Esercizi 1.6 - ex6 /1

Esercizio

The first Pascal environments included:

- A Pascal compiler, written in Pascal, which produced P-code (code for the intermediate machine);
- The same compiler, translated into P-code;
- An interpreter for P-code written in Pascal.

To implement the Pascal language in an interpretative way on a new host machine means (manually) translating the P-code interpreter into the language on the host machine. Given such an interpretative implementation, how can one obtain a compiled implementation for the same host machine, minimising the effort required? (Hint: think about a modification to the compiler for Pascal also written in Pascal.)

Soluzione

- $\mathcal{C}^{\texttt{Pascal}}_{\texttt{Pascal},\texttt{Pcode}}$ il compilatore in (a).
- $\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pascal},\texttt{Pcode}}$ il compilatore in (b).
- $\mathcal{I}^{\texttt{Pascal}}_{\texttt{Pcode}}$ l'interprete in (c).
- $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}$ l'interprete suggerito per la soluzione interpretativa, permette di eseguire i programmi Pascal in due passi: $\forall \texttt{p} \in \mathcal{P}^{\texttt{Pascal}}$, $\forall \texttt{x}$ a cui vogliamo applicare p,
  - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pascal},\texttt{Pcode}}, \texttt{p}) = \texttt{q} \in \mathcal{P}^{\texttt{Pcode}}$
  - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\texttt{q}, \texttt{x})$ fornisce il valore calcolato dall'applicazione (p,x) in Pascal

Questa soluzione è una soluzione mista di tipo interpretativo.

Esercizi 1.6 - ex6 /2

- Risposta. Indichiamo con
  - $\mathcal{C}^{\texttt{Pascal}}_{\texttt{Pascal},\texttt{Pcode}}$ il compilatore in (a).
  - $\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pascal},\texttt{Pcode}}$ il compilatore in (b).
  - $\mathcal{I}^{\texttt{Pascal}}_{\texttt{Pcode}}$ l'interprete in (c).
  - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}$ l'interprete suggerito per la soluzione interpretativa, permette di eseguire i programmi Pascal in due passi: $\forall \texttt{p} \in \mathcal{P}^{\texttt{Pascal}}$, $\forall \texttt{x}$ a cui vogliamo applicare $\texttt{p}$,
    - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pascal},\texttt{Pcode}}, \texttt{p}) = \texttt{q} \in \mathcal{P}^{\texttt{Pcode}}$
    - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\texttt{q}, \texttt{x})$ fornisce il valore calcolato dall'applicazione $\texttt{(p,x)}$ in Pascal.

  Questa soluzione è una soluzione mista di tipo interpretativo.

  Allora:
  - Modifichiamo $\mathcal{C}^{\texttt{Pascal}}_{\texttt{Pascal},\texttt{Pcode}}$ in un compilatore che traduce da $\texttt{Pcode}$ in $\mathcal{L}_0$, otteniamo: $\mathcal{C}^{\texttt{Pascal}}_{\texttt{Pcode},\mathcal{L}_0}$.
  - Eseguiamo una sola volta la seguente applicazione *Compiler-Compiler*:
  $$\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pascal},\texttt{Pcode}}, \mathcal{C}^{\texttt{Pascal}}_{\texttt{Pcode},\mathcal{L}_0}) = \mathcal{C}^{\texttt{Pcode}}_{\texttt{Pcode},\mathcal{L}_0} \in \mathcal{P}^{\texttt{Pcode}}$$
  - $\mathcal{C}_{\texttt{Pascal},\mathcal{L}_0}$ è ottenuto componendo i due passi seguenti: $\forall \texttt{p} \in \mathcal{P}^{\texttt{Pascal}}$
    - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pascal},\texttt{Pcode}}, \texttt{p}) = \texttt{q} \in \mathcal{P}^{\texttt{Pcode}}$
    - $\mathcal{I}^{\mathcal{L}_0}_{\texttt{Pcode}}(\mathcal{C}^{\texttt{Pcode}}_{\texttt{Pcode},\mathcal{L}_0}, \texttt{q}) = \texttt{w} \in \mathcal{P}^{\mathcal{L}_0}$ fornisce il programma oggetto da eseguire su $\mathcal{M}_{\mathcal{L}_0}$

