Lezione 21-22

April 16, 2012

Logic Programming Fundamentals

- Costrutti: Formule e Controllo della Refutazione
- Calcolo dei predicati, Clausole, Formule Atomiche, Termini
- Semantica: Model Theoretic, Resolution (LUSH and SLD)
- Semantica: Resolution e Programmazione Deduttiva

Costrutti: Formule e Controllo della dimostrazione/1

Un programma Prolog è:

- **Sintatticamente**:
  - Teoria Logica del I° Ordine
    - un insieme di formule che considereremo gli assiomi
    - tutti contemporaneamente veri nella teoria
  - Formula del I° Ordine
- **Semanticamente**,
  - **estensionale** (Teoria):
    il modello minimo tra quelli che soddisfano la teoria
  - **intensionale** (Formula):
    la refutazione generata dalla (SLD) Resolution applicata alla Formula (in forma a clausole)

Costrutti: Formule /2

- **Sintatticamente**, una Teoria Logica del I° Ordine
  - connettivi logici: $\wedge, \vee, \neg, \Rightarrow, \Leftrightarrow$
  - costanti: `true`, `false`
  - variabili: un numerabile (e.g. identificatori con prima lettera Maiuscola)
  - quantificatori: $\exists, \forall$
  - simboli propri della teoria:
    - simboli di funzione: un numerabile Σ con arità
    - simboli di predicato: un numerabile Π con arità
- **Forma Prenessa a Clausole**
- **Ristretta a Clausole Horn**

Costrutti: Formule /3

Forma Prenessa a Clausole:

- Prenessa a Clausole: $\forall \overline{\mathtt{X}}_1\ \mathtt{C}_1\ \ldots\ \forall \overline{\mathtt{X}}_k\ \mathtt{C}_\mathtt{k}$ (con $\mathtt{Var}(\mathtt{C}_\mathtt{i}) \in \overline{\mathtt{X}}_\mathtt{i}$)
- Clausola: $\mathtt{C} \equiv \mathtt{L}_1 \vee\ \ldots\ \vee\ \mathtt{L}_\mathtt{n}$
- Letterali Generalizzati:
  - $\mathtt{L} \equiv \neg(\mathtt{t}_1 = \mathtt{t}_2)$ (con $\mathtt{t}_1, \mathtt{t}_2 \in \mathtt{HU}_\Sigma$)
  - $\mathtt{L} \equiv \mathtt{A}$ oppure $\mathtt{L} \equiv \neg\mathtt{A}$
- Atomi: $\mathtt{A} \equiv \mathrm{q}(\mathtt{t}_1, \ldots, \mathtt{t}_\mathtt{n})$ (con $\mathrm{q} \in \Pi$, $\mathtt{t}_\mathtt{i} \in \mathtt{HU}_\Sigma$)

Predicato di uguaglianza sintattica: $\mathtt{E} \equiv \mathtt{t}_1 = \mathtt{t}_2$ uno speciale letterale che indichiamo anche con $\mathtt{E}$ per distinguerlo dagli atomi (anche positivi)

Example

$\mathtt{H}_1 \vee\ \ldots\ \vee \mathtt{H}_\mathtt{k} \vee\ \neg\mathtt{A}_1 \vee\ \ldots\ \vee \neg\mathtt{A}_\mathtt{n}\ \overline{\vee\ \neg\mathtt{E}}$ – ordine inessenziale

in forma implicativa: – $\mathtt{H}_\mathtt{i}$ sono atomi

$\mathtt{H}_1 \vee\ \ldots\ \vee \mathtt{H}_\mathtt{k} \leftarrow \mathtt{A}_1 \wedge\ \ldots\ \wedge \mathtt{A}_\mathtt{n}\ \overline{\wedge\ \mathtt{E}}$

in logic programming

$\mathtt{H}_1, \ldots, \mathtt{H}_\mathtt{k} :- \mathtt{A}_1, \ldots, \mathtt{A}_\mathtt{n}\ \overline{, \mathtt{E}}$

Costrutti: Trasformazione in Clausal Form /4

Forma Prenessa a Clausole: Sia $\mathtt{T}=\{\overline{\mathtt{F}}\}$. Ad ogni $\mathtt{F}_i$, applichiamo:

