Principles of Programming Languages

http://www.di.unipi.it/~andrea/Didattica/PLP-14/

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Lesson 21

- Type systems
- Type safety
- Type checking
  - Equivalence, compatibility and coercion
- Primitive and composite types
  - Discrete and scalar types
  - Tuples and records
  - Arrays

What is a Data Type?

- A (*data*) *type* is a **homogeneous collection of values**, effectively presented, equipped with a set of **operations** which manipulate these values
- Various perspectives:
  - collection of values from a "domain" (the denotational approach)
  - internal structure of a bunch of data, described down to the level of a small set of fundamental types (the structural approach)
  - collection of well-defined operations that can be applied to objects of that type (the abstraction approach)

Advantages of Types

- Program organization and documentation
  - Separate types for separate concepts
    - Represent concepts from problem domain
  - Document intended use of declared identifiers
    - Types can be checked, unlike program comments
- Identify and prevent errors
  - Compile-time or run-time checking can prevent meaningless computations such as 3 + true – "Bill"
- Support implementation and optimization
  - Example: short integers require fewer bits
  - Access components of structures by known offset

Type system

A **type system** consists of

1. The set of **predefined types** of the language.
2. The mechanisms which permit the **definition of new types**.
3. The mechanisms for the **control** (**checking**) **of types**, which include:
    1. **Equivalence rules** which specify when two formally different types correspond to the same type.
    2. **Compatibility rules** specifying when a value of a one type can be used in given context.
    3. Rules and techniques for **type inference** which specify how the language assigns a type to a complex expression based on information about its components (and sometimes on the context).
4. The specification as to whether (or which) constraints are **statically** or **dynamically checked**.

Type errors

- A **type error** occurs when a value is used in a way that is inconsistent with its definition
- Type errors are *type system* (thus *language*) *dependent*
- Implementations can react in various ways
  - Hardware interrupt, *e.g. apply fp addition to non-legal bit configuration*
  - OS exception, *e.g. segmentation fault when dereferencing* 0 in C
  - Continue execution possibly with wrong values
- Examples
  - Array out of bounds access
    - C/C++: runtime errors
    - Java: dynamic type error
  - Null pointer dereference
    - C/C++: run-time errors
    - Java: dynamic type error
    - Haskell/ML: pointers are hidden inside datatypes
      - Null pointer dereferences would be incorrect use of these datatypes, therefore static type errors

Type safety

- A language is **type safe** (**strongly typed**) when no program can violate the distinctions between types defined in its type system
- In other words, a type system is safe when no program, during its execution, can generate an unsignalled type error
- Also: if code accesses data, it is handled with the type associated with the creation and previous manipulation of that data

Safe and *not safe* languages

- Not safe: C and C++
  - Casts, pointer arithmetic
- Almost safe (aka "weakly typed"): Algol family, Pascal, Ada.
  - Dangling pointers.
    - Allocate a pointer p to an integer, deallocate the memory referenced by p, then later use the value pointed to by p.
    - No language with explicit deallocation of memory is fully type-safe.
- Safe (aka "strongly typed"): Lisp, Smalltalk, ML, Haskell, Java, JavaScript
  - Dynamically typed: Lisp, Smalltalk, JavaScript
  - Statically typed: ML, Haskell, Java

Type checking

- To prevent type errors, before any operation is performed, its operands must be **type-checked** to ensure that they comply with the compatibility rules of the type system
  - *mod* operation: check that both operands are integers
  - *and* operation: check that both operands are booleans
  - *indexing operation*: check that the left operand is an array, and that the right operand is a value of the array's index type.
- **Statically typed** languages: (most) type checking is done during compilation
- **Dynamically typed** languages: type checking is done at runtime

Static vs dynamic typing

- In a **statically typed** PL:
  - all variables and expressions have fixed types (either stated by the programmer or inferred by the compiler)
  - most operands are type-checked at *compile-time*.
- Most PLs are called "statically typed", including Ada, C, C++, Java, Haskell, … even if some type-checking is done at run-time (e.g. access to arrays)
- In a **dynamically typed** PL:
  - values have fixed types, but variables and expressions do not
  - operands must be type-checked when they are computed at *run-time*.
- Some PLs and many scripting languages are dynamically typed, including Smalltalk, Lisp, Prolog, Perl, Python.

