# Logica per la Programmazione - LPP 2013/14

## Leggi sul calcolo proposizionale

| | | |
|---|---|---|
| $A \vee \mathsf{F} \equiv A$ | $A \wedge \mathsf{T} \equiv A$ | (unità) |
| $A \vee \mathsf{T} \equiv \mathsf{T}$ | $A \wedge \mathsf{F} \equiv \mathsf{F}$ | (assorbimento o zero) |
| $A \wedge A \equiv A$ | $A \vee A \equiv A$ | (idempotenza) |
| $A \wedge B \equiv B \wedge A$ | $A \vee B \equiv B \vee A$ | (commutatività) |
| $A \wedge (B \wedge C) \equiv (A \wedge B) \wedge C$ | $A \vee (B \vee C) \equiv (A \vee B) \vee C$ | (associatività) |
| $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ | $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ | (distributività) |
| $\neg \mathsf{T} \equiv \mathsf{F}$ | $\neg\neg A \equiv A$ | ($\mathsf{T}$ : $\mathsf{F}$ / doppia negazione) |
| $A \vee \neg A \equiv \mathsf{T}$ | $A \wedge \neg A \equiv \mathsf{F}$ | (terzo escluso / contraddizione) |
| $\neg(A \vee B) \equiv \neg A \wedge \neg B$ | $\neg(A \wedge B) \equiv \neg A \vee \neg B$ | (De Morgan) |
| $(A \equiv B) \equiv (A \Rightarrow B) \wedge (A \Leftarrow B)$ | $(A \equiv B) \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$ | (elim.-$\equiv$ / elim.-$\equiv$-bis ) |
| $(A \Leftarrow B) \equiv (B \Rightarrow A)$ | $(A \Rightarrow B) \equiv \neg A \vee B$ | (elim.-$\Leftarrow$ / elim.-$\Rightarrow$) |
| $\neg(A \Rightarrow B) \equiv A \wedge \neg B$ | $\neg(A \equiv B) \equiv (A \equiv \neg B)$ | ($\neg$-$\Rightarrow$ / $\neg$-$\equiv$) |
| $A \vee (\neg A \wedge B) \equiv A \vee B$ | $A \wedge (\neg A \vee B) \equiv A \wedge B$ | (complemento) |
| $A \vee (A \wedge B) \equiv A$ | $A \wedge (A \vee B) \equiv A$ | (assorbimento) |
| $(A \Rightarrow B) \equiv (\neg B \Rightarrow \neg A)$ | $(A \Rightarrow B) \wedge A \Rightarrow B$ | (contronominale / Modus Ponens) |
| $A \wedge B \Rightarrow A$ | $A \Rightarrow A \vee B$ | (semplif.-$\wedge$ / introd.-$\vee$) |
| $(A \Rightarrow B) \wedge (B \Rightarrow C) \Rightarrow (A \Rightarrow C)$ | $(A \vee B) \wedge (\neg B \vee C) \Rightarrow A \vee C$ | (transitività-$\Rightarrow$ / risoluzione) |

## Leggi sui quantificatori

| | | |
|---|---|---|
| $(\forall x . P) \Rightarrow P[^{\mathbf{t}}/_x]$ | $P[^{\mathbf{t}}/_x] \Rightarrow (\exists x . P)$ | con **t** termine (elim-$\forall$ / intro-$\exists$) |
| $\neg(\exists x . A) \equiv (\forall x . \neg A)$ | $\neg(\forall x . A) \equiv (\exists x . \neg A)$ | (De Morgan) |
| $(\forall x . A \wedge B) \equiv (\forall x . A) \wedge (\forall x . B)$ | $(\exists x . A \vee B) \equiv (\exists x . A) \vee (\exists x . B)$ | ($\forall : \wedge$ / $\exists : \vee$) |
| $(\forall x . A) \vee (\forall x . B) \Rightarrow (\forall x . A \vee B)$ | $(\exists x . A \wedge B) \Rightarrow (\exists x . A) \wedge (\exists x . B)$ | ($\forall : \vee$ / $\exists : \wedge$) |
| | $x = y \Rightarrow P \equiv P[^y/_x]$ | (Leibniz) |
| $(\forall x . x = y \Rightarrow P) \equiv P[^y/_x]$ | $(\exists x . x = y \wedge P) \equiv P[^y/_x]$ | (singoletto) |
| $(\forall x . A) \equiv A \quad (\exists x . A) \equiv A$ | se $x$ non è libera in $A$ e il dominio non è vuoto | (costante) |

## Leggi su dominio e intervalli

$$(\forall x . A \vee B \Rightarrow P) \equiv (\forall x . A \Rightarrow P) \wedge (\forall x . B \Rightarrow P) \qquad \text{(dominio-}\forall\text{)}$$

$$(\exists x . (A \vee B) \wedge P) \equiv (\exists x . A \wedge P) \vee (\exists x . B \wedge P) \qquad \text{(dominio-}\exists\text{)}$$

$$(\forall x . x \in [a,b] \Rightarrow P) \equiv (\forall x . x \in [a,b) \Rightarrow P) \wedge P[^b/_x], \text{ se } a \leq b \qquad \text{(interv-}\forall\text{)}$$

$$(\exists x . x \in [a,b] \wedge P) \equiv (\exists x . x \in [a,b) \wedge P) \vee P[^b/_x], \text{ se } a \leq b \qquad \text{(interv-}\exists\text{)}$$