- eliminazione $\Leftrightarrow$: $\mathtt{g} \Leftrightarrow \mathtt{f} \sim (\mathtt{g} \Rightarrow \mathtt{f}) \wedge (\mathtt{f} \Rightarrow \mathtt{g})$
- eliminazione $\Rightarrow$: $\mathtt{g} \Rightarrow \mathtt{f} \sim (\neg \mathtt{g}) \vee \mathtt{f}$
- rigth S. negation: $\neg(\forall \overline{\mathtt{X}} \mathtt{g}) \sim \exists \overline{\mathtt{X}}(\neg \mathtt{g})$, ed anche $\neg(\exists \overline{\mathtt{X}} \mathtt{g}) \sim \forall \overline{\mathtt{X}}(\neg \mathtt{g})$
- eliminazione $\exists$: $\exists \overline{\mathtt{X}} \mathtt{g} \sim_{\mathcal{M}} \mathtt{g}[\mathtt{k}(\overline{\mathtt{Y}})/\mathtt{X}]$[1]
- left Shifting $\forall$: $(\forall \overline{\mathtt{X}} \mathtt{g}) \sqcap (\forall \overline{\mathtt{Y}} \mathtt{f}) \sim \forall \overline{\mathtt{X}} \forall \overline{\mathtt{Y'}} \mathtt{g} \sqcap \mathtt{f}[\overline{\mathtt{Y'}}/\overline{\mathtt{Y}}]$[2].
- conjuctive form[3]: $\mathtt{f} \vee (\mathtt{g} \wedge \mathtt{h}) \sim (\mathtt{f} \vee \mathtt{g}) \wedge (\mathtt{f} \vee \mathtt{h})$
  $\mathtt{f} \wedge (\mathtt{g} \vee \mathtt{h}) \sim (\mathtt{f} \wedge \mathtt{g}) \vee (\mathtt{f} \wedge \mathtt{h})$
- eliminazione $\wedge$: $\mathtt{C}_1 \wedge \ldots \wedge \mathtt{C}_n \sim_{\mathcal{M}} \{C_1, \ldots, C_n\}$

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[1] con $\mathtt{k}(\overline{\mathtt{Y}})$un nuovo simbolo di funzione e $\overline{\mathtt{Y}}$ la sequenza di variabili quantificate universalmente aventi $\exists \overline{\mathtt{X}} \mathtt{g}$ nel proprio scope. (skolemizzazione)

[2] con $\overline{\mathtt{Y'}}$ fresh variabile e $\sqcap \in \{\wedge, \vee\}$

[3] ripetere fino ad ottenere congiunzioni di disgiunzioni di letterali

Costrutti: Formule /5

- Clausola:

$$\mathtt{H}_1 \lor \ \dots \ \lor \mathtt{H}_k \lor \ \neg \mathtt{A}_1 \lor \ \dots \ \lor \neg \mathtt{A}_n \ \overline{\lor \neg \mathtt{E}}$$
$$\mathtt{H}_1 \lor \ \dots \ \lor \mathtt{H}_k \leftarrow \mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land \ \mathtt{E}}$$

- Clausole Horn:
    - al più un letterale positivo

      $$\mathtt{H} \leftarrow \mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land \ \mathtt{E}}$$
      $$\mathtt{H} :- \mathtt{A}_1, \dots, \mathtt{A}_n \ \overline{, \mathtt{E}}$$
    - header: H è l'header della clausola
    - tre forme di clausole:
        - Unit: solo un letterale positivo

          $$\mathtt{H} \leftarrow$$
          $$\mathtt{H}$$
        - Definite: un letterale positivo e almeno uno negativo

          $$\mathtt{H} \leftarrow \mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land \ \mathtt{E}}$$
          $$\mathtt{H} :- \mathtt{A}_1, \dots, \mathtt{A}_n \ \overline{, \mathtt{E}}$$
        - Goal: solo letterali negativi

          $$??? \leftarrow \mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land \ \mathtt{E}}$$

- Unit: solo un letterale positivo

  $\mathtt{H} \leftarrow$

  $\mathtt{H}$

- Definite: un letterale positivo e almeno uno negativo

  $\mathtt{H} \leftarrow \mathtt{A}_1 \wedge \ \dots \ \wedge \mathtt{A}_n \ \overline{\wedge \ \mathtt{E}}$

  $\mathtt{H} :- \mathtt{A}_1, \dots, \mathtt{A}_n \ \overline{, \mathtt{E}}$

- Goal: solo letterali negativi

  $\mathtt{false} \leftarrow \mathtt{A}_1 \wedge \ \dots \ \wedge \mathtt{A}_n \ \overline{\wedge \ \mathtt{E}}$

  $:- \mathtt{A}_1, \dots, \mathtt{A}_n \ \overline{, \mathtt{E}}$

  in forma prenessa esplicita (con $\overline{\mathtt{X}}$ variabili nella formula):

  $\forall \overline{\mathtt{X}} \neg (\mathtt{A}_1 \wedge \ \dots \ \wedge \mathtt{A}_n \ \overline{\wedge \ \mathtt{E}})$

- Calcolare=Dimostrare: Proviamo che tale formula è falsa nella teoria definita dall'insieme di clausole date.