Example: Ada static typing

- Ada function definition:

```
function is_even (n: Integer)
     return Boolean is
begin
   return (n mod 2 = 0);
end;
```

Knowing that n’s type is Integer, the compiler infers that the type of “n **mod** 2 = 0” will be Boolean.

- Call:

```
p: Integer;
…
if is_even(p+1) …
```

Knowing that p’s type is Integer, the compiler infers that the type of “p+1” will be Integer.

- Even without knowing the values of variables and parameters, the Ada compiler can guarantee that no type errors will happen at run-time.

Example: Python dynamic typing

- Python function definition:

```
def even (n):
   return (n % 2 == 0)
```

The type of n is unknown. So the "%" (*mod*) operation must be protected by a run-time type check.

- The types of variables and parameters are not declared, and cannot be inferred by the Python compiler. So run-time type checks are needed to detect type errors.

Static vs dynamic type checking

- Static typing is **more efficient**
  - No run-time checks
  - Values do not need to be tagged at run-time
- Static typing is often considered **more secure**
  - The compiler guarantees that the object program contains no type errors. With dynamic typing you rely on the implementation.
- Dynamic typing is **more flexible**
  - Needed by some applications where the types of the data are not known in advance.
    - JavaScript array: elements can have different types
    - Haskell list: all elements must have same type
- Note: **type safety** is independent of dynamic/static

Static typing is conservative

- In JavaScript, we can write a function like

```
function f(x) { return x < 10 ? x : x(); }
```

Some uses will produce type error, some will not.

- Static typing must be *conservative*

```
    if  (possibly-non-terminating-boolean-expression)
        then  f(5);
        else  f(15);
```

Cannot decide at compile time if run-time error will occur!

Type Checking: how does it work

- Checks that each operator is applied to arguments of the right type. It needs:
  - ***Type inference***, to infer the type of an expression given the types of the basic constituents
  - ***Type compatibility***, to check if a value of type A can be used in a context that expects type B
    - *Coercion rules*, to transform silently a type into a compatible one, if needed
  - ***Type equivalence**,* to know if two types are considered the same

Towards Type Equivalence: Type Expressions

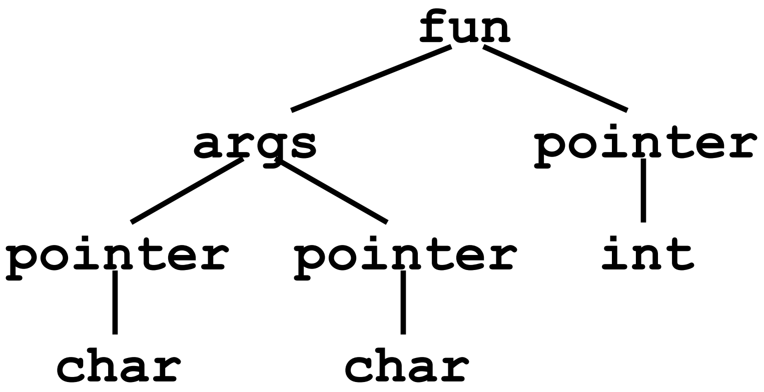
- **Type expressions** are used in declarations and type casts to define or refer to a type

Type ::= **int** | **bool** | … | X | Tname |pointer-to(Type) |
array(*num*, Type) | record(Fields) | class(…) |
Type → Type | Type x Type

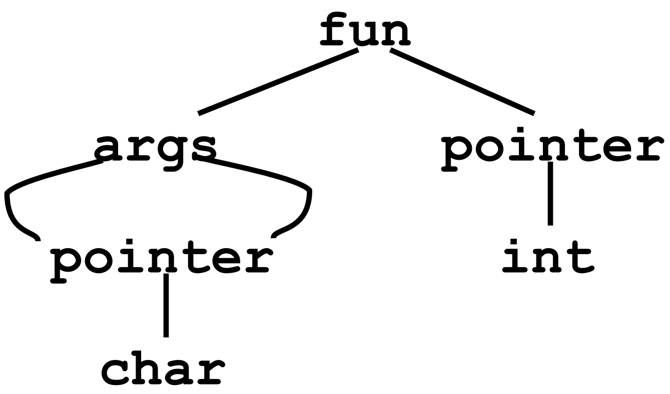
  - *Primitive types*, such as **int** and **bool**
  - *Type constructors*, such as pointer-to, array-of, records and classes, and functions
  - *Type names*, such as typedefs in C and named types in Pascal, refer to type expressions

