Principles of Programming Languages

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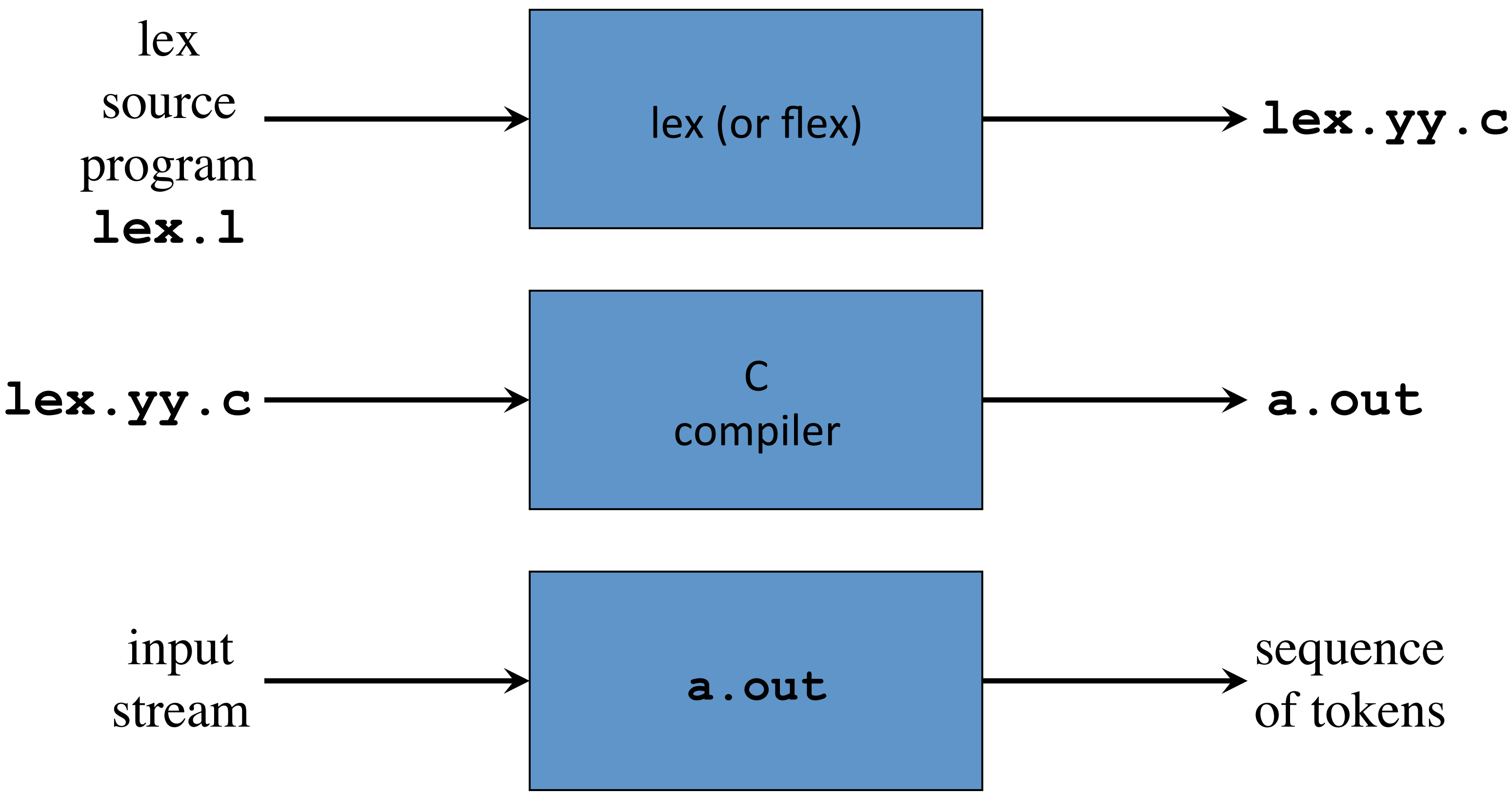
Lesson 5

- Generation of Lexical Analyzers

Creating a Lexical Analyzer with Lex and Flex



Lex Specification

- A *lex specification* consists of three parts:

  regular definitions, C declarations in `%{ %}`

  `%%`

  translation rules

  `%%`

  user-defined auxiliary procedures

- The *translation rules* are of the form:

  p_1 { $action_1$ }

  p_2 { $action_2$ }

  …

  p_n { $action_n$ }

Regular Expressions in Lex

`x` match the character **`x`**
`\.` match the character **`.`**
"*string*" match contents of string of characters
`.` match any character except newline
`^` match beginning of a line
`$` match the end of a line
`[xyz]` match one character **`x`**, **`y`**, or **`z`** (use **`\`** to escape **`-`**)
`[^xyz]` match any character except **`x`**, **`y`**, and **`z`**
`[a-z]` match one of **`a`** to **`z`**
r`*` closure (match zero or more occurrences)
r`+` positive closure (match one or more occurrences)
r`?` optional (match zero or one occurrence)
r_1r_2 match r_1 then r_2 (concatenation)
r_1`|`r_2 match r_1 or r_2 (union)
`(` *r* `)` grouping
r_1`\`r_2 match r_1 when followed by r_2
`{`*d*`}` match the regular expression defined by *d*

Example Lex Specification 1

Translation rules

Contains the matching lexeme

```
%{
#include <stdio.h>
%}
%%
[0-9]+  { printf("%s\n", yytext); }
.|\n    { }
%%
main()
{ yylex();
}
```

Invokes the lexical analyzer

```
lex spec.l
gcc lex.yy.c -ll
./a.out < spec.l
```

Example Lex Specification 2

Translation rules

Regular definition

```
%{
#include <stdio.h>
int ch = 0, wd = 0, nl = 0;
%}
delim     [ \t]+
%%
\n          { ch++; wd++; nl++; }
^{delim}    { ch+=yyleng; }
{delim}     { ch+=yyleng; wd++; }
.           { ch++; }
%%
main()
{ yylex();
  printf("%8d%8d%8d\n", nl, wd, ch);
}
```

Example Lex Specification 3

Regular definitions

Translation rules

```
%{
#include <stdio.h>
%}
digit      [0-9]
letter     [A-Za-z]
id         {letter}({letter}|{digit})*
%%
{digit}+   { printf(“number: %s\n”, yytext); }
{id}       { printf(“ident: %s\n”, yytext); }
.          { printf(“other: %s\n”, yytext); }
%%
main()
{ yylex();
}
```

Example Lex Specification 4

```
%{ /* definitions of manifest constants */
#define LT (256)
…
%}
delim       [ \t\n]
ws          {delim}+
letter      [A-Za-z]
digit       [0-9]
id          {letter}({letter}|{digit})*
number      {digit}+(\.{digit}+)?(E[+\-]?{digit}+)?
%%
{ws}        { }
if          {return IF;}
then        {return THEN;}
else        {return ELSE;}
{id}        {yylval = install_id(); return ID;}
{number}    {yylval = install_num(); return NUMBER;}
“<“         {yylval = LT; return RELOP;}
“<=“        {yylval = LE; return RELOP;}
“=“         {yylval = EQ; return RELOP;}
“<>“        {yylval = NE; return RELOP;}
“>“         {yylval = GT; return RELOP;}
“>=“        {yylval = GE; return RELOP;}
%%
int install_id()
…
```