Costrutti: Clausole Horn/2

Calcolare=Dimostrare: Proviamo che tale formula è falsa nella teoria definita dall'insieme di clausole date.

- Formula falsa in ogni modello
- Formula falsa nel modello minimo (incluso in tutti i modelli)
- Formula falsa nel modello liberamente generato da $\mathtt{HU}_\Sigma$

Table 17: Modello Minimo liberamente generato su HU
Sia $\mathtt{P}$ un insieme di clausole (unit e definite) su (Π, Σ) $\Sigma = \cup \Sigma^{\mathtt{n}}$, con $\mathtt{n}$ l'arità dei simboli di funzione e Σ^0 non vuoto $\mathtt{HU}_\Sigma =_{\mathtt{min}} \Sigma^0 \cup \{\mathtt{f}(\mathtt{t}_1, ..., \mathtt{t}_\mathtt{n}) \mid \mathtt{f} \in \Sigma^\mathtt{n}, \mathtt{t}_\mathtt{i} \in \mathtt{HU}_\Sigma\}$ $\mathcal{M}(\mathtt{P}) =_{min} \{\mathtt{A}[\overline{\mathtt{T}}/\overline{\mathtt{X}}] \mid \mathtt{C} \in \mathtt{P}, \mathtt{C} \equiv \mathtt{A} :- \mathtt{A}_1, .., \mathtt{A}_\mathtt{k}, \overline{\mathtt{X}} = \mathtt{Var}(\mathtt{C})$ $\overline{\mathtt{T}} \in \mathtt{HU}_\Sigma^{\lvert\overline{\mathtt{X}}\rvert}, (\forall \mathtt{i})\mathtt{A}_\mathtt{i} \in \mathcal{M}(\mathtt{P})\}$

Esempio: Modello Minimo

Example

$\mathtt{P} \equiv \{\mathtt{add}(\mathtt{0}, \mathtt{Y}, \mathtt{Y}); \mathtt{add}(\mathtt{s}(\mathtt{X}), \mathtt{Y}, \mathtt{s}(\mathtt{Z})) :- \mathtt{add}(\mathtt{X}, \mathtt{Y}, \mathtt{Z})\}$
$\mathtt{G} \equiv \mathtt{add}(\mathtt{X}, \mathtt{Y}, \mathtt{s}(\mathtt{0})\mathtt{ovvero\ la\ formula}\ \forall(\mathtt{X}, \mathtt{Y})\neg\mathtt{add}(\mathtt{X}, \mathtt{Y}, \mathtt{s}(\mathtt{0}))$
$\Pi = \Pi^3 = \{add\}; \ \ \Sigma = \Sigma^0 \cup \Sigma^1 = \{\mathtt{0}\} \cup \{s\}$
$\mathcal{M}(\mathtt{P}) = \{\mathtt{add}(\mathtt{0}, \mathtt{0}, \mathtt{0}), \mathtt{add}(\mathtt{0}, \mathtt{s}(\mathtt{0}), \mathtt{s}(\mathtt{0}), \ldots$
$\mathcal{M}(\mathtt{P}) = \{\mathtt{add}(\mathtt{0}, \mathtt{0}, \mathtt{0}), \mathtt{add}(\mathtt{0}, \mathtt{s}(\mathtt{0}), \mathtt{s}(\mathtt{0}), \ldots, \mathtt{add}(\mathtt{s}(\mathtt{0}), \mathtt{0}, \mathtt{s}(\mathtt{0})), \ldots$

E vediamo che almeno 2 coppie di valori falsificano l'asserto

Costrutti: Clausole Horn/3

- **Goal**: solo letterali negativi

  $$\mathtt{false} \leftarrow \mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land\ \mathtt{E}}$$

  $$\mathtt{:-A}_1, \dots, \mathtt{A}_n \ \overline{,\mathtt{E}}$$

  in forma prenessa esplicita (con $\overline{\mathtt{X}}$ variabili nella formula):

  $$\forall\overline{\mathtt{X}}\neg(\mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land\ \mathtt{E}})$$

- Calcolare=Dimostrare:
  - Proviamo che tale formula è falsa nella teoria definita dall'insieme di clausole date.
  - Proviamo che tale formula è non soddisfacibile in tutti i modelli della teoria ...
  - Diamo una refutazione della formula: ovvero proviamo (resolution):

$$\neg(\forall\overline{\mathtt{X}}\neg(\mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land\ \mathtt{E}}))$$

$$\boxed{\exists\overline{\mathtt{X}}(\mathtt{A}_1 \land \ \dots \ \land \mathtt{A}_n \ \overline{\land\ \mathtt{E}})}$$

