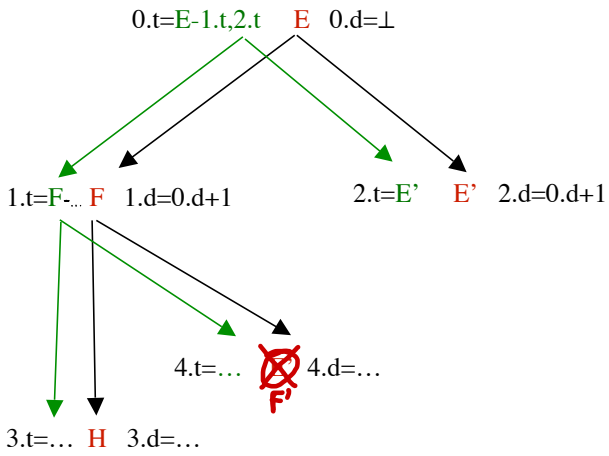
Apply Attribute Grammars to Parse Trees: An Exercise

Let *mk-tree* be an arity-variant procedure that applies to a *label* and to n *trees* and results the tree rooted on a node having, as label, the first argument, and, as sons, the n trees, in the order of appearance from left. Say: 1) the type of the attributes; 2) the family $L_{\{ai\}}$; 3) the value of each attribute relating to 2+3.

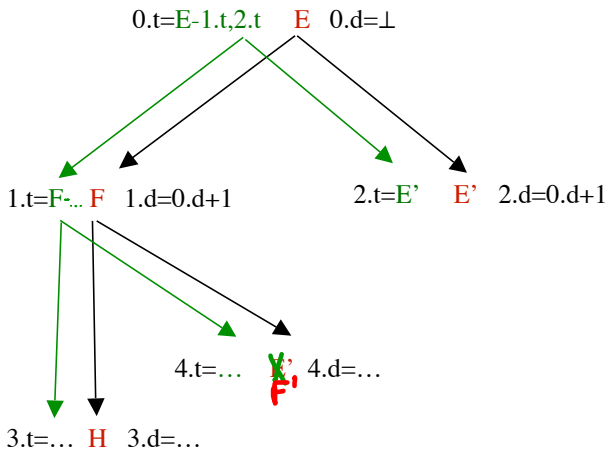
E::FE' [0] E'::=+E [1] | [2] ε
F::=HF' [3] F'::=*F [4] | [5] ε
H::=num [6] | [7] (E)



E::= F E' [0]	**E.tree:= mk-tree('E', F.tree, E'.tree), F.depth:=E.depth+1, E'.depth:=E.depth+1**
E'::= + E [1]	**E'.tree:= mk-tree('E'', mk-leaf('+'), E.tree), +.depth:=E'.depth+1,E.depth:=E'.depth+1**
E'::= ε [2]	**E'.tree:= mk-tree('E'', mk-leaf('ε')) ε.depth:=E'.depth+1**
F::= H F' [3]	**F.tree:= mk-tree('F', H.tree, F'.tree), H.depth:