Graph Representations for Type Expressions

- Internal compiler representation, built during parsing
- Example: `int *f(char*,char*)`



Tree forms

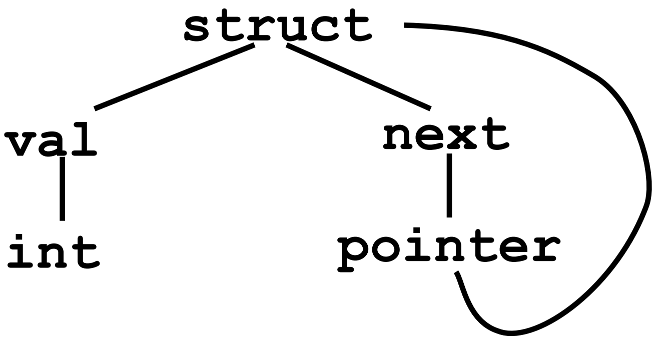


DAGs

Cyclic Graph Representations

Source program

```
struct Node
{ int val;
  struct Node *next;
};
```



struct

val

next

int

pointer

Internal compiler representation
of the **Node** type: cyclic graph

Equivalence of Type Expressions

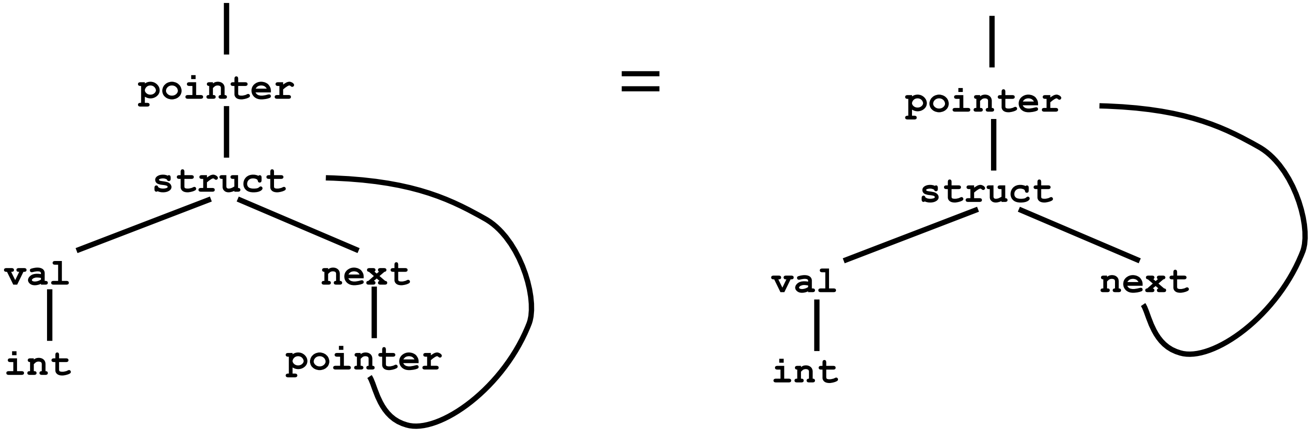
- Two different notions: **name equivalence** and **structural equivalence**
  - Two types are ***structurally equivalent*** if
    1. They are the same basic types, or
    2. They have the form **TC(T1,…, Tn)** and **TC(S1, …, Sn)**, where **TC** is a type constructor and **Ti** is structurally equivalent to **Si** for all 1 <= i <= n, or
    3. One is a type name that denotes the other.
  - Two types are ***name equivalent*** if they satisfy 1. and 2.

