa, \{a\}, aa, \emptyset$
$(a \to (c, here)(b, in_2), I_1I_2I'_2cc)$
acceptable!
$\emptyset, I_1I_2I'_2, \emptyset, (2, b)$
$a, \{a\}, aa, \emptyset$
$(a \to (c, here)(b, in_2), I_1I_2I'_2c)$

Inference Rules for Membranes and Juxtapositions

$$\frac{x \xrightarrow[u,U,u,v']{M,I,O^{\uparrow},O^{\downarrow}} y \quad \delta \notin O^{\uparrow}}{[_l x\,]_l \xrightarrow{M,\{(l,I)\},O^{\uparrow},O^{\downarrow}} [_l y\,]_l} \quad (m1)$$

$$\frac{x \xrightarrow[u,U,u,v']{M,I,O^{\uparrow},O^{\downarrow}} y \quad \delta \in O^{\uparrow}}{[_l x\,]_l \xrightarrow{M,\{(l,I)\},O^{\uparrow},O^{\downarrow}} \mathbf{v}} \quad (m2)$$

- information under the arrow is no longer necessary
- the transition from x to y must be *acceptable*
  - the first and the third label under the arrow are the same
  - this is important to ensure maximal parallelism

$$\frac{x_1 \xrightarrow{M_1,\mathcal{I}_1,O_1^{\uparrow},\varnothing} y_1 \quad x_2 \xrightarrow{M_2,\mathcal{I}_2,O_2^{\uparrow},\varnothing} y_2 \quad \delta \notin O_1^{\uparrow}O_2^{\uparrow}}{x_1|x_2 \xrightarrow{\varnothing,\mathcal{I}_1\mathcal{I}_2,O_1^{\uparrow}O_2^{\uparrow},\varnothing} y_1|y_2} \quad (jux1)$$

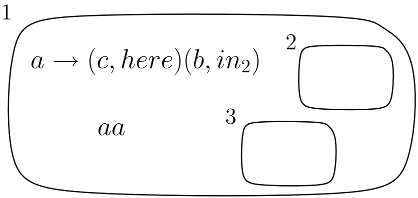
+ similar rules to handle dissolution of x_1 or/and x_2

Inference Rules for Hierarchical Compositions

$$\frac{x_1 \xrightarrow{M_1,\{(l_1,I_1)\},O_1^{\uparrow},O_1^{\downarrow}} y_1 \qquad x_2 \xrightarrow{M_2,\mathcal{I}_2,O_2^{\uparrow},\varnothing} y_2 \qquad O_1^{\downarrow} \doteq \mathcal{I}_2 \quad O_2^{\uparrow} \subseteq I_1 \quad M_1 \cap \mathsf{Labels}(\mathcal{I}_2) = \varnothing \quad \delta \notin O_1^{\uparrow}O_2^{\uparrow}}{\mu(x_1,x_2) \xrightarrow{\varnothing,(l_1,I_1 \setminus O_2^{\uparrow}),O_1^{\uparrow},\varnothing} \mu(y_1,y_2)} \quad (h1)$$

+ similar rules to handle dissolution of x_1 or/and x_2

- $O_1^{\downarrow} \doteq \mathcal{I}_2$ means that the two sets of pairs are the same apart from some $(l_i, \varnothing)$



$$(a \to (c, here)(b, in_2)\ ,\ aa) \xrightarrow[aa,\{a\},aa,\varnothing]{\varnothing,I_1,\varnothing,(2,bb)} (a \to (c, here)(b, in_2)\ ,\ Icc)$$

$$[_1\ (a \to c_{here}b_{in_2}\ ,\ aa)\ ]_1 \xrightarrow{\varnothing,\{(1,I_1)\},\varnothing,(2,bb)} [_1\ (a \to c_{here}b_{in_2}\ ,\ Icc)\ ]_1$$

$$[_2(\varnothing, \varnothing)]_2 \xrightarrow{\varnothing,\{(2,bb)\},\varnothing,\varnothing} [_2(\varnothing, bb)]_2 \qquad [_3(\varnothing, \varnothing)]_3 \xrightarrow{\varnothing,\{(3,\varnothing)\},\varnothing,\varnothing} [_3(\varnothing, \varnothing)]_3$$

$$[_2(\varnothing, \varnothing)]_2 \mid [_3(\varnothing, \varnothing)]_3 \xrightarrow{\varnothing,\{(2,bb),(3,\varnothing)\},\varnothing,\varnothing} [_2(\varnothing, bb)]_2 \mid [_3(\varnothing, \varnothing)]_3$$

$$\mu([_1\ldots]_1, [_2\ldots]_2 \mid [_3\ldots]_3) \xrightarrow{\varnothing,\{(1,I_1)\},\varnothing,\varnothing} \mu([_1(\_, Icc)]_1, [_2(\varnothing, bb)]_2 \mid [_3(\varnothing, \varnothing)]_3)$$