Semantica: Unificazione

Table 17: Unificazione e LUSH Resolution
Unification
$\dfrac{\vdash \ \overline{T} = \overline{T}' \ \rightarrow_{mgu} \ \overline{X} = \overline{R}}{\vdash \ X = X, \overline{T} = \overline{T}' \ \rightarrow_{mgu} \ \overline{X} = \overline{R}}$ $\qquad$ $\dfrac{\vdash X = R, \overline{T} = \overline{T}' \rightarrow_{mgu} \ X = R, \overline{X} = \overline{R}}{\vdash R = X, \overline{T} = \overline{T}' \rightarrow_{mgu} \ X = R, \overline{X} = \overline{R}}$
$\dfrac{X \notin Var(R)^* \qquad \vdash (\overline{T} = \overline{T}')[R/X] \rightarrow_{mgu} \ \overline{X} = \overline{R}}{\vdash X = R, \overline{T} = \overline{T}' \rightarrow_{mgu} \ X = R, \overline{X} = \overline{R}}$
$\dfrac{\vdash \overline{T}_f = \overline{T}'_f \rightarrow_{mgu} \ \overline{X}_f = \overline{R}_f \qquad \begin{matrix} f = g^* \\ \vdash (\overline{T} = \overline{T}')[\overline{R}_f/\overline{X}_f] \rightarrow_{mgu} \ \overline{X} = \overline{R} \end{matrix} \qquad f \in \Sigma_{\lvert \overline{T}_f \rvert}}{\vdash f(\overline{T}_f) = g(\overline{T}'_f), \overline{T} = \overline{T}' \rightarrow_{mgu} \ \overline{X}_f = \overline{R}_f, \overline{X} = \overline{R}}$
Legenda: $\quad \overline{Z} \equiv Z_1, \ldots, Z_n \,; \qquad \overline{Z}, \overline{Z}' \equiv Z_1, \ldots, Z_n, Z'_1, \ldots, Z'_m$

Failure Quando applicando le regole 3 e 4 le *condizioni** non sono soddsfatte si ha un fallimento.

Semantica: Unificazione

Example

```
h(f(X,X),Y) = h(Y,f(g(Z),W))
f(X,X)=Y,Y=f(g(Z),W)
Y=f(X,X),Y=f(g(Z),W)
```

Y=f(X,X),`f(X,X)=f(g(Z),W)`

```
          f(X,X)=f(g(Z),W)
          X=g(Z),X=W
```

X=g(Z),`g(Z)=W`

```
                  g(Z)=W
                  W=g(Z)
```

W=g(Z)

I termini unificano con most general unifier:

$\sigma = \{$`Y=f(X,X),X=g(Z),W=g(Z)`$\}$

Semantica: LUSH Resolution

Table 17: Unificazione e LUSH Resolution

Clauses

$$\overline{\mathtt{P}}_1, \mathtt{C}, \overline{\mathtt{P}}_2 \vdash \mathtt{C}\ \mathtt{OK}$$

LUSH Resolution

$$\frac{\vdash\ \overline{\mathtt{T}} = \overline{\mathtt{T}}' \longrightarrow_{\mathtt{mgu}} \overline{\mathtt{X}} = \overline{\mathtt{R}} \qquad \overline{\mathtt{P}} \vdash\ (\overline{\mathtt{G}}_1, \overline{\mathtt{G}}_2)[\overline{\mathtt{R}}/\overline{\mathtt{X}}] \longrightarrow_{\mathtt{lush}} \overline{\mathtt{X}}_\mathtt{G} = \overline{\mathtt{R}}_\mathtt{G}}{\overline{\mathtt{P}} \vdash\ \overline{\mathtt{G}}_1, \overline{\mathtt{T}} = \overline{\mathtt{T}}', \overline{\mathtt{G}}_2 \longrightarrow_{\mathtt{lush}} \overline{\mathtt{X}} = \overline{\mathtt{R}}, \overline{\mathtt{X}}_\mathtt{G} = \overline{\mathtt{R}}_\mathtt{G}}$$