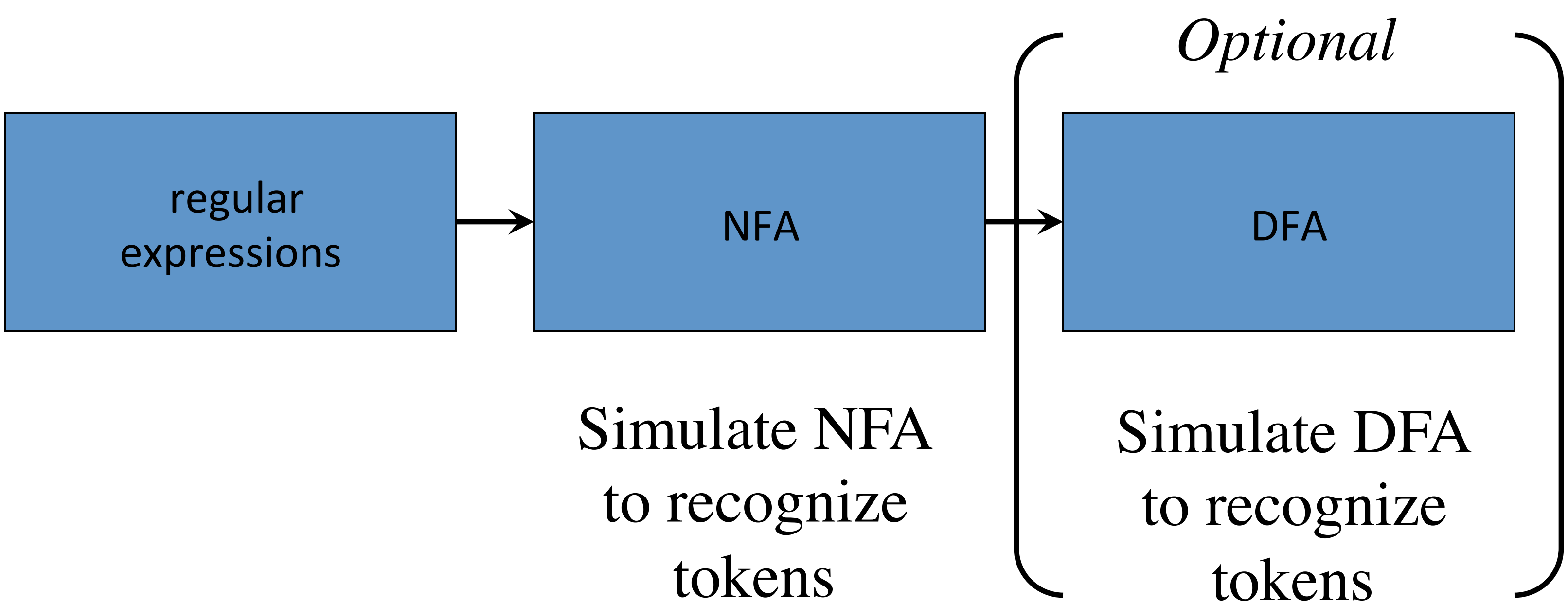
Return token to parser

Token attribute

Install **`yytext`** as identifier in symbol table

Design of a Lexical Analyzer Generator

- Translate regular expressions to NFA
- Translate NFA to an efficient DFA



Nondeterministic Finite Automata

- An NFA is a 5-tuple $(S, \Sigma, \delta, s_0, F)$ where

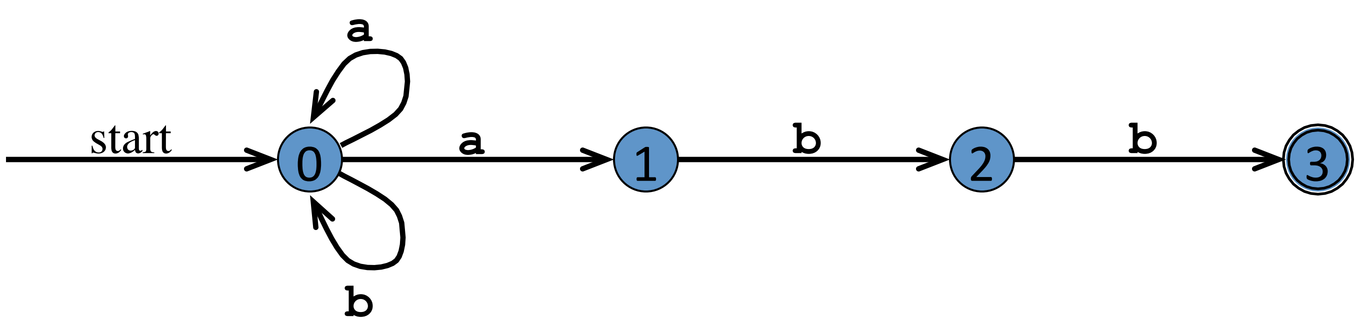
  S is a finite set of *states*
  Σ is a finite set of symbols, the *alphabet*
  δ is a *mapping* from $S \times \Sigma$ to a set of states

  $$\delta : S \times \Sigma \rightarrow P(S)$$

  $s_0 \in S$ is the *start state*
  $F \subseteq S$ is the set of *accepting* (or *final) states*

Transition Graph

- An NFA can be diagrammatically represented by a labeled directed graph called a *transition graph*



$S = \{0,1,2,3\}$
$\Sigma = \{\mathbf{a},\mathbf{b}\}$
$s_0 = 0$
$F = \{3\}$

Transition Table

- The mapping δ of an NFA can be represented in a *transition table*

$\delta(0,\mathbf{a}) = \{0,1\}$
$\delta(0,\mathbf{b}) = \{0\}$
$\delta(1,\mathbf{b}) = \{2\}$
$\delta(2,\mathbf{b}) = \{3\}$

⟶

State	*Input* **a**	*Input* **b**
0	{0, 1}	{0}
1		{2}
2		{3}

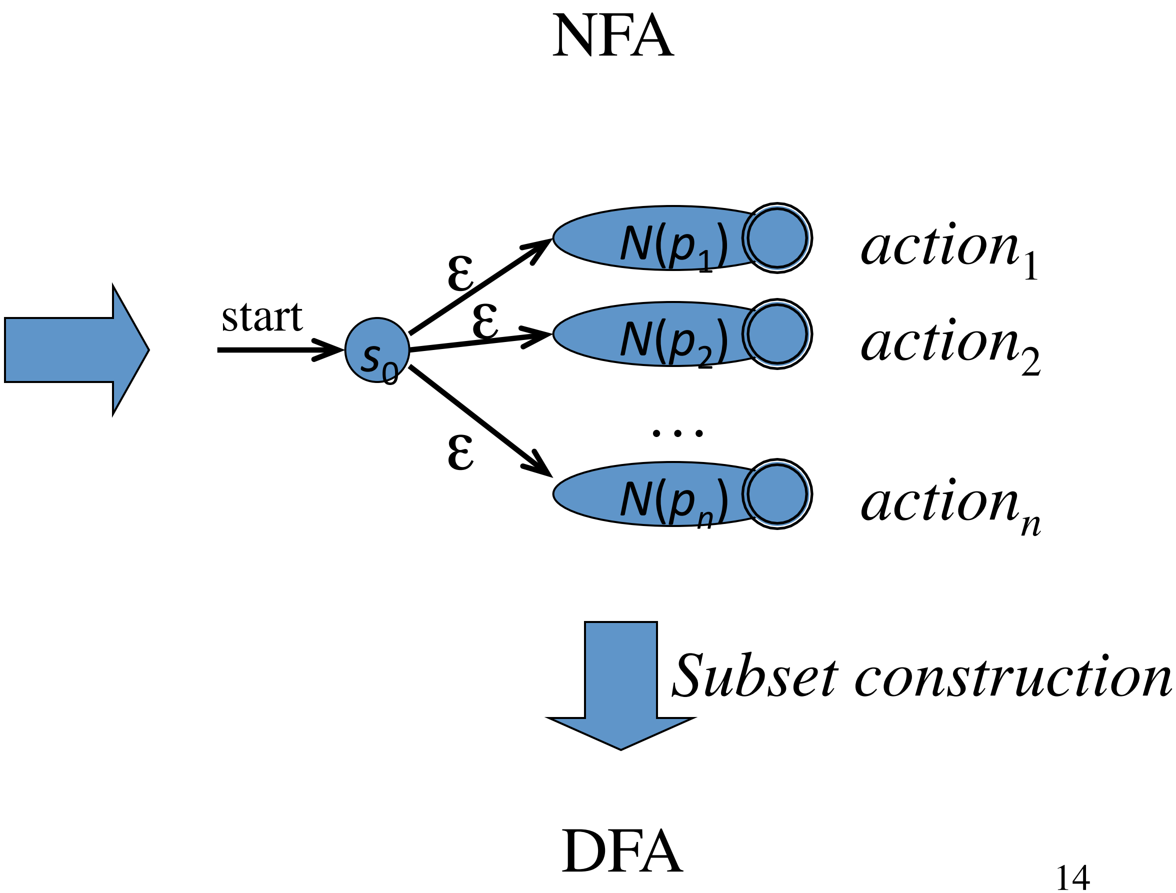
The Language Defined by an NFA

- An NFA *accepts* an input string *x* (over *Σ*) if and only if there is some path with edges labeled with symbols from *x* in sequence from the start state to some accepting state in the transition graph
- A state transition from one state to another on the path is called a *move*
- The *language defined by* an NFA is the set of input strings it accepts
- What is the language accepted by the example NFA?
  - (**a**|**b**)***abb**

Design of a Lexical Analyzer Generator: RE to NFA to DFA

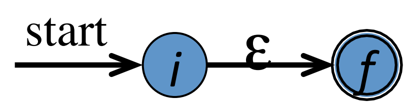
Lex specification with regular expressions

p_1 { $action_1$ }
p_2 { $action_2$ }
…
p_n { $action_n$ }

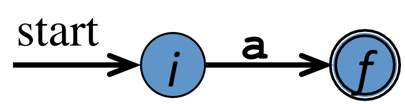


From Regular Expression to NFA (Thompson's Construction)