Esercizi Vari

Esercizio

Si dia una grammatica non ambigua per il linguaggio:

$$\mathtt{L} = \{\mathtt{a}^{\mathtt{n}_1}\,\mathtt{b}^{\mathtt{m}_1} \cdots \mathtt{a}^{\mathtt{n}_k}\,\mathtt{b}^{\mathtt{m}_k}\,\mathtt{c}\,\mathtt{d}^{\mathtt{p}} \mid \mathtt{n}_i, \mathtt{m}_i \geq 0,\ \mathtt{p} = \mathtt{m}_1 + \cdots + \mathtt{m}_k\}.$$

Soluzione

soluzione 1:

$$\mathtt{S} \longrightarrow \mathtt{a}\ \mathtt{S} \mid \mathtt{b}\ \mathtt{S}\ \mathtt{d} \mid \mathtt{c}$$

soluzione 2:

$$\mathtt{S} \longrightarrow \mathtt{A}\ \mathtt{b}\ \mathtt{S}\ \mathtt{d} \mid \mathtt{A}\ \mathtt{c}$$
$$\mathtt{A} \longrightarrow \mathtt{a}\ \mathtt{A} \mid \epsilon$$

variante di 2 con fattorizzazione di A:

$$\mathtt{S} \longrightarrow \mathtt{A}\ \mathtt{B}$$
$$\mathtt{A} \longrightarrow \mathtt{a}\ \mathtt{A} \mid \epsilon$$
$$\mathtt{B} \longrightarrow \mathtt{b}\ \mathtt{S}\ \mathtt{d} \mid \mathtt{c}$$

Esercizi.

Esercizio

Si mostri la sequenza di $\alpha - \mathtt{red}$ *applicate per ridurre il termine dato ad un termine contenente sempre, identificatori diversi per variabili diverse:* $\lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x})\mathrm{x})(\lambda\,\mathrm{y}.\mathrm{x}(\lambda\,\mathrm{x}.\mathrm{yx})\mathrm{y})$

Soluzione

$\lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x})\mathrm{x})(\lambda\,\mathrm{y}.\mathrm{x}(\lambda\,\mathrm{x}.\mathrm{yx})\mathrm{y})$

$\rightarrow_\alpha \lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}_1.\{\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x})\mathrm{x}\}[\mathrm{x}_1/\mathrm{x}])(\lambda\,\mathrm{y}.\mathrm{x}(\lambda\,\mathrm{x}.\mathrm{yx})\mathrm{y})$

$\quad \lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}_1.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x}_1)\mathrm{x}_1)(\lambda\,\mathrm{y}.\mathrm{x}(\lambda\,\mathrm{x}.\mathrm{yx})\mathrm{y})$

$\rightarrow_\alpha \lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}_1.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x}_1)\mathrm{x}_1)(\lambda\,\mathrm{y}_1.\{\mathrm{x}(\lambda\,\mathrm{x}.\mathrm{yx})\mathrm{y})\}[\mathrm{y}_1/\mathrm{y}]$

$\quad \lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}_1.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x}_1)\mathrm{x}_1)(\lambda\,\mathrm{y}_1.\mathrm{x}(\lambda\,\mathrm{x}.\mathrm{y}_1\mathrm{x})\mathrm{y}_1)$

$\rightarrow_\alpha \lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}_1.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x}_1)\mathrm{x}_1)(\lambda\,\mathrm{y}_1.\mathrm{x}(\lambda\,\mathrm{x}_2.\{\mathrm{y}_1\mathrm{x}\}[\mathrm{x}_2/\mathrm{x}])\mathrm{y}_1)$

$\quad \lambda\,\mathrm{x}.\lambda\,\mathrm{y}.(\lambda\,\mathrm{x}_1.\mathrm{y}(\lambda\,\mathrm{y}.\mathrm{x}_1)\mathrm{x}_1)(\lambda\,\mathrm{y}_1.\mathrm{x}(\lambda\,\mathrm{x}_2.\mathrm{y}_1\mathrm{x}_2)\mathrm{y}_1)$

Esercizi.