$$(\Sigma x : x \in [a,b] \wedge P . E) = \begin{cases} (\Sigma x : x \in [a,b) \wedge P . E) + E[^b/_x] & \text{se } a \leq b \text{ e } P[^b/_x] \\ (\Sigma x : x \in [a,b) \wedge P . E) & \text{se } a \leq b \text{ e } \neg P[^b/_x] \end{cases} \qquad \text{(interv-}\Sigma\text{)}$$

$$\#\{x : x \in [a,b] \mid P\} = \begin{cases} \#\{x : x \in [a,b) \mid P\} + 1 & \text{se } a \leq b \text{ e } P[^b/_x] \\ \#\{x : x \in [a,b) \mid P\} & \text{se } a \leq b \text{ e } \neg P[^b/_x] \end{cases} \qquad \text{(interv-\#)}$$

$$(\mathbf{m}\, x : x \in [a,b] \wedge P . E) = \begin{cases} (\mathbf{m}\, x : x \in [a,b) \wedge P . E)\ m\ E[^b/_x] & \text{se } a \leq b \text{ e } P[^b/_x] \\ (\mathbf{m}\, x : x \in [a,b) \wedge P . E) & \text{se } a \leq b \text{ e } \neg P[^b/_x] \end{cases} \qquad \mathbf{m} \in \{\mathbf{max}, \mathbf{min}\} \ \text{(interv-}\mathbf{m}\text{)}$$

**Regole di inferenza per Calcolo Proposizionale e Logica dei Predicati**

$$\frac{P \equiv Q}{R \equiv R[^{Q}/_{P}]}$$ (Principio di sostituzione per $\equiv$)

$$\frac{P \Rightarrow Q \quad P \text{ occorre positivamente in } R}{R \Rightarrow R[^{Q}/_{P}]}$$ (Principio di sostituzione per $\Rightarrow$ (1))

$$\frac{P \Rightarrow Q \quad P \text{ occorre negativamente in } R}{R \Leftarrow R[^{Q}/_{P}]}$$ (Principio di sostituzione per $\Rightarrow$ (2))

$$\frac{\Gamma \vdash P[^{d}/_{x}], \text{ con } d \text{ nuova costante}}{\Gamma \vdash (\forall x.P)}$$ (Generalizzazione)

$$\frac{(\exists x.P) \in \Gamma \quad \Gamma, P[^{d}/_{x}] \vdash Q, \text{ con } d \text{ nuova costante}}{\Gamma \vdash Q}$$ (Skolemizzazione)

$$\frac{R \Rightarrow P[^{d}/_{x}], \text{ con } d \text{ nuova costante}}{R \Rightarrow (\forall x.P)}$$ (Generalizzazione-$\Rightarrow$)

$$\frac{(\exists x.P) \wedge P[^{d}/_{x}] \wedge R \Rightarrow Q, \text{ con } d \text{ nuova costante}}{(\exists x.P) \wedge R \Rightarrow Q}$$ (Skolemizzazione-$\Rightarrow$)

**Triple di Hoare: Assiomi**

$\{R\}$ **skip** $\{R\}$ (SKIP) $\{def(E) \wedge P[^{E}/_{x}]\}\ x := E\ \{P\}$ (ASS)

$\{def(E_1) \wedge \ldots \wedge def(E_k) \wedge P[^{E_1,\ldots,E_k}/_{x_1,\ldots,x_k}]\}\ x_1,\ldots,x_k := E_1,\ldots,E_k\ \{P\}$ (ASS-MULT)

$\{def(E) \wedge def(E') \wedge E \in dom(\mathbf{a}) \wedge P[^{\mathbf{b}}/_{\mathbf{a}}]\}\ \mathbf{a}[E] := E'\ \{P\}$, dove $\mathbf{b} = \mathbf{a}[^{E'}/_{E}]$ (AGG-SEL)

**Triple di Hoare: Regole di Inferenza**

$$\frac{P \Rightarrow P' \quad \{P'\}\ C\ \{R'\} \quad R' \Rightarrow R}{\{P\}\ C\ \{R\}}$$ (PRE-POST)

$$\frac{P \Rightarrow R}{\{P\}\ \mathbf{skip}\ \{R\}}$$ (SKIP)

$$\frac{P \Rightarrow def(E) \wedge R[^{E}/_{x}]}{\{P\}\ x := E\ \{R\}}$$ (ASS)

$$\frac{\{P\}\ C_1\ \{R\} \quad \{R\}\ C_2\ \{Q\}}{\{P\}\ C_1 ; C_2\ \{Q\}}$$ (SEQ)

$$\frac{P \Rightarrow def(E) \quad \{P \wedge E\}\ C_1\ \{R\} \quad \{P \wedge \neg E\}\ C_2\ \{R\}}{\{P\}\ \mathbf{if}\ E\ \mathbf{then}\ C_1\ \mathbf{else}\ C_2\ \mathbf{fi}\ \{R\}}$$ (COND)

$$\frac{\begin{array}{c} P \Rightarrow Inv \wedge def(E) \quad Inv \wedge \neg E \Rightarrow R \quad Inv \Rightarrow t \geq 0 \\ \{Inv \wedge E\}\ C\ \{Inv \wedge def(E)\} \quad \{Inv \wedge E \wedge t = V\}\ C\ \{t < V\} \end{array}}{\{P\}\ \mathbf{while}\ E\ \mathbf{do}\ C\ \mathbf{endw}\ \{R\}}$$ (WHILE)