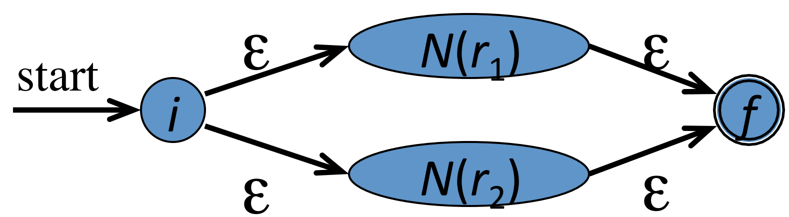
ε



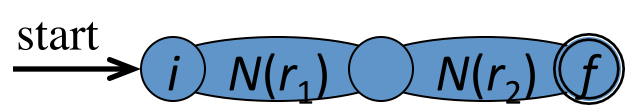
a



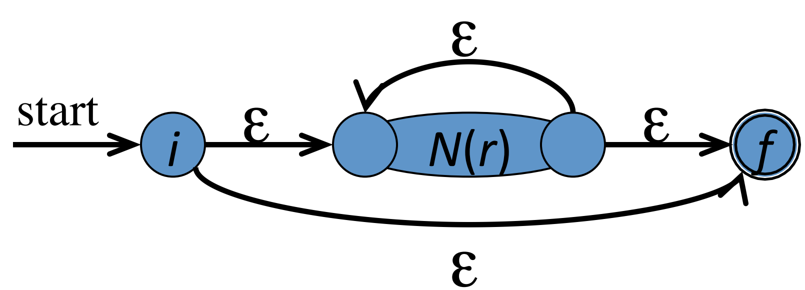
$r_1 \mid r_2$



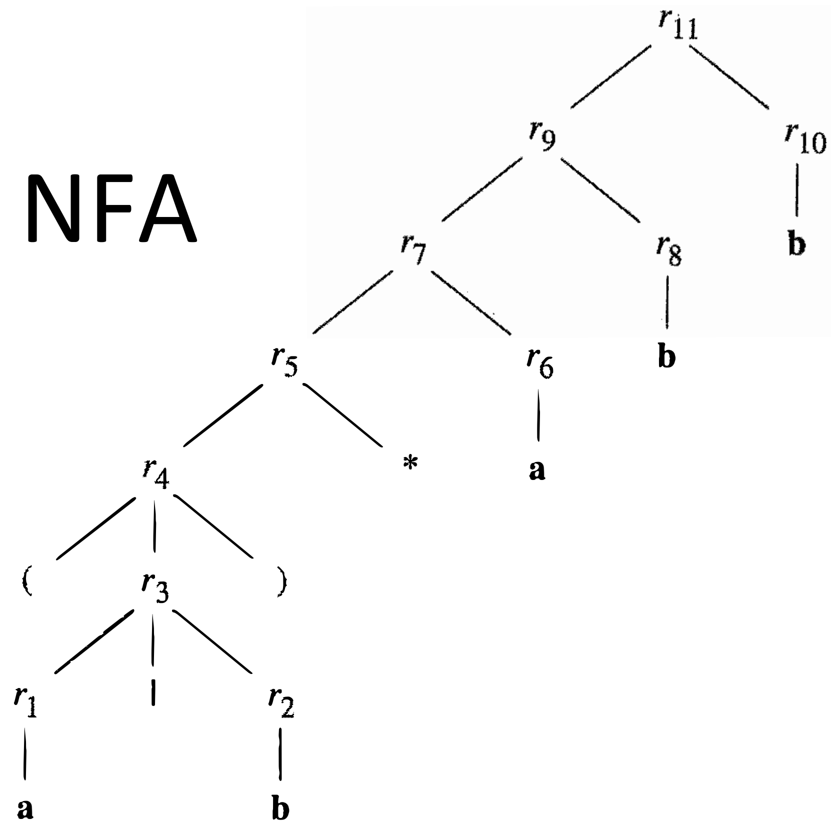
$r_1 r_2$



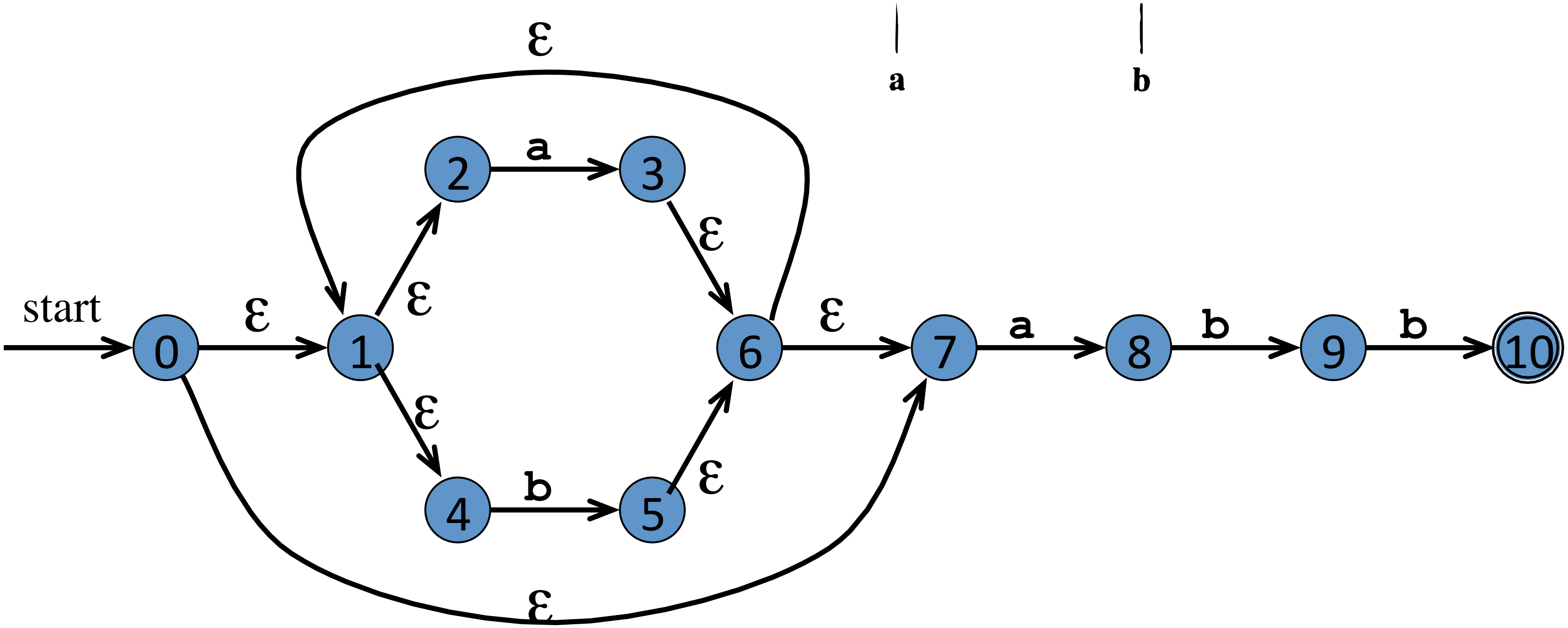
r^*



An example:
RE -> Parse Tree -> NFA

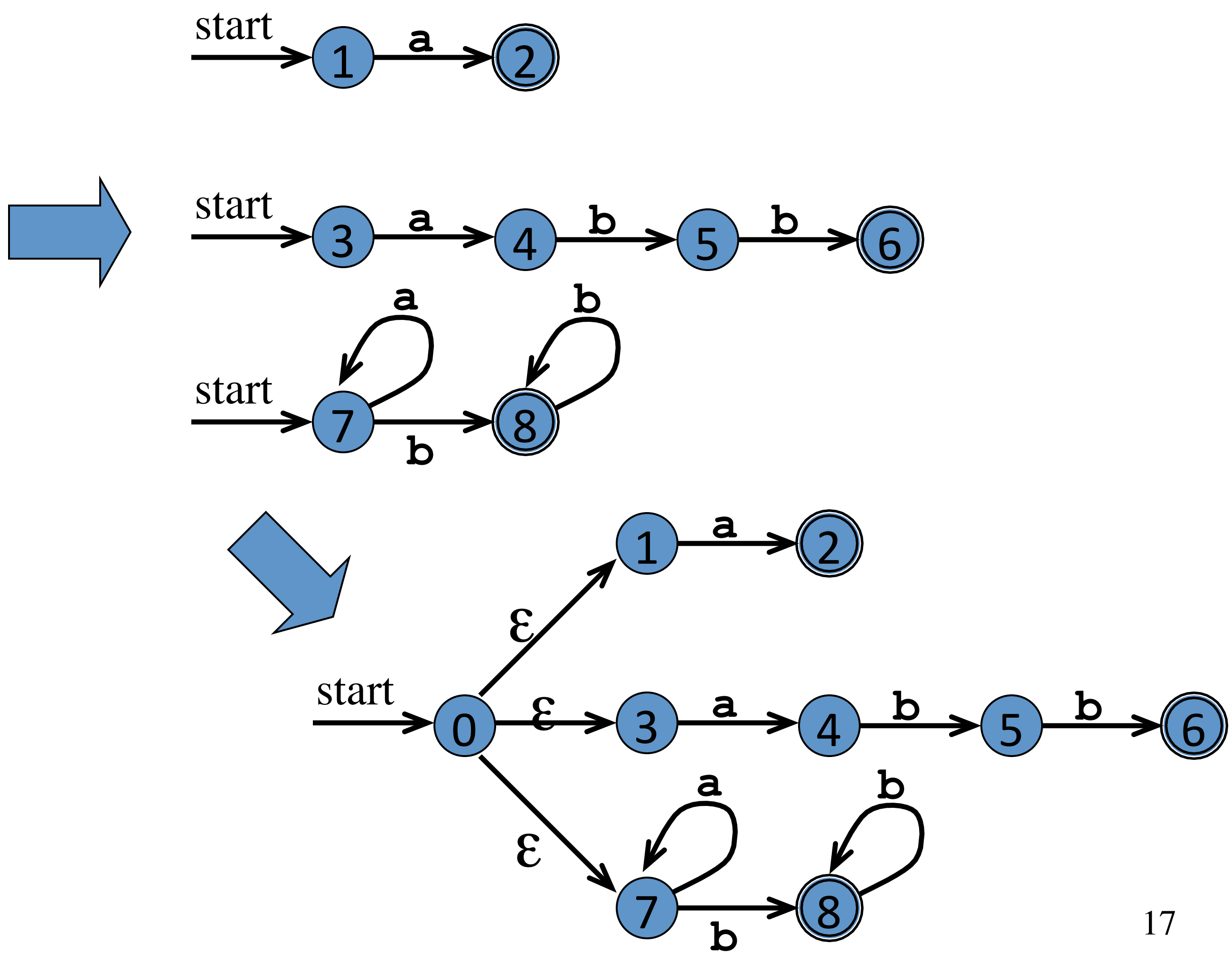


(a|b)*abb

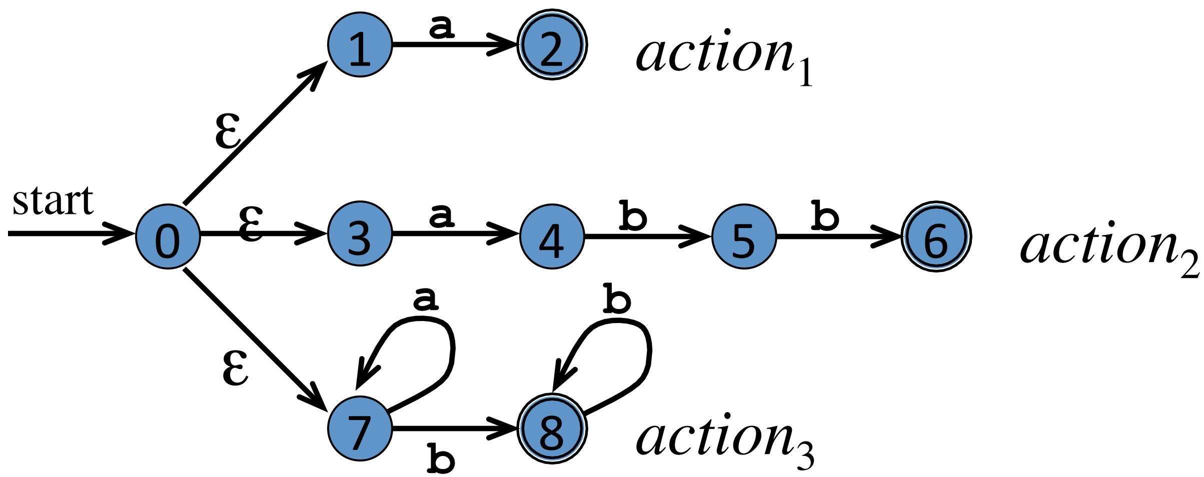


