A Driver for NFA (DFA): How to remove Backtracking - 1

$$f(\gamma) = \begin{cases} \textit{accept} \ \textbf{if} \ \text{move1}^*(\gamma) \cap F \neq \{\} \\ \textit{noaccept} \ \textbf{otherwise} \end{cases}$$

move

	a	b	ε
0	{1}		
1	{2}		
2	{0}	{1,2}	

move1

	a	b	ε
{0}	{1}		
{1}	{2}		
{2}	{0}	{1,2}	
{1,2}	?	?	

The states of move1 are **Sets** of the states of move

How to remove Backtracking

Function *move1*: S x Σ -> S

How to computes transitions using Set of States instead of single States

Let $move1(S,c) = Clos(\cup_{s \in Clos(S)} \{move(s,c)\})$

remember: $Clos(S) = S \cup Clos(\cup_{s \in S}\ move(s,\varepsilon))$

move

	a	b	ε
0	{1}		
1	{2}		
2	{0}	{1,2}	

move1

	a	b	ε
{0}	{1}		
{1}	{2}		
{2}	{0}	{1,2}	
{1,2}	{0,2}	?	

A Converter from NFA to DFA

based on *compiling move1* into a *Transition Table*

Preliminaries Definitions:

Structures and Notational Conventions

Star: Set -> (Σ -> Set) is a function representing tables

- with **Set**-indexed rows and **Σ**-indexed columns
- **Star[s]** - is the **s**-indexed row of table **Star**
- **Star[s]c** - is the value at the row **s** and column **c**
- **Star** does not contain any **ε**-indexed column

A Basic Operation on Tables

merge(move,S) = merge-row({move[s] | s∈S})

$\forall_{c \in \Sigma}$ merge-row($\{R_1,\ldots,R_k\}$) c = $(\cup_{1 \le i \le k} \mathrm{Clos}(R_i c))$

	digit	.	ε
0	1		1
1	3	2	{1,3}
2		2	
3	0	4	
4	4	5	
5	6		
6	6		

move

merge-row({0,1,3})=

{0,1,3}	{2,4}

A Converter from NFA to DFA

based on *compiling move1* into a *Transition Table*

Implementation: A Routine

Table ***movestar***() **//DFA Transition Table.**

```
{EntryStar = ∅;
  List = Clos({s0});
  while (List ≠ emptylist) {
        S = firstout(List);
        if (S ∉ EntryStar) {
              add(S,EntryStar);
              Star[S]= merge(move,S);
              List=List+{Star[S]c | c∈Σ};
              }
        }
   return Star;
}
```

EntryStar = list of the state sets already cosidered for the row indices of Star
List = list of the reached state sets to be considered for inclusion in the row indices of Star
Firstout = selection and removal of the first element
Add = adds a state set to the current list EntryStar

A suitable renaming, of the resulting Table states, should be always used

Applying it to automaton A

$<S= \{0,1,2\}, \Sigma= \{a,b\},$
move$= \{<<0,a>\{1\}>,$
$<<1,b>\{1,2\}>\}$
$s_0=0, F= \{2\} >$

Results automaton A'

$<S= \{0,1,2\}, \Sigma= \{a,b\},$
move$= \{<<0,a>1>,<<1,b>2>,$
$<<2,b>2>\}$
$s_0=0, F= \{2\} >$
Renaming {1,2} with 2

Exercise 2

A Driver for DFA

Answer ***StarDriver***()

{s= s_0;
nextchar(c);
while(c≠**eof**) **and** (s≠{}) {
s=Star[s]c;
nextchar(c);}
if ((s∉F) **or** (c≠**eof**)) **return** 'noaccept'
return 'accept';
}

Linear Time
O(1) operations

(All steps run in constant time)