$$\frac{\begin{array}{c}\overline{\mathtt{P}} \vdash \mathtt{q}(\overline{\mathtt{T}}'_\mathtt{A}) \mathrel{:-} \overline{\mathtt{A}}' \qquad \mathtt{q}(\overline{\mathtt{T}}_\mathtt{A}) \mathrel{:-} \overline{\mathtt{A}} = \mathtt{Rename}(\mathtt{q}(\overline{\mathtt{T}}'_\mathtt{A}) \mathrel{:-} \overline{\mathtt{A}}') \\ \vdash \overline{\mathtt{T}}_\mathtt{A} = \overline{\mathtt{T}}_\mathtt{B} \longrightarrow_{\mathtt{mgu}} \overline{\mathtt{X}} = \overline{\mathtt{R}} \qquad \overline{\mathtt{P}} \vdash\ (\overline{\mathtt{G}}_1, \overline{\mathtt{G}}_2)[\overline{\mathtt{R}}/\overline{\mathtt{X}}] \longrightarrow_{\mathtt{lush}} \overline{\mathtt{X}}_\mathtt{G} = \overline{\mathtt{R}}_\mathtt{G}\end{array}}{\overline{\mathtt{P}} \vdash\ \overline{\mathtt{G}}_1, \mathtt{q}(\overline{\mathtt{T}}_\mathtt{B}), \overline{\mathtt{G}}_2 \longrightarrow_{\mathtt{lush}} \overline{\mathtt{X}} = \overline{\mathtt{R}}, \overline{\mathtt{X}}_\mathtt{G} = \overline{\mathtt{R}}_\mathtt{G}}$$

Legenda:

$\overline{\mathtt{Z}} \equiv \mathtt{Z}_1, \ldots, \mathtt{Z}_n; \qquad \overline{\mathtt{Z}}, \overline{\mathtt{Z}}' \equiv \mathtt{Z}_1, \ldots, \mathtt{Z}_n, \mathtt{Z}'_1, \ldots, \mathtt{Z}'_m$

`Rename(C)` rinomina le variabili nella clausola `C`

Goal Risolto Quando contiene solo `E` in forma risolta (unificati).

Semantica: Risoluzione

Example

$\mathtt{P} \equiv \{\mathtt{add}(0,0,0); \mathtt{add}(\mathtt{s}(\mathtt{X}),\mathtt{Y},\mathtt{s}(\mathtt{Z})) :\!-\mathtt{add}(\mathtt{X},\mathtt{Y},\mathtt{Z})\}$

$\mathtt{G} \equiv \mathtt{add}(\mathtt{X},\mathtt{Y},\mathtt{s}(0))$ $\quad$ [ovvero la formula $\neg\exists(\mathtt{X},\mathtt{Y})\mathtt{add}(\mathtt{X},\mathtt{Y},\mathtt{s}(0))$]

R(C1,L1(G)):

$\quad \mathtt{Rename}(\mathtt{C1}) = \mathtt{add}(0,0,0)$

$\quad \mathtt{Unify}\{\mathtt{X} = 0, \mathtt{Y} = 01, \mathtt{s}(0) = 0\} : \mathtt{fail}$

R(C2,L1(G)):

$\quad \mathtt{Rename}(\mathtt{C2}) = \mathtt{add}(\mathtt{s}(\mathtt{X1}),\mathtt{Y1},\mathtt{s}(\mathtt{Z1})) :\!-\mathtt{add}(\mathtt{X1},\mathtt{Y1},\mathtt{Z1})$

$\quad \mathtt{Unify}\{\mathtt{X}=\mathtt{s}(\mathtt{X1}),\mathtt{Y}=\mathtt{Y1},\mathtt{s}(0)=\mathtt{s}(\mathtt{Z1})\}\rightarrow_{\mathtt{mgu}}\{\mathtt{X}=\mathtt{s}(\mathtt{X1}),\mathtt{Y}=\mathtt{Y1},\mathtt{Z1}=0\}$

$\quad \mathtt{G1} \equiv \mathtt{X} = \mathtt{s}(\mathtt{X1}), \mathtt{Y} = \mathtt{Y1}, \mathtt{Z1} = 0, \mathtt{add}(\mathtt{X1},\mathtt{Y1},0)$

R(C1,L4(G1)):

$\quad \mathtt{Rename}(\mathtt{C1}) = \mathtt{add}(0,0,0)$

$\quad \mathtt{Unify}\{\mathtt{X1} = 0, \mathtt{Y1} = 0\}\rightarrow_{\mathtt{mgu}}\{\mathtt{X1} = 0, \mathtt{Y1} = 0\}$

$\quad \mathtt{G2} \equiv \mathtt{X} = \mathtt{s}(0), \mathtt{Y} = 0, \mathtt{Z1} = 0, \mathtt{X1} = 0, \mathtt{Y1} = 0)$

La risoluzione produce un *goal risolto*: i valori di X e Y sono quelli di "falsificano" G

Altre soluzioni?: Il *risolvente* R(C2,L4(G1))?