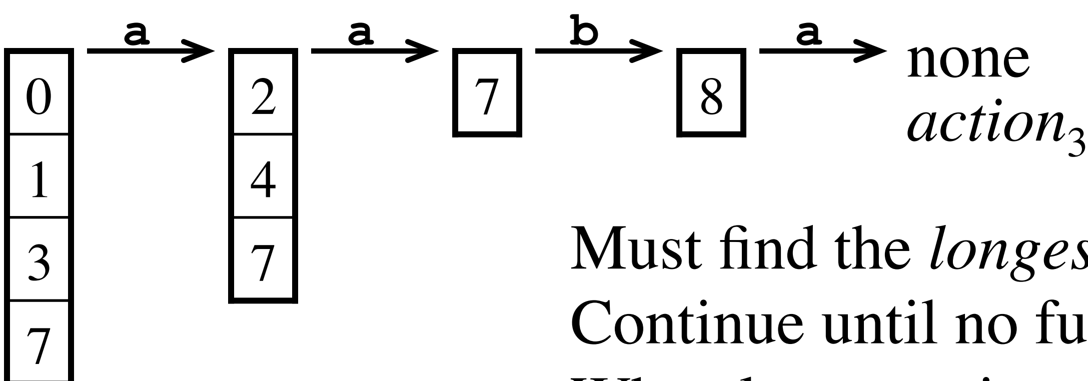
Combining the NFAs of a Set of Regular Expressions

a { *action*$_1$ }
abb { *action*$_2$ }
a*b+ { *action*$_3$ }



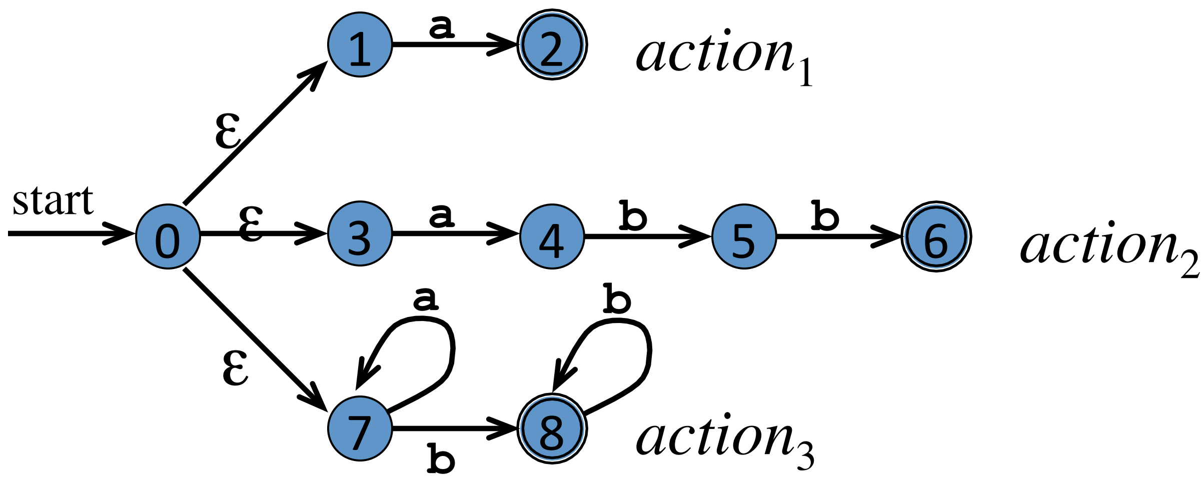
Simulating the Combined NFA
Example 1

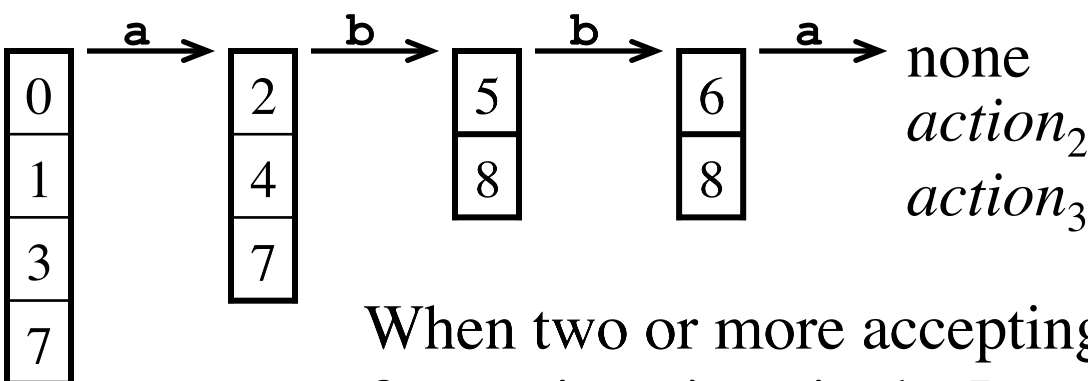




Must find the *longest match*:
Continue until no further moves are possible
When last state is accepting: execute action