Apply it to automaton B

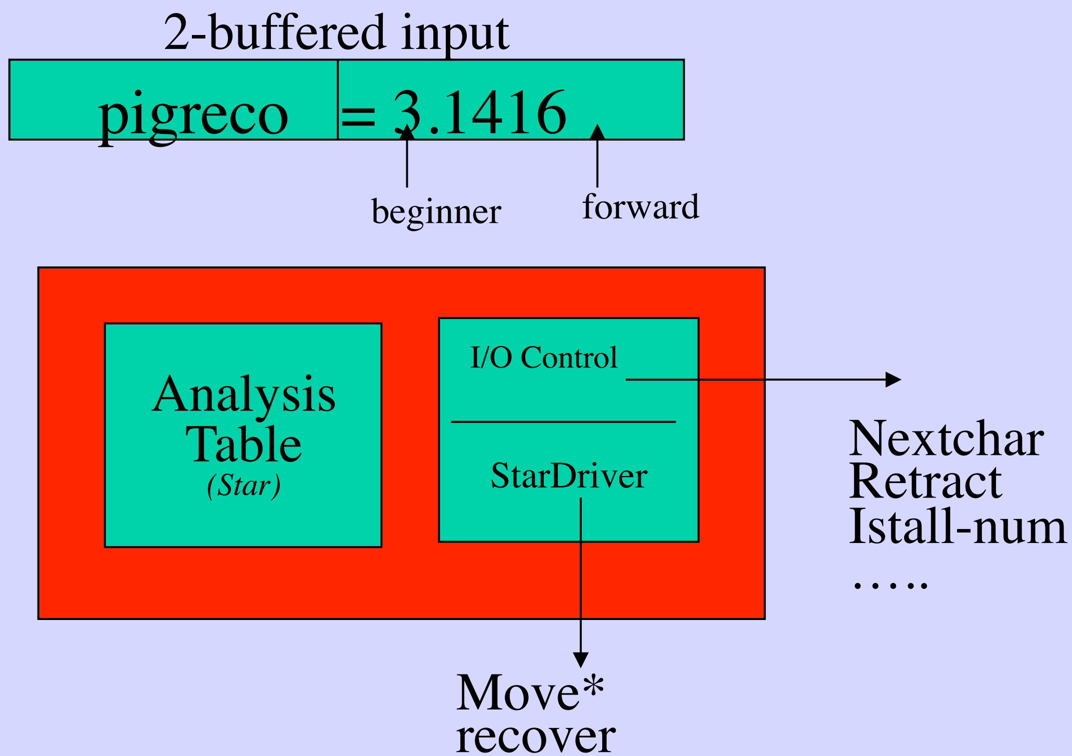
<**S**= {0,1,2}, **Σ**= {a,b},
move= {<<0,a>1>,<<1,b>2>,
<<2,b>2>}
s_0=0, **F**= {2} >

when scanning: abb$

How to build Scanner for a Lexics L

Use the following steps in order to obtain the structure on the right side:

1. Define a Regular Grammar G for L;
2. Check L(G)=L holds;
3. Define an Automaton A for G (using a safe transformation technique)
4. Convert A into a deterministic B having a Transition Table T.
5. Use the following setting:
   Analysis Table = T



How to do step 3? By extending Thompson's transformation to work on grammars, or by using other techniques (dotted automata)

Conclusions

- *Lexics* is a Regular Language
- *Regular Languages* is a subset $\Re \subseteq 2^{\Sigma^*}$ of languages on Σ:
    - let $F = \{I \subseteq \Sigma^* \mid \#I < n$, for some natural $n\}$ be the set of all finite languages on Σ:

      $\Re$ can be obtained from F by finite *unions* and *concatenations*, and *Kleene's star*

      $$\Re = F_{\{\cup, X_\bullet, *\}}$$
- *FSA* furnishes linear analyzers for $\Re$

Basic Properties (1)

Let L1 and L2 be Regular.

Union: is $L1 \cup L2$ Regular? yes

Product: is $L1 \times L2$ Regular? yes

Intersection: is $L1 \cap L2$ Regular? yes

Complement: is C(L1) Regular? yes

Decidability: Exists g computable such that g(L)=yes iff L is Regular?