Esercizio

Si dimostri che: $\forall \mathtt{F}, \quad \Psi\,\mathtt{F} = \mathtt{F}(\Psi\,\mathtt{F})$

Soluzione

$$\begin{aligned}
\Psi\,\mathtt{F} &= (\lambda\,\mathtt{g}.(\lambda\,\mathtt{x}.\mathtt{g}(\mathtt{xx}))(\lambda\,\mathtt{x}.\mathtt{g}(\mathtt{xx})))\mathtt{F} \\
&= (\lambda\,\mathtt{x}.\mathtt{F}(\mathtt{xx}))(\lambda\,\mathtt{x}.\mathtt{F}(\mathtt{xx})) \\
&= (\mathtt{F}((\lambda\,\mathtt{x}.\mathtt{F}(\mathtt{xx}))(\lambda\,\mathtt{x}.\mathtt{F}(\mathtt{xx})))) \\
&= (\mathtt{F}(\Psi\,\mathtt{F}))
\end{aligned}$$

Esercizi.

Esercizio

Si dia una semantica SOS per il Lambda Calcolo

Soluzione

Table 1 : Lambda − Calcolo

Sintassi

$\Lambda ::= \mathtt{X} \mid \Pi \mid \lambda \mathtt{X}.\Lambda \mid \Lambda\Lambda$

Semantica

$\lambda\, \mathtt{x.a} \longrightarrow \lambda \mathtt{y.a[y/x]}$	provided $\mathtt{y} \notin \mathtt{FV(a)}$	$(\alpha - \mathit{red})$
$(\lambda\, \mathtt{x.a})\mathtt{a}' \longrightarrow \mathtt{a[a'/x]}$	provided $\mathtt{FV(a')} \cap \mathtt{FV(a)} = \{\}$	$(\beta - \mathit{red})$
$\lambda\, \mathtt{x.(ax)} \longrightarrow \lambda \mathtt{a}$	provided $\mathtt{x} \notin \mathtt{FV(a)}$	$(\eta - \mathit{red})$

SOS

$$\langle \mathtt{x}, \sigma\rangle \rightsquigarrow \langle \mathtt{x}, \sigma(\mathtt{x})\rangle \qquad (\mathit{bound\ var})$$

$$\langle \mathtt{p}, \sigma\rangle \rightsquigarrow \langle \mathtt{p}, \sigma(\mathtt{x})\rangle \qquad (\mathit{constant}, \mathtt{p} \in \Pi)$$

$$\frac{\langle \mathtt{a}, \sigma(\mathtt{x}, \mathtt{a}')\rangle \rightsquigarrow \langle \mathtt{a}'', \sigma(\mathtt{x}, \mathtt{a}')\rangle \quad \mathtt{FV(a')} = \{\} \quad \mathtt{x} \notin \mathtt{FV(a'')}}{\langle (\lambda \mathtt{x.a})\mathtt{a}', \sigma\rangle \rightsquigarrow \langle \mathtt{a}'', \sigma\rangle} \qquad (\lambda - \mathit{application})$$

$$\frac{\langle \mathtt{a}', \sigma\rangle \rightsquigarrow \langle \mathtt{a}'', \sigma\rangle}{\langle \mathtt{aa}', \sigma\rangle \rightsquigarrow \langle \mathtt{aa}'', \sigma\rangle} \qquad \frac{\langle \mathtt{a}', \sigma\rangle \rightsquigarrow \langle \mathtt{a}'', \sigma\rangle}{\langle \mathtt{a}'\mathtt{a}, \sigma\rangle \rightsquigarrow \langle \mathtt{a}''\mathtt{a}, \sigma\rangle} \qquad (\mathit{application})$$

σ: $(\mathtt{x}_1, \mathtt{a}_1)\ldots(\mathtt{x}_k, \mathtt{a}_k)$ k identificatori anche stesso nome

ambiente vuoto : ϵ

selezione $\sigma(\mathtt{x}_i) = \mathtt{a}_i$, if $\forall \mathtt{j} > \mathtt{i}, \mathtt{x_j} \neq \mathtt{x_i}$

aggiunta di un legame, nuovo σ': $\sigma(\mathtt{x}_{k+1}, \mathtt{a}_{k+1})$

Esercizi.