Simulating the Combined NFA Example 2





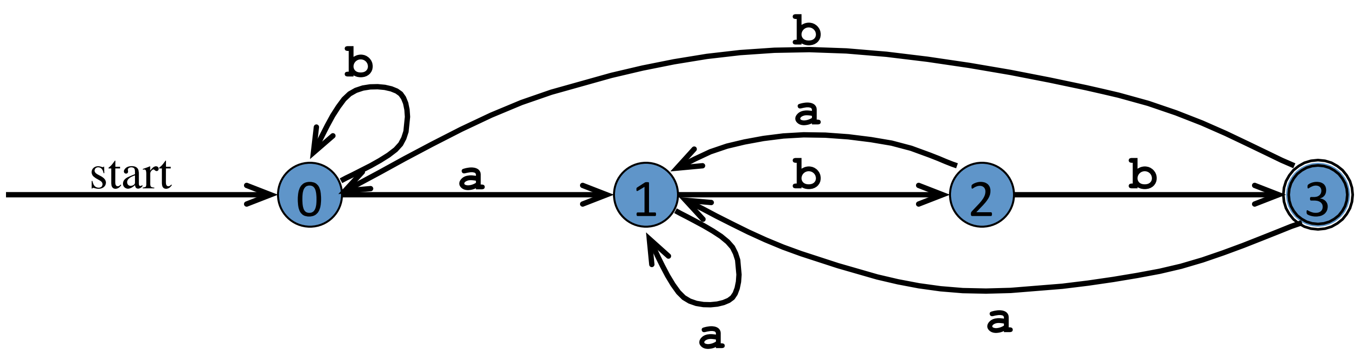
When two or more accepting states are reached, the first action given in the Lex specification is executed

Deterministic Finite Automata

- A *deterministic finite automaton* is a special case of an NFA
  - No state has an ε-transition
  - For each state *s* and input symbol *a* there is at most one edge labeled *a* leaving *s*
- Each entry in the transition table is a single state
  - At most one path exists to accept a string
  - Simulation algorithm is simple

Example DFA

A DFA that accepts (**a** | **b**)***abb**



Conversion of an NFA into a DFA

- The *subset construction algorithm* converts an NFA into a DFA using:

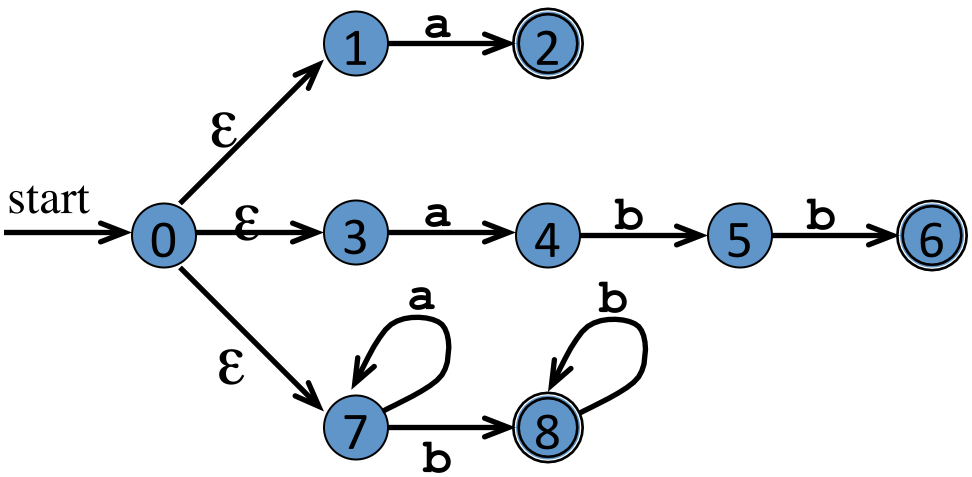
  $\varepsilon\text{-}closure(s) = \{s\} \cup \{t \mid s \rightarrow_\varepsilon \ldots \rightarrow_\varepsilon t\}$

  $\varepsilon\text{-}closure(T) = \cup_{s \in T}\, \varepsilon\text{-}closure(s)$

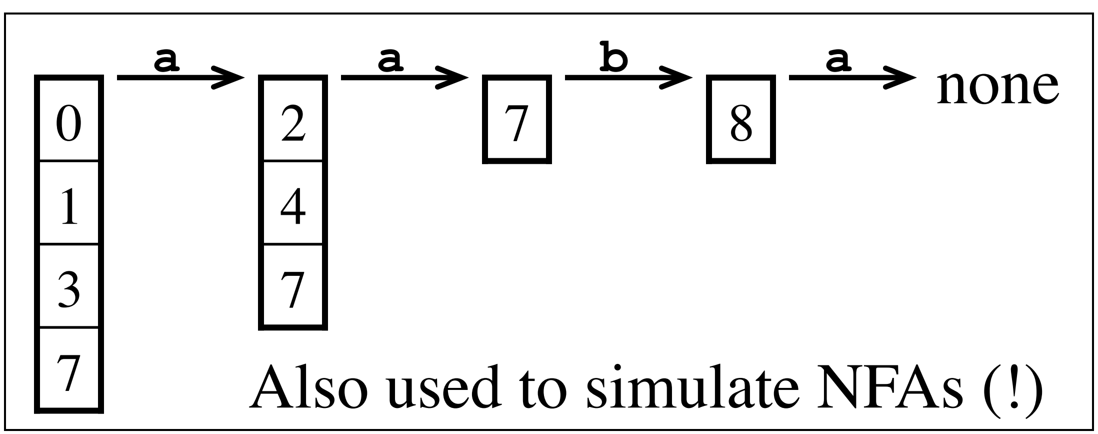
  $move(T, a) = \{t \mid s \rightarrow_a t \text{ and } s \in T\}$

- The algorithm produces:
  Dstates is the set of states of the new DFA consisting of sets of states of the NFA
  Dtran is the transition table of the new DFA

ε-*closure* and *move* Examples



ε-*closure*({0}) = {0,1,3,7}
move({0,1,3,7},**a**) = {2,4,7}
ε-*closure*({2,4,7}) = {2,4,7}
move({2,4,7},**a**) = {7}
ε-*closure*({7}) = {7}
move({7},**b**) = {8}
ε-*closure*({8}) = {8}
move({8},**a**) = $\emptyset$



Simulating an NFA using ε-*closure* and *move*

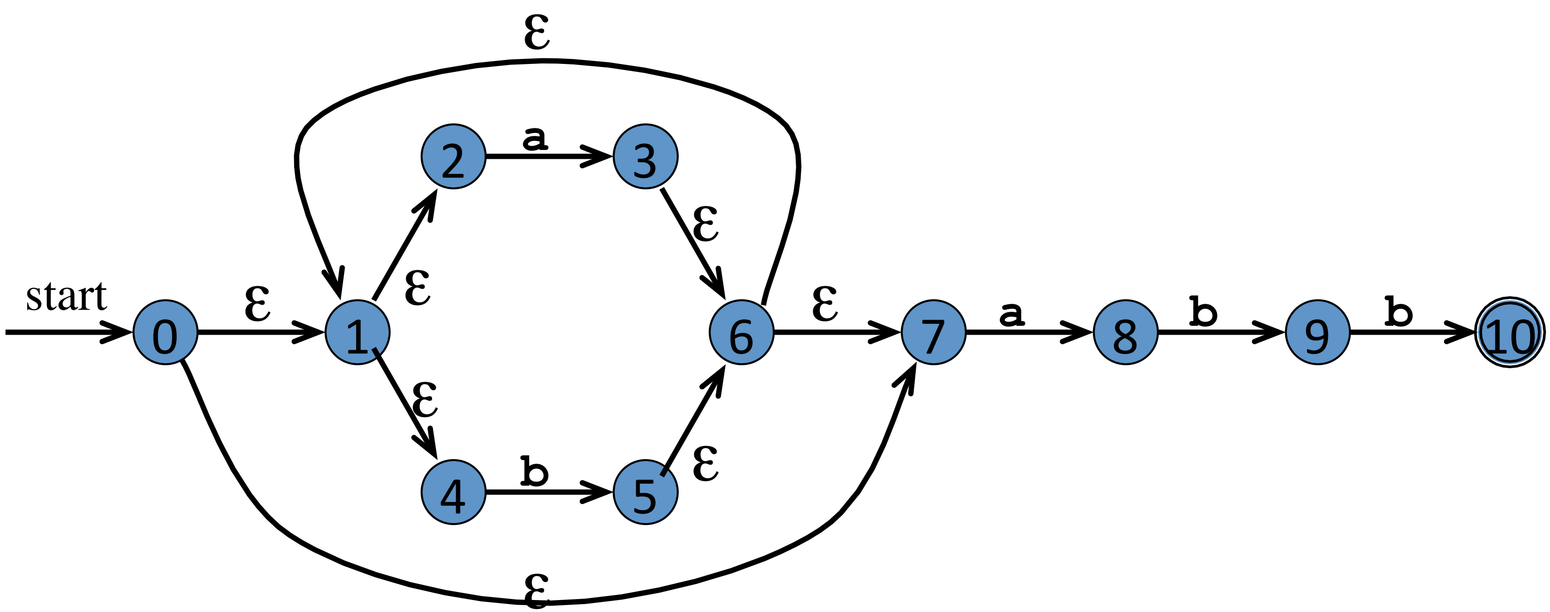
$S := \varepsilon\text{-}closure(\{s_0\})$
$S_{prev} := \emptyset$
$a := nextchar()$
while $S \neq \emptyset$ **do**
    $S_{prev} := S$
    $S := \varepsilon\text{-}closure(move(S,a))$
    $a := nextchar()$
end do
if $S_{prev} \cap F \neq \emptyset$ **then**
    execute *action in* S_{prev}
    return "yes"
else **return** "no"