NO

Basic Properties (2)

Let **L** be Regular, and

$A=<S,\sum,m,s_0,F>$ be $L(A)=L$

Th.(iteration: --- Pumping Lemma --- $\forall$ L, $\exists$ #S$\in\aleph$)

If $x\in L$, $|x|>\#S$,

then: $\exists u,w,v$ such that $0<|w|\leq\#S$, $x=u.w.v$,

$u.w^k.v \in L$, *for each natural k*

Corollary: --- $\forall$ L, $\exists$ #S

If $x\in L$, $|x|>\#S$,

then: $\exists u,w,v$ such $0<|uw|\leq\#S$, $x=u.w.v$,

$u.w^k.v \in L$, *for each natural k*

Apply it to prove that

$\boxed{\mathbf{L = \{a^n b^n \mid n \text{ natural}\}}}$

is not Regular

Exercises

Exercise: Application of the Iteration Theorem

Problem. Prove that the language $L = \{a^n b^n \mid n \in \aleph, a,b \in \Sigma\}$, for given Σ, is not a regular language

Proof. (by contradiction, using pumping lemma)

- Assume L be regular.
- Then, the pumping lemma applies to L. Let m be the characteristic constant #S, of L, mentioned in the lemma.
- Then, let $x = a^m b^m$:
  - $x \in L$, since $m \in \aleph$
  - $|x| = 2m > m$, since $m>0$
  - hence, $\exists$ u,w, v:
    - $x = uwv$
    - $|uw| \leq m$ & $|w| \neq 0$:
    - hence, $\exists$ m1,m2,m3:
      - $m2 \neq 0$ & $m=m1+m2+m3$ & $w=a^{m2}$ & $x= a^{m1} a^{m2} a^{m3} b^m$
      - hence:
        - $a^{m1} a^{m3} b^m \in L$ according Lemma since $k*m2=0$ for $k=0$
        - $a^{m1} a^{m3} b^m \notin L$ by definition of L since $m1+m3 \neq m1+m2+m3$ when $m2 \neq 0$

Exercises

Consider the lexics L of the numerals for integer and fixed-point numbers in decimal notation and arbitrary number of digits.

1. Give separate regular expressions for: integers, S, fixed point integers, F, the union of the two ones.
2. Give a regular grammar for lexics L.
3. Give an automaton for lexics L.
4. Give a deterministic automaton for L.
5. Give deterministic recognizer Y for lexics L
6. Modify Y for recognizing word sequences of L, separated by any character not in {0,..,9,'.'}. The new recognizer generates a sequence of <pi,li> where pi=position, li=length of the i-th recognized word. (words not in L are ignored)
7. Modify Y so that it recognizes the first, longer (7.1- shorter) word of the lexics that occur in a string on an alphabet containing {0,..,9,'.'}.

Exercise - 1

1. Give separate regular expressions for: integers, S, fixed point integers, F, the union of the two ones.

S = digit digit*
F = digit*.digit digit*
Digit = 0|1|…|9
...

Exercise - 2

2. Give a regular grammar for lexics L.

S = digit digit*
F = digit*.digit digit*
Digit = 0|1|…|9

It is Regular since: digit < S,F

or

F = digit*.S
S = digit digit*
Digit = 0|1|…|9

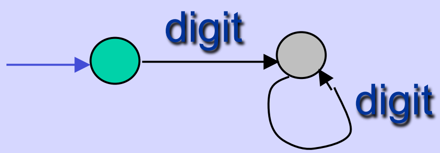
It is Regular since: digit < S < F

Exercise - 3

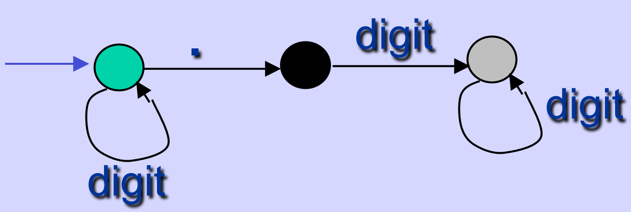
2. Give an automaton for lexics L.

Remark. We simplify the construction by using *digit* as a character, thus avoiding the use of 10 distinct edges for each...

