LOGICA PER LA PROGRAMMAZIONE – a.a. 2014/15
Esercizi sulla Logica del Primo Ordine

1. Dimostrare che le seguenti formule *non* sono *valide* (fornendo un controesempio ovvero una interpretazione che le rende false)

   (a) $(\forall x.(\exists y.P(x,y))) \equiv (\exists y.(\forall x.P(x,y)))$,

   (b) $(\exists x.P(x)) \wedge (\exists x.Q(x)) \Rightarrow (\exists x.P(x) \wedge Q(x))$,

   (c) $(\forall x.P(x) \vee Q(x)) \Rightarrow (\forall x.P(x)) \vee (\forall x.Q(x))$.

2. Dimostrare che le seguenti formule sono *valide*

   (a) $(\exists x.(P(x) \vee Q(x)) \wedge R(x)) \equiv (\exists x.P(x) \wedge R(x)) \vee (\exists x.Q(x) \wedge R(x))$.

   (b) $\big(\forall x\,.\,\neg(B(x) \Rightarrow \neg A(x))\big) \vee \neg\big(\exists x\,.\,A(x) \vee (\neg B(x) \Rightarrow A(x))\big) \;\Rightarrow\; (\forall x\,.\,A(x) \Rightarrow B(x))$

   (c) $(\forall x\,.\,\neg Q(x)) \wedge (\exists x\,.\,\neg Q(x) \wedge \neg P(x) \Rightarrow R(x)) \;\Rightarrow\; (\exists x\,.\,P(x) \vee R(x))$

   (d) $\neg\big((\exists x\,.\,P(x) \vee Q(x)) \wedge (\exists x\,.\,R(x))\big) \wedge (\forall x\,.\,P(x)) \;\Rightarrow\; (\forall x\,.\,R(x) \Rightarrow Q(x))$

   (e) $(\exists x.R(x) \wedge Q(x)) \wedge ((\forall x.P(x)) \vee (\forall x.\neg R(x))) \Rightarrow (\exists x.P(x)) \wedge (\exists x.R(x))$,

   (f) $(\exists x.\ P(x)) \vee (\exists x.P(x) \wedge Q(x)) \Rightarrow (\exists x.P(x) \Rightarrow Q(x))$,

   (g) $(\forall x.\neg P(x) \Rightarrow R(x)) \wedge \neg(P(a) \vee Q(a)) \Rightarrow \neg(\forall x.\neg R(x) \vee Q(x))$.