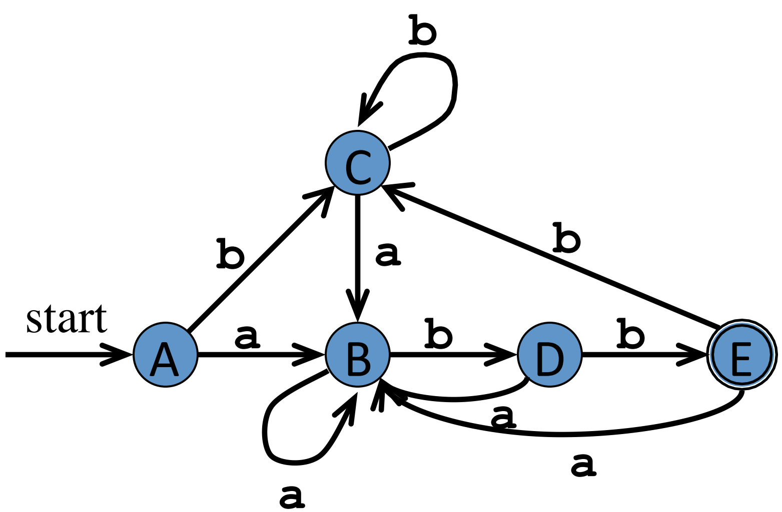
The Subset Construction Algorithm: from a NFA to an equivalent DFA

- Initially, ε-*closure*(s_0) is the only state in *Dstates* and it is unmarked

```
while there is an unmarked state T in Dstates do
    mark T
    for each input symbol a ∈ Σ do
        U := ε-closure(move(T,a))
        if U is not in Dstates then
            add U as an unmarked state to Dstates
        end if
        Dtran[T, a] := U
    end do
end do
```