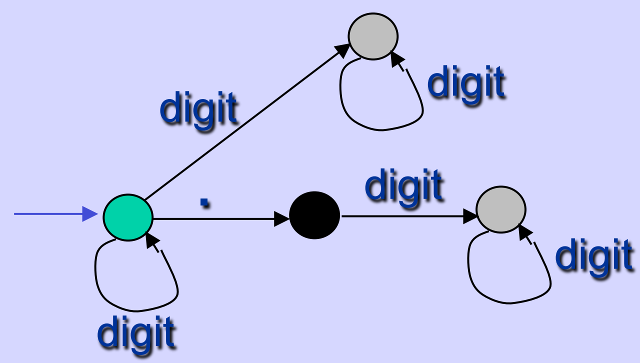
S = digit digit*



F = digit*. digit digit*



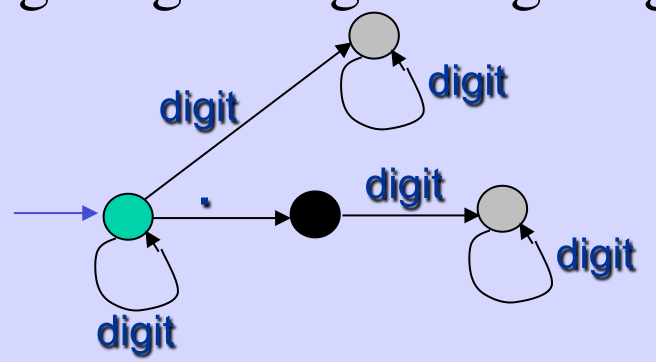
digit digit*| digit*. digit digit*



Exercise - 4

4. Give a deterministic automaton for L.

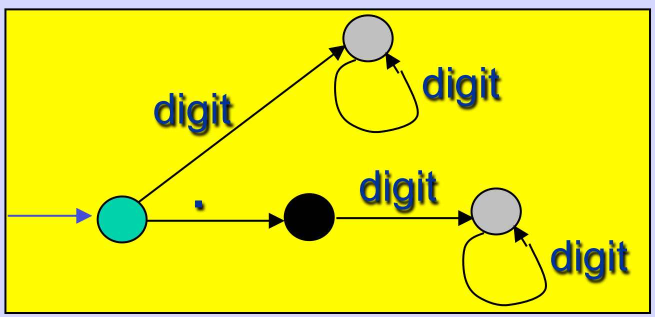
digit digit*| digit*. digit digit*



	digit	.
0	{0,1}	{2}
1	{1}	
2	{3}	
3	{3}	

Move*

	digit	.
{0}	{0,1}	{2}
{0,1}	{0,1}	{2}
{2}	{3}	
{3}	{3}	



Exercise - 5

5. Give deterministic recognizer Y for lexics L

Answer ***StarDriver***()
{s= s_0;
nextchar(c);
while(c≠**eof**) **and** (s≠{}) {
s=Star[s]c;
nextchar(c);}
if ((s∉F) **or** (c≠**eof**)) answer='noaccept'
else answer='accept';
return (answer);}

Tabella di analisi: **Star**

	digit	.
{0}	{0,1}	{2}
{0,1}	{0,1}	{2}
{2}	{3}	
{3}	{3}	

Exercise - 6

6. Modify Y for recognizing word sequences of L, separated by any character not in {0,..,9,' .' }. The new recognizer generates a sequence of <pi,li> where pi=position, li=length of the i-th recognized word. (words not in L are ignored)

```
Answer StarDriver()
{input=0;
  while(nextchar(c) != eof){
     length=0; cur=input;
     s= s0;
     while(c ∈Σ && s≠{}) {
              s=Star[s]c;
              nextchar(c); //incr. anche input
              length++;}
     if (s∈F) answer= answer  ++ <cur,length>;}
return (answer);}
```

Tabella di analisi: **Star**

	digit	.
{0}	{0,1}	{2}
{0,1}	{0,1}	{2}
{2}	{3}	
{3}	{3}	