On Structural Equivalence

- **Structural equivalence**: unravel all type constructors obtaining type expressions containing only primitive types, then check if they are equivalent
- Used in C/C++, C#

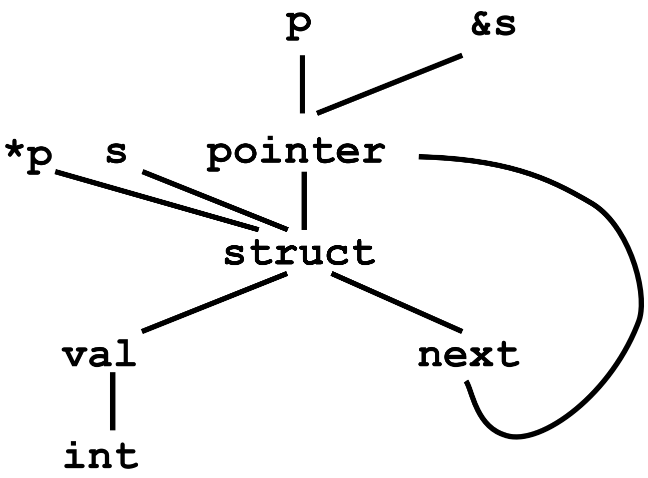
```
-- pseudo Pascal
type Student = record
     name, address : string
     age : integer

type School = record
     name, address : string
     age : integer

x : Student;
y : School;

x:= y;
--ok with structural equivalence
--error with name equivalence
```

Structural Equivalence of Recursive Type Expressions



- Two structurally equivalent type expressions have the same pointer address when constructing graphs by (maximally) sharing nodes

```
struct Node
{ int val;
  struct Node *next;
};
struct Node s, *p;
p = &s; // OK
*p = s; // OK
p = s;  // ERROR
```



On Name Equivalence

- Each ***type name*** is a distinct type, even when the type expressions that the names refer to are the same
- Types are identical only if names match
- Used for Abstract Data Types and by OO languages
- Used by Pascal (inconsistently)

```
type link = ^node;
var next : link;
    last : link;
       p : ^node;
    q, r : ^node;
```

With name equivalence in Pascal:

```
p := next         FAIL
last := p         FAIL
q := r            OK
next := last      OK
p := q            FAIL !!!
```

On Name Equivalence

- **Name equivalence**: sometimes “aliases” needed

```
TYPE stack_element = INTEGER;
MODULE stack;
IMPORT stack_element;
EXPORT push, pop;
(* alias *)
       ...
   PROCEDURE push(elem : stack_element);
       ...
   PROCEDURE pop() : stack_element;
...

var st:stack;
st.push(42);     // this should be OK
```

Type compatibility and Coercion

- **Type compatibility** rules vary a lot
  - Integers as reals OK
  - Subtypes as supertypes OK
  - Reals as integers ???
  - Doubles as floats ???
- When an expression of type **A** is used in a context where a compatible type **B** is expected, an automatic implicit conversion is performed, called **coercion**

Type checking with attributed grammars

A simple language example

P → *D* **;** *S*
D → *D* **;** *D*
| **id :** *T*
T → **boolean**
| **char**
| **integer**
| **array [num] of** *T*
| **^** *T*
S → **id :=** *E*
| **if** *E* **then** *S*
| **while** *E* **do** *S*
| *S* **;** *S*

Pointer to *T*

E → **true**
| **false**
| **literal**
| **num**
| **id**
| *E* **and** *E*
| *E* **+** *E*
| *E* **[** *E* **]**
| *E* **^**

Pascal-like pointer dereference operator

Synthesized attributes
T.type : type expression
E.type : type of expression or *type_error*
S.type : *void* if statement is well-typed, *type_error* otherwise

Declarations

$D \rightarrow$ **id** : T	{ *addtype*(**id**.entry, T.type) }
$T \rightarrow$ **boolean**	{ T.type := *boolean* }
$T \rightarrow$ **char**	{ T.type := *char* }
$T \rightarrow$ **integer**	{ T.type := *integer* }
$T \rightarrow$ **array [num] of** T_1	{ T.type := *array*(1..**num**.val, T_1.type) }
$T \rightarrow$ **^** T_1	{ T.type := *pointer*(T_1) }

Parametric types:
type constructor

Checking Statements

S → **id :=** *E* { *S*.type := (**if id**.type = *E*.type **then** *void* **else** *type_error*) }

- Note: the type of **id** is determined by scope's environment:
  id.type = *lookup*(**id**.entry)

S → **if** *E* **then** S_1 { *S*.type := (**if** *E*.type = *boolean* **then** S_1.type
else *type_error*) }