Subset Construction Example 1





Dstates

A = {0,1,2,4,7}

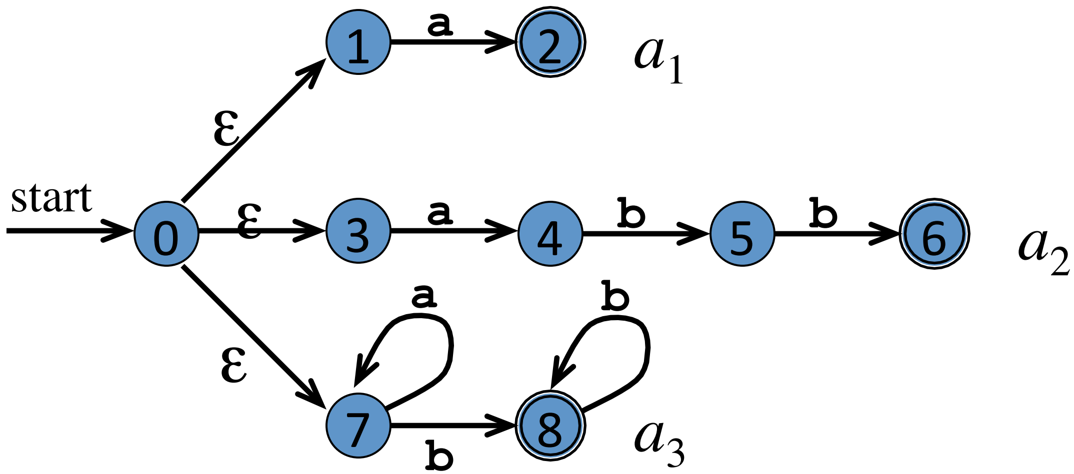
B = {1,2,3,4,6,7,8}

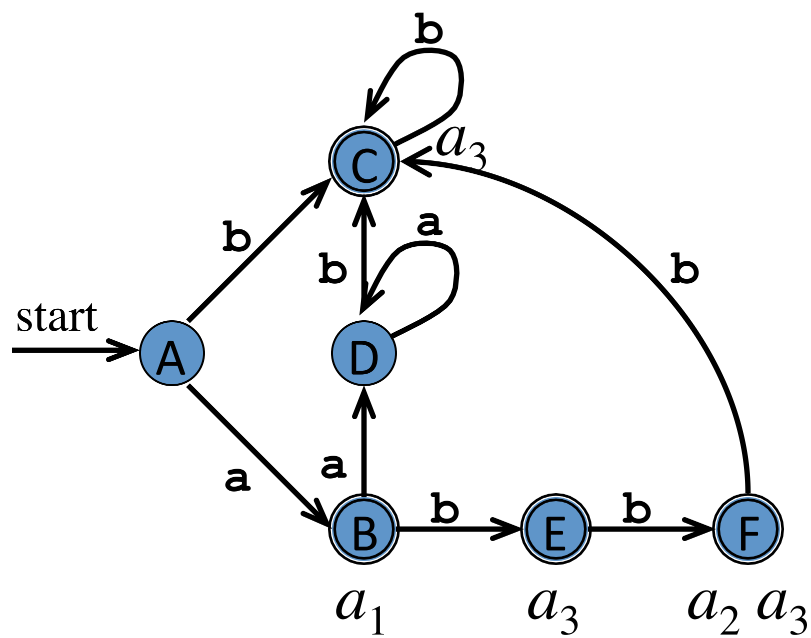
C = {1,2,4,5,6,7}

D = {1,2,4,5,6,7,9}

E = {1,2,4,5,6,7,10}

Subset Construction Example 2





Dstates

A = {0,1,3,7}

B = {2,4,7}

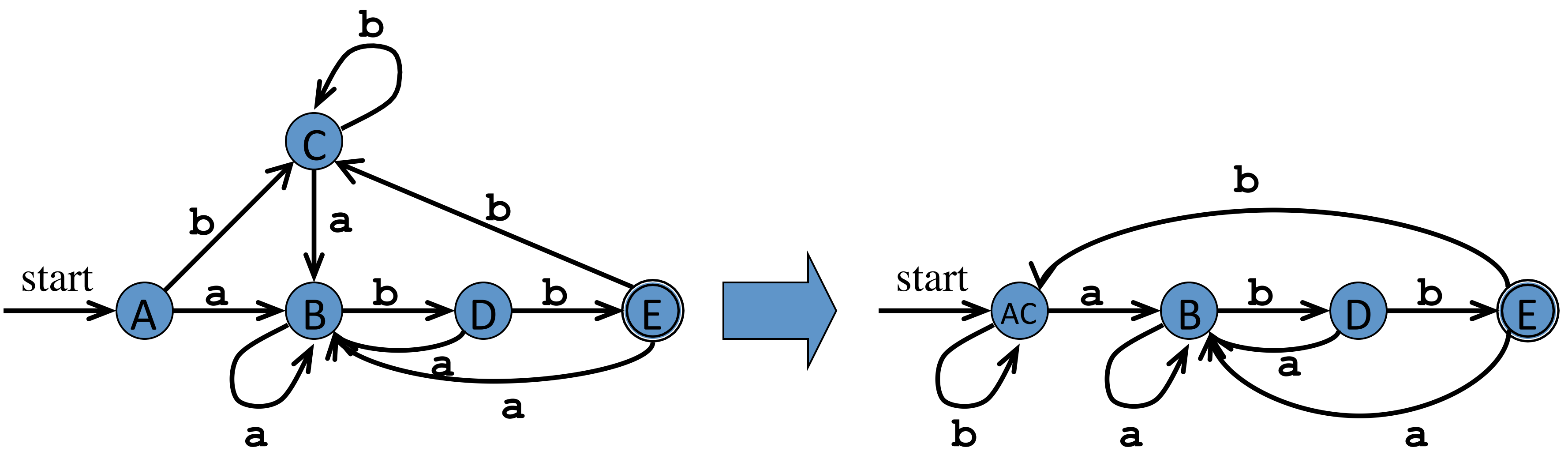
C = {8}

D = {7}

E = {5,8}

F = {6,8}

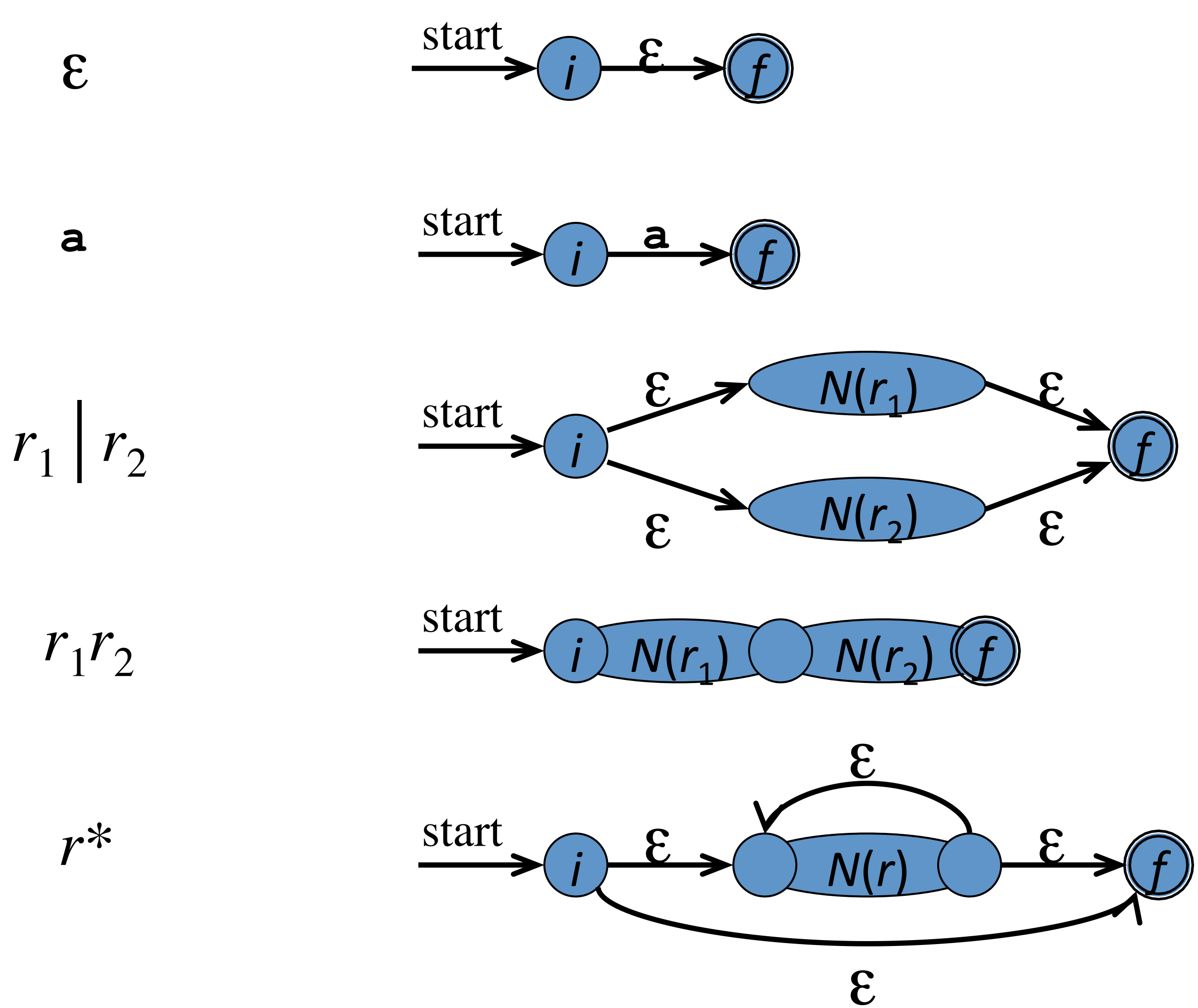
Minimizing the Number of States of a DFA



From Regular Expression to DFA Directly

- The "*important states*" of an NFA are those without an ε-transition, that is if $move(\{s\}, a) \neq \emptyset$ for some a then s is an important state
- The subset construction algorithm uses only the important states when it determines ε-*closure*(*move*(*T*, *a*))

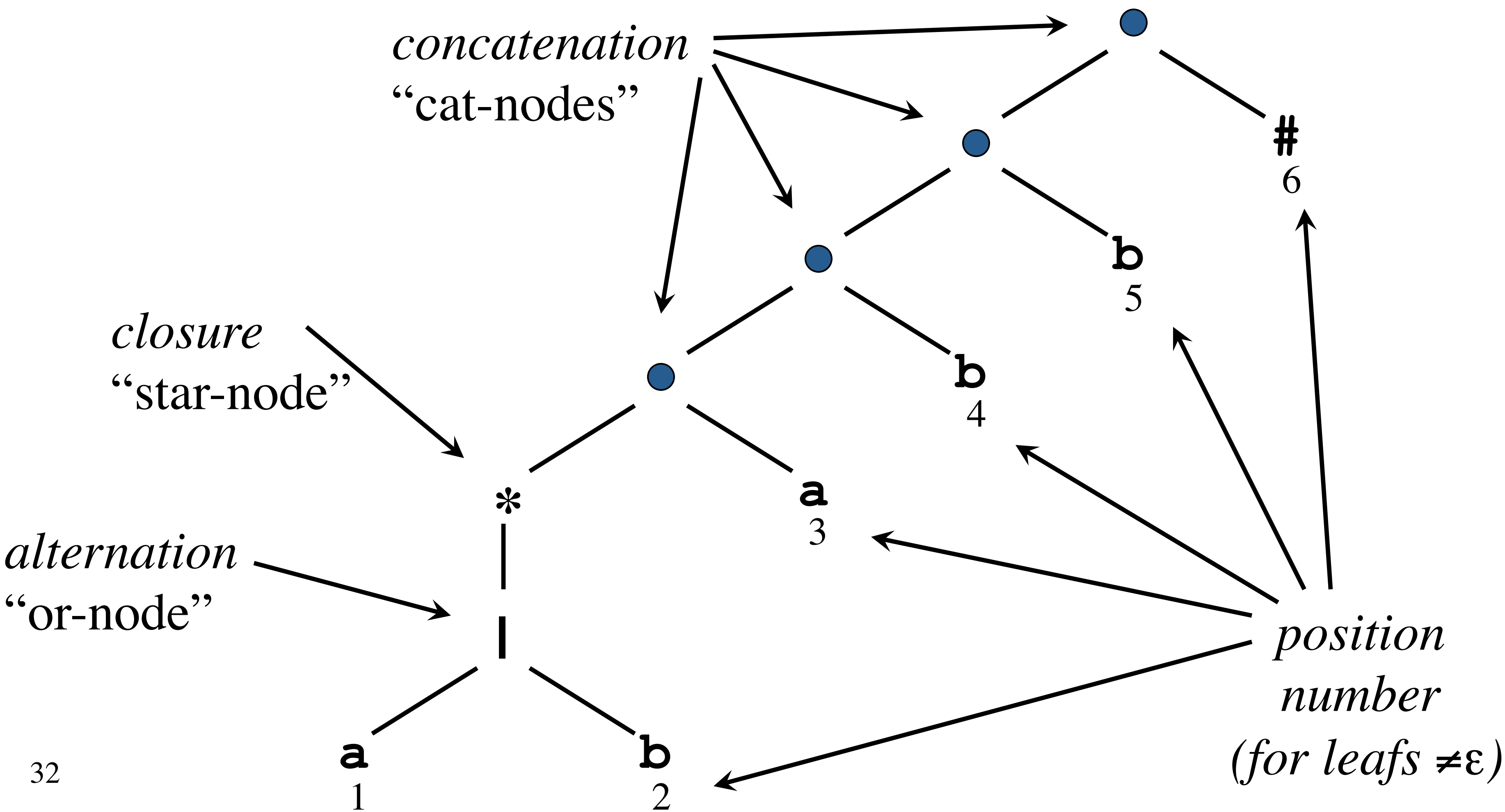
What are the “important states” in the NFA built from Regular Expression?



From Regular Expression to DFA Directly (Algorithm)

- The only accepting state (via the Thompson algorithm) is not important
- Augment the regular expression *r* with a special end symbol # to make accepting states important: the new expression is *r#*
- Construct a syntax tree for *r#*
- Attach a unique integer to each node not labeled by ε