Esercizio

Si completi la computazione definita dal seguente stato:

$$\langle(\lambda \mathtt{y}.(+\mathtt{yx}))(*\mathtt{xy}), (\mathtt{y}, 3)(\mathtt{x}, 5)\rangle$$

utilizzando la semantica SOS data per il Lambda-Calcolo nella slide precedente. Si assuma che i simboli `+, *, 3, 5` *abbiano l'usuale interpretazione di somma, prodotto, naturali 3 e 5, ovvero il Calcolo considerato (e la semantica, SOS) sia esteso con le operazioni primitive di somma e prodotto e i numeri naturali (quindi,* `3*5` *è valutato* `15`*).*

Soluzione

$\langle(\lambda \mathtt{y}.(+\mathtt{yx}))(*\mathtt{xy}), (\mathtt{y}, 3)(\mathtt{x}, 5)\rangle \rightsquigarrow \ldots$

Esercizi.

Esercizio

Si dimostri che nell'aritmetica data per il Lambda-Calcolo, la somma è correttamente espressa da:

$$\mathtt{plus} \equiv \lambda\,\mathtt{n}.\lambda\,\mathtt{m}.\lambda\mathtt{y}.\lambda\mathtt{x}.\mathtt{ny}(\mathtt{myx}).$$

Soluzione

Ricordiamo come sono fatti i numerali per i naturali:

$[0] \equiv \lambda\mathtt{y}.\lambda\mathtt{x}.\mathtt{x}$

$[\mathtt{n}+1] \equiv \lambda\mathtt{y}.\lambda\mathtt{x}.\mathtt{y}([\mathtt{n}]\mathtt{yx})$

allora:

Procediamo per induzione sulle coppie di naturali {con (n,m)<(n,m+1)<(n+1,m+1)}:

$$\begin{aligned}\mathtt{plus}[0][0] &= \lambda\mathtt{y}.\lambda\mathtt{x}.[0]\mathtt{y}([0]\mathtt{yx})\\ &= \lambda\mathtt{y}.\lambda\mathtt{x}.(\lambda\mathtt{x}.\mathtt{x})([0]\mathtt{yx}) = \lambda\mathtt{y}.\lambda\mathtt{x}.[0]\mathtt{yx}\\ &= \lambda\mathtt{y}.\lambda\mathtt{x}.\mathtt{x}\\ &= [0]\end{aligned}$$

{con n,m>0}:

$$\begin{aligned}\mathtt{plus}[\mathtt{n}][\mathtt{m}+1] &= \lambda\mathtt{y}.\lambda\mathtt{x}.[\mathtt{n}]\mathtt{y}([\mathtt{m}+1]\mathtt{yx})\\ &= \lambda\mathtt{y}.\lambda\mathtt{x}.[\mathtt{n}]\mathtt{y}((\lambda\mathtt{y}.\lambda\mathtt{x}.\mathtt{y}([\mathtt{m}]\mathtt{yx}))\mathtt{yx})\\ &= \lambda\mathtt{y}.\lambda\mathtt{x}.[\mathtt{n}]\mathtt{y}(\mathtt{y}([\mathtt{m}]\mathtt{yx}))\end{aligned}$$

Esercizi.

Esercizio

Si scriva, in Lambda-Calcolo, un programma per la funzione `prod` *introdotta nell'aritmetica data per tale linguaggio.*

Soluzione

Ricordiamo come sono fatti i numerali per i naturali:

$[0] \equiv \lambda \mathtt{y}.\lambda \mathtt{x}.\mathtt{x}$

$[\mathtt{n}+1] \equiv \lambda \mathtt{y}.\lambda \mathtt{x}.\mathtt{y}([\mathtt{n}]\mathtt{yx})$

allora:

$\mathtt{prod} \equiv \lambda\, \mathtt{n}.\lambda\, \mathtt{m}.\lambda \mathtt{y}.\lambda \mathtt{x}.\mathtt{n}(\lambda \mathtt{x}.\mathtt{myx})\mathtt{x}$

Esercizio

Si dia una semantica SOS per la Logica Combinatoria

Soluzione

Esercizi.

Esercizio

Si dimostri che `S K K = I`

Soluzione

Esercizio

Si scriva, in Logica Combinatoria, un programma per la funzione `plus` *introdotta nell'aritmetica data per tale linguaggio.*

Soluzione