S → **while** *E* **do** S_1 { *S*.type := (**if** *E*.type = *boolean* **then** S_1.type
else *type_error*) }

S → S_1 **;** S_2 { *S*.type := (**if** S_1.type = *void* **and** S_2.type = *void*
then *void* **else** *type_error*) }

Checking Expressions

$E \rightarrow$ **true** { E.type = *boolean* }

$E \rightarrow$ **false** { E.type = *boolean* }

$E \rightarrow$ **literal** { E.type = *char* }

$E \rightarrow$ **num** { E.type = *integer* }

$E \rightarrow$ **id** { E.type = *lookup*(**id**.entry) }

$E \rightarrow E_1$ **+** E_2 { E.type := (**if** E_1.type = *integer* **and** E_2.type = *integer*
then *integer* **else** *type_error*) }

$E \rightarrow E_1$ **and** E_2 { E.type := (**if** E_1.type = *boolean* **and** E_2.type = *boolean*
then *boolean* **else** *type_error*) }

$E \rightarrow E_1$ **[** E_2 **]** { E.type := (**if** E_1.type = *array*(*s*, ***t***) **and** E_2.type = *integer*
then ***t*** **else** *type_error*) }

- Parameter ***t*** is set with the unification of E_1.type = *array*(*s*, ***t***)

$E \rightarrow E_1$ **^** { E.type := (**if** E_1.type = *pointer*(***t***) **then** ***t***
else *type_error*) }

- Parameter ***t*** is set with the unification of E_1.type = *pointer*(***t***)

Type Conversion and Coercion

- **Type conversion** is explicit, for example using type casts
- **Type coercion** is implicitly performed by the compiler to generate code that converts types of values at runtime (typically to *narrow* or *widen* a type)
- Both require a *type system* to check and infer types from (sub)expressions

On Coercion

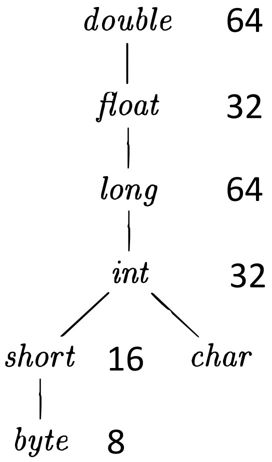
- Coercion may change the representation of the value or not
  - Integer → Real *binary representation is changed*

    `{int x = 5; double y = x; …}`
  - A → B subclasses *binary representation not changed*

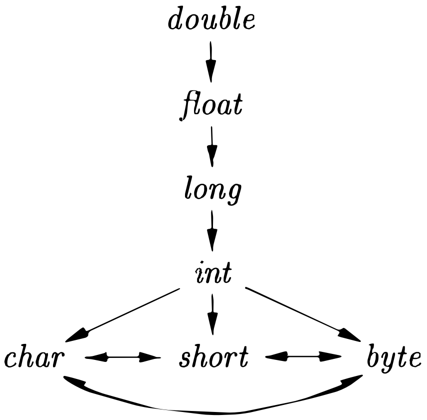
    ```
    class A extends B{ … }
    {B myBobject = new A(…); … }
    ```
- Coercion may cause loss of information, in general
  - Not in Java, with the exception of **long** as **float**
- In statically typed languages coercion instructions are inserted during semantic analysis (type checking)
- Popular in Fortran/C/C++, tends to be replaced by overloading and polymorphism
- Popular again in modern scripting languages

Example: Type Coercion and Cast in Java among numerical types

- Coercion (implicit, widening)
  - No loss of information (almost…)
- Cast (explicit, narrowing)
  - Some information can be lost
- Explicit cast is always allowed when coercion is



(a) Widening conversions



(b) Narrowing conversions

Handling coercion during translation

Translation of sum without type coercion:

$E \rightarrow E_1$ **+** E_2 { *E*.place := *newtemp*();
gen(*E*.place ':=' E_1.place '+' E_2.place) }

With type coercion:

$E \rightarrow E_1$ **+** E_2 { *E*. type = **max**(E_1.type,E_2.type);
a_1 = **widen**(E_1.addr, E_1.type, E.type) ;
a_2 = **widen**(E_2.addr, E_2.type, E.type);
E.addr = new Temp();
gen(E.addr '=' a_1 '+' a_2); }

where:

- **max(T_1,T_2)** returns the least upper bound of T_1 and T_2 in the widening hierarchy
- **widen(addr, T_1, T_2)** generate the statement that copies the value of type T_1 in addr to a new temporary, casting it to T_2

Pseudocode for widen

```
Addr widen(Addr a, Type t, Type w){
    temp = new Temp();
    if(t = w) return a;  //no coercion needed
    elseif(t = integer and w = float){
        gen(temp '=' '(float)' a);
    elseif(t = integer and w = double){
        gen(temp '=' '(double)' a);
    elseif ...
    else error;
    return temp; }
}
```

Built-in primitive types

- Typical built-in primitive types:

Boolean = {*false*, *true*}

Character = {…, ‘A’ , …, ‘Z’ ,
…, ‘0’ , …, ‘9’ ,
…}

PL- or implementation-defined set of characters (ASCII, ISO-Latin, or Unicode)

Integer = {…, −2, −1,
0, +1, +2, …}

PL- or implementation-defined set of whole numbers

Float = {…, −1.0, …,
0.0, +1.0, …}

PL- or implementation-defined set of real numbers

- *Note:* In some PLs (such as C), booleans and characters are just small integers.
- Names of types vary from one PL to another: not significant.

Terminology

- Discrete types – countable
  - integer, boolean, char
  - enumeration
  - subrange

```
type Color is (red, green, blue);
```

```
type Population is range 0 .. 1e10;
```

- Scalar types - one-dimensional
  - discrete
  - real

Composite types

- Types whose values are *composite*, that is composed of other values (simple or composite):
  - records (unions)
  - Arrays (Strings)
  - algebraic data types
  - sets
  - pointers
  - lists
- Most of them can be understood in terms of a few concepts:
  - Cartesian products (records)
  - mappings (arrays)
  - disjoint unions (algebraic data types, unions, objects)
  - recursive types (lists, trees, etc.)
- Different names in different languages.
- Defined applying *type constructors* to other types (eg *struct, array, record,…*)

An brief overview of composite types

- We review type constructors in Ada, Java and Haskell corresponding to the following mathematical concepts:
  - Cartesian products (records)
  - mappings (arrays)
  - disjoint unions (algebraic data types, unions)
  - recursive types (lists, trees, etc.)

Cartesian products (1)

- In a **Cartesian product**, values of several types are grouped into tuples.
- Let (x, y) be the **pair** whose first component is x and whose second component is y.
- $S \times T$ denotes the Cartesian product of S and T:

  $$S \times T = \{ (x, y) \mid x \in S; y \in T \}$$

- Cardinality:

  $$\#(S \times T) = \#S \times \#T$$

  hence the “×” notation

Cartesian products (2)

- We can generalise from pairs to **tuples**. Let $S_1 \times S_2 \times \ldots \times S_n$ stand for the set of all *n*-tuples such that the *i*th component is chosen from S_i:

  $$S_1 \times S_2 \times \ldots \times S_n = \{ (x_1, x_2, \ldots, x_n) \mid x_1 \in S_1; x_2 \in S_2; \ldots; x_n \in S_n \}$$
- Basic operations on tuples:
  - **construction** of a tuple from its component values
  - **selection** of an *explicitly-designated* component of a tuple
    - we can select the 1st or 2nd (but not the *i*th) component
- **Records** (Ada), **structures** (C), and **tuples** (Haskell) can all be understood in terms of Cartesian products.

Example: Ada records (1)

- Type declarations:

```
type Month is (jan, feb, mar, apr, may, jun,
    jul, aug, sep, oct, nov, dec);
type Day_Number is range 1 .. 31;
type Date is record
       m: Month;
       d: Day_Number;
    end record;
```

- Application code:

record construction

```
someday: Date := (jan, 1);
…
put(someday.m+1); put("/"); put(someday.d);
someday.d := 29; someday.m := feb;
```

component selection

Example: Haskell tuples

- Declarations:

```
data Month = Jan | Feb | Mar | Apr
           | May | Jun | Jul | Aug
           | Sep | Oct | Nov | Dec
type Date = (Month, Int)
```

- Set of values:

  Date = Month × Integer
  = {*Jan*, *Feb*, …, *Dec*} × {…, –1, 0, 1, 2, …}

- Application code:

```
someday = (jan, 1)      // tuple construction
m, d = someday          // component selection
                        // (by pattern matching)
anotherday = (m + 1, d)
```

Mappings

- We write $m : S \rightarrow T$ to state that m is a **mapping** from set S to set T. In other words, m maps every value in S to some value in T.
- If m maps value x to value y, we write $y = m(x)$. The value y is called the **image** of x under m.
- Some of the mappings in $\{u, v\} \rightarrow \{a, b, c\}$:

$$m_1 = \{u \rightarrow a, v \rightarrow c\}$$

$$m_2 = \{u \rightarrow c, v \rightarrow c\}$$

$$m_3 = \{u \rightarrow c, v \rightarrow b\}$$

image of u is c,
image of v is b

Arrays (1)

- **Arrays** (found in all imperative and OO PLs) can be understood as mappings.
- If the array's elements are of type *T* (*base type*) and its index values are of type *S*, the array's type is $S \rightarrow T$.
- An array's **length** is the number of components, #*S*.
- Basic operations on arrays:
  - **construction** of an array from its components
  - **indexing** – using a *computed* index value to select a component
    - we *can* select the *i*th component

Arrays (2)

- An array of type $S \to T$ is a *finite* mapping.
- Here S is nearly always a finite range of consecutive values $\{l, l+1, \ldots, u\}$. This is called the array's **index range**.

lower bound

upper bound

- In C and Java, the index range must be $\{0, 1, \ldots, n-1\}$.
In Pascal and Ada, the index range may be any scalar (sub)type other than real/float.

- We can generalise to n-dimensional arrays. If an array has index ranges of types $S_1, \ldots, S_n$, the array's type is $S_1 \times \ldots \times S_n \to T$.

When is the index range known?

- A **static array** is an array variable whose index range is fixed by the program code.
- A **dynamic array** is an array variable whose index range is fixed at the time when the array variable is created.
  - In Ada, the definition of an array type must fix the index *type*, but need not fix the index *range*. Only when an array variable is created must its index range be fixed.
  - Arrays as formal parameters of subroutines are often dynamic (eg. *conformant arrays* in Pascal)
- A **flexible** (or **fully dynamic**) **array** is an array variable whose index range is not fixed at all, but may change whenever a new array value is assigned.

Example: C static arrays

- Array variable declarations:

```
float v1[] = {2.0, 3.0, 5.0, 7.0};
float v2[10];
```

index range is {0, …, 3}

index range is {0, …, 9}

- Function:

```
void print_vector (float v[], int n) {
// Print the array v[0], …, v[n-1] in the form “[…  …]”.
   int i;
   printf("[%f", v[0]);
   for (i = 1; i < n; i++)
      printf(" %f", v[i]);
   printf("]");
}
…
print_vector(v1, 4);  print_vector(v2, 10);
```

A C array doesn’t know its own length!

Example: Ada dynamic arrays

- Array type and variable declarations:

```
type Vector is
     array (Integer range <>) of Float;
v1: Vector(1 .. 4) := (1.0, 0.5, 5.0, 3.5);
v2: Vector(0 .. m) := (0 .. m => 0.0);
```

- Procedure:

```
procedure print_vector (v: in Vector) is
-- Print the array v in the form “[… … …]” .
begin
   put('[');  put(v(v'first));
   for i in v'first + 1 .. v'last loop
      put(' ');  put(v(i));
   end loop;
   put(']');
end;
…
print_vector(v1);  print_vector(v2);
```

Example: Java flexible arrays

- Array variable declarations:

```
float[] v1 = {1.0, 0.5, 5.0, 3.5};
float[] v2 = {0.0, 0.0, 0.0};
…
v1 = v2;
```

index range is {0, …, 3}

index range is {0, …, 2}

v1’s index range is now {0, …, 2}

- Method:

```
static void printVector (float[] v) {
// Print the array v in the form “[… … …]”.
     System.out.print("[" + v[0]);
     for (int i = 1; i < v.length; i++)
        System.out.print(" " + v[i]);
     System.out.print("]");
}
…
printVector(v1);  printVector(v2);
```

Enhanced for:

```
for  (float f : v)
System.out.print(" " + f)
```