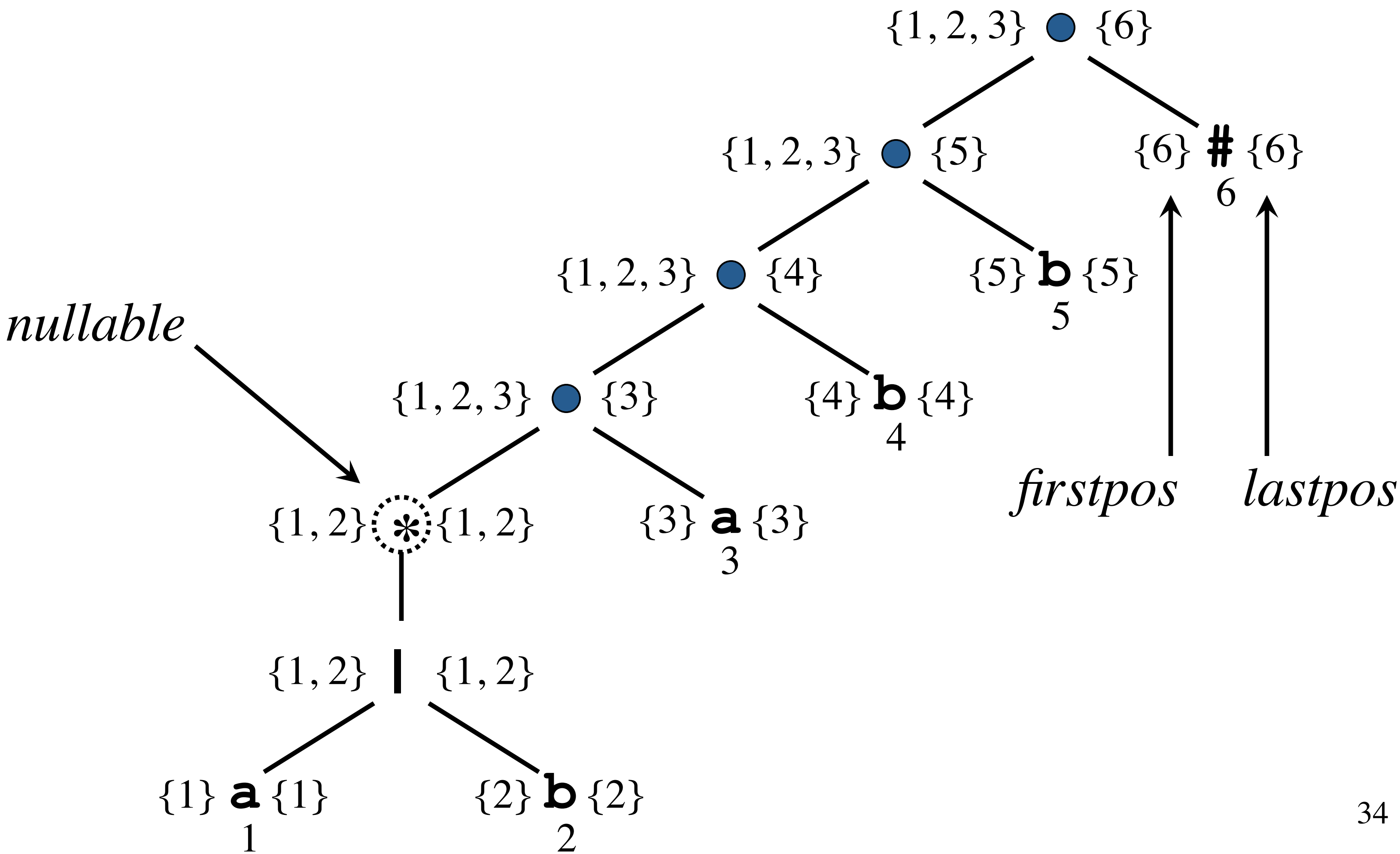
From Regular Expression to DFA Directly: Syntax Tree of **(a|b)*abb#**



From Regular Expression to DFA Directly: Annotating the Tree

- Traverse the tree to construct functions *nullable*, *firstpos*, *lastpos*, and *followpos*
- For a node *n*, let *L(n)* be the language generated by the subtree with root *n*
- *nullable*(*n*): *L(n)* contains the empty string ε
- *firstpos*(*n*): set of *positions* under *n* that can match the first symbol of a string in *L(n)*
- *lastpos*(*n*): the set of *positions* under *n* that can match the last symbol of a string in *L(n)*
- *followpos*(*i*): the set of positions that can follow position *i* in the tree

From Regular Expression to DFA Annotating the Syntax Tree of (a|b)*abb#



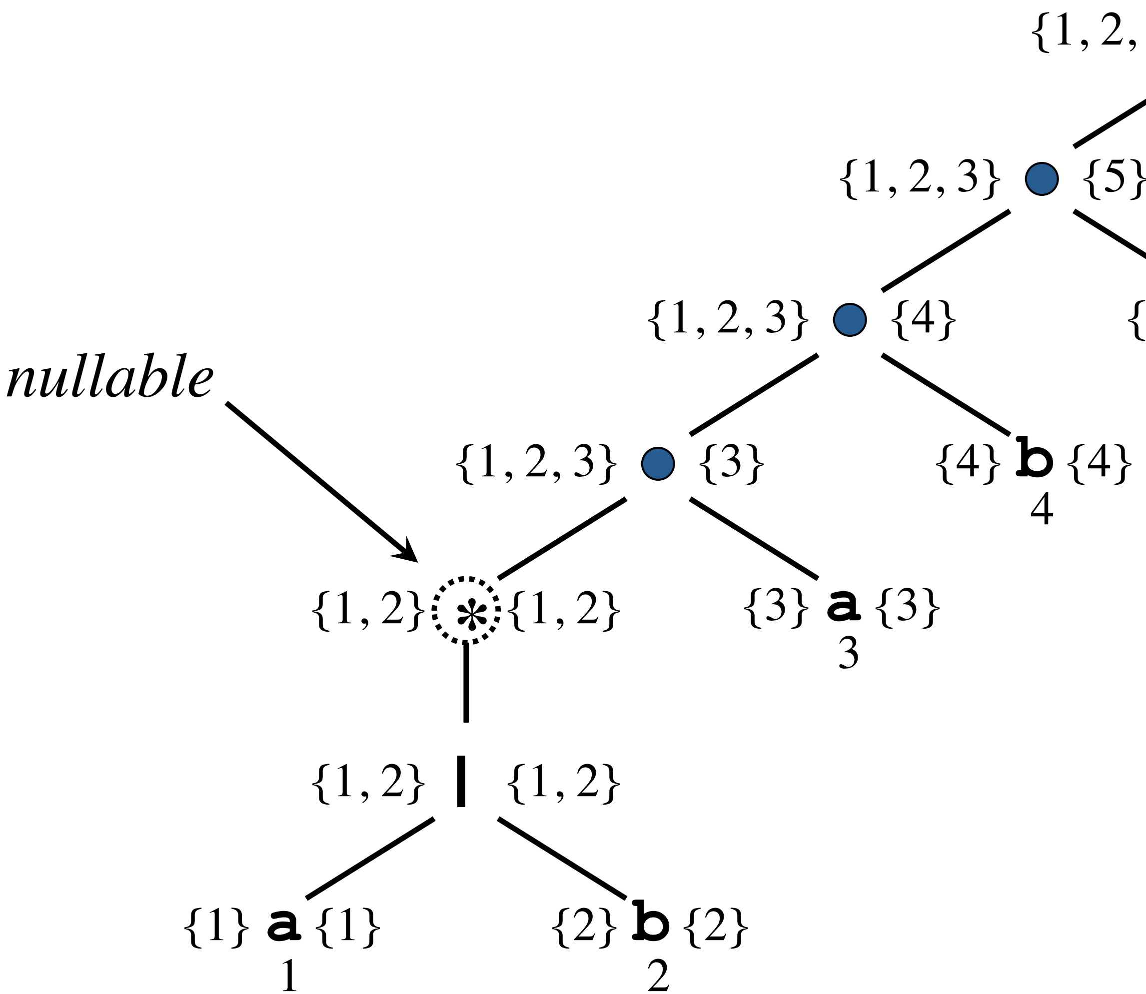
From Regular Expression to DFA Directly: Annotating the Tree

Node *n*	*nullable*(*n*)	*firstpos*(*n*)	*lastpos*(*n*)
Leaf ε	true	∅	∅
Leaf *i*	false	{*i*}	{*i*}
| / \\ c_1 c_2	*nullable*(c_1) or *nullable*(c_2)	*firstpos*(c_1) ∪ *firstpos*(c_2)	*lastpos*(c_1) ∪ *lastpos*(c_2)
• / \\ c_1 c_2	*nullable*(c_1) and *nullable*(c_2)	**if** *nullable*(c_1) **then** *firstpos*(c_1) ∪ *firstpos*(c_2) **else** *firstpos*(c_1)	**if** *nullable*(c_2) **then** *lastpos*(c_1) ∪ *lastpos*(c_2) **else** *lastpos*(c_2)
* | c_1	true	*firstpos*(c_1)	*lastpos*(c_1)

From Regular Expression to DFA Directly: *followpos*

```
for each node n in the tree do
    if n is a cat-node with left child c1 and right child c2 then
        for each i in lastpos(c1) do
            followpos(i) := followpos(i) ∪ firstpos(c2)
        end do
    else if n is a star-node
        for each i in lastpos(n) do
            followpos(i) := followpos(i) ∪ firstpos(n)
        end do
    end if
end do
```

From Regular Expression to DFA
followpos on the Syntax Tree of (**a|b**)***abb#**



NODE n	*followpos*(n)
1	$\{1, 2, 3\}$
2	$\{1, 2, 3\}$
3	$\{4\}$
4	$\{5\}$
5	$\{6\}$
6	$\emptyset$

From Regular Expression to DFA Directly: Algorithm

s_0 := *firstpos*(*root*) where *root* is the root of the syntax tree for ***(r)#***
Dstates := {s_0} and is unmarked
while there is an unmarked state *T* in *Dstates* **do**
    mark *T*
    for each input symbol $a \in \Sigma$ **do**
        let *U* be the union of *followpos*(*p*) for all positions *p* in *T*
            such that the symbol at position *p* is *a*
        if *U* is not empty and not in *Dstates* **then**
            add *U* as an unmarked state to *Dstates*
        end if
        Dtran[*T*, *a*] := *U*
    end do
end do

From Regular Expression to DFA Directly: Example

Node	*followpos*
1 a	{1, 2, 3}
2 b	{1, 2, 3}
3 a	{4}
4 b	{5}
5 b	{6}
6 #	-