Maximal Parallelism Theorem

Theorem (Maximality)

$$(\mathcal{R}, u) \xrightarrow[u',U,u',v']{M,I,O_1^{\uparrow},O_1^{\downarrow}} x \qquad \text{implies} \qquad (\mathcal{R}, v') \not\xrightarrow[u'',U',u'',v'']{M,I',O_2^{\uparrow},O_2^{\downarrow}}$$

for any $u'' \neq \varnothing$

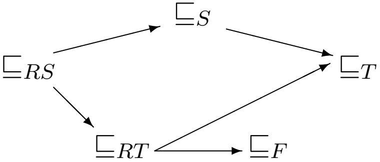
Well–known Behavioral Preorders (1)

Let $(\mathcal{S}, \mathcal{L}, \{\xrightarrow{\ell} \mid \ell \in \mathcal{L}\})$ be an LTS. A relation $R \subseteq \mathcal{S} \times \mathcal{S}$

- is a *simulation* ($\sqsubseteq_S$ the largest) if, for each pair $s_1 \, R \, s_2$, if $s_1 \xrightarrow{\ell} s'_1$ then there is a transition $s_2 \xrightarrow{\ell} s'_2$ such that $s'_1 \, R \, s'_2$;
- is a *ready simulation* ($\sqsubseteq_{RS}$ the largest) if it is a simulation and, for each pair $s_1 \, R \, s_2$, if $s_1 \not\xrightarrow{\ell}$ then $s_2 \not\xrightarrow{\ell}$;
- is a *ready trace preorder* ($\sqsubseteq_{RT}$ the lar.) if, for each pair $s_1 R s_2$, any ready trace of s_1 is a ready trace of s_2 (a sequence $L_0 \ell_1 L_1 \dots \ell_n L_n$ with $L_i \subseteq \mathcal{L}$ and $\ell_i \in \mathcal{L}$ is a *ready trace* of a state s_0 if $s_0 \xrightarrow{\ell_1} s_1 \xrightarrow{\ell_2} \dots s_{n-1} \xrightarrow{\ell_n} s_n$ and $\mathsf{Initials}(s_i) = L_i$ for $i = 0, \dots, n$);
- is a *failure preorder* ($\sqsubseteq_F$ the largest)if, for each pair $s_1 R s_2$, any failure of s_1 is a failure of s_2 (a pair $(\ell_1 \dots \ell_n, L)$ with $\ell_1 \dots \ell_n \in \mathcal{L}$ and $L \subseteq \mathcal{L}$ is a *failure* of a state s if $s \xrightarrow{\ell_1} \dots \xrightarrow{\ell_n} s'$ for some state s' such that $\mathsf{Initials}(s') \cap L = \varnothing$);
- is a *trace preorder* ($\sqsubseteq_T$ the largest)if, for each pair $s_1 R s_2$, any trace of s_1 is a trace of s_2 (a sequence $\ell_1 \dots \ell_n$ with $\ell_i \in \mathcal{L}$ is a *trace* of a state s_0 if $s_0 \xrightarrow{\ell_1} \dots \xrightarrow{\ell_n} s_n$ for some state s_n).

Well–known Behavioral Preorders (2)

It is well–known that the considered preorders are structured as follows (where $\rightarrow$ is $\subseteq$)



In the case of the P Algebra all the inclusions are strict

Well–known Behavioral Equivalences

The kernels of the preorders (the largest equivalence each of them contains) are well–known behavioral equivalences

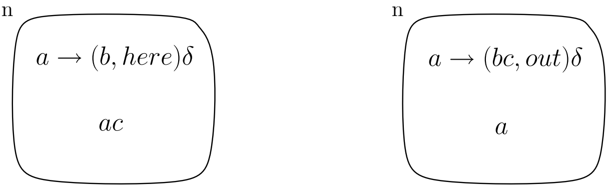
- bisimulation $\approx$ is the kernel of $\sqsubseteq_S$
- trace equivalence $\approx_T$ is the kernel of $\sqsubseteq_T$

The inference rules of the semantics of the P Algebra satisfy *de Simone* format.

Theorem All of the considered preorders are precongruences

Corollary All of the kernels of the considered preorders are congruences

Examples of Equivalent Membranes (1)



Their semantics are

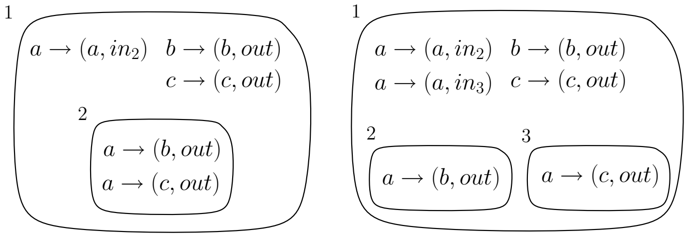
$$[_n \ (a \to (b, here)\delta \ , \ ac) \ ]_n \xrightarrow{\varnothing, I, bc, \varnothing} \mathbf{v}$$

and

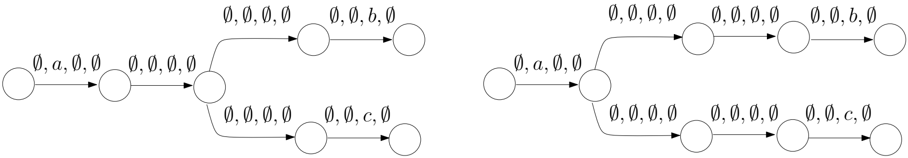
$$[_n \ (a \to (bc, out)\delta \ , \ a) \ ]_n \xrightarrow{\varnothing, I, bc, \varnothing} \mathbf{v}$$

that are (obviously) both trace equivalent $\approx_T$ and bisimilar $\approx$

Examples of Equivalent Membranes (2)



Portions of their semantics are



that are trace equivalent $\approx_T$ but not bisimilar $\approx$

Outline of the talk

1. Introduction
   - P Systems
   - Behavioral Equivalences for P Systems
2. The P Algebra
   - Syntax
   - Semantics
   - Behavioral Preorders and Equivalences
3. The PP Algebra
   - Few Words
4. Conclusions and Future Work

Priorities

As for maximal parallelism, rule priorities are difficult to be described compositionally

The definition becomes easier if we choose a different notation

$$a \rightarrow (b, in_2) > c \rightarrow (d, out)$$

becomes

$$a \rightarrow (b, in_2) \qquad \{(a, 2)\}\ c \rightarrow (d, out)$$

where $(a, 2)$ is called *priority pair* and contains information on the applicability of the rule with higher priority

The PP Algebra

Def. (PP Algebra) The syntax of *membrane contents* c, *membranes* m, and *membrane systems* ms is given by the following grammar:

$$
\begin{aligned}
c ::= & (\varnothing, \varnothing) \;\big|\; (\{(u_i, M_i)\}\, u \rightarrow v_h v_o \{v_{l_i}\}, \varnothing) \\
& \;\big|\; (\{(u_i, M_i)\}\, u \rightarrow v_h v_o \{v_{l_i}\} \delta, \varnothing) \;\big|\; (\varnothing, a) \;\big|\; c \cup c \\
m ::= & [_l c\,]_l \\
ms ::= & m \;\big|\; ms \mid ms \;\big|\; \mu(m, ms) \;\big|\; \mathbf{v}
\end{aligned}
$$

where l and l_i range over $\mathbb{N}$ and a ranges over V.

- $\{(u_i, M_i)\}\, u \rightarrow v_h v_o \{v_{l_i}\}$ stands for $\{(u_1, M_1), \ldots, (u_n, M_n)\}\, u \rightarrow (v_h, here)(h_o, out)(v_{l_1}, in_{l_1}) \ldots (v_{l_m}, in_{l_m})$

Transitions with Priorities

$$(\mathcal{R}, u) \xrightarrow[u, U, v, v']{M, I, O^{\uparrow}, O^{\downarrow}, M_P, A_P} (\mathcal{R}', u')$$

- M: Assumptions on child membranes
- $I, O^{\uparrow}, O^{\downarrow}$: Observable behavior (input/output)
- M_P, A_P: Information about priorities
- u, U, v, v': Information for maximal parallelism

Conclusions and Future Work

We defined a compositional semantics of P Systems

- we proved that it correctly describes maximal parallelism (and priorities)

We considered some well–known behavioral preorders and equivalences

- we proved that they are different relations in the case of P Systems
- we proved them to be (pre)congruences

As future work we will develop *axiomatic semantics*

- syntactical transformations between terms being correct and complete w.r.t. a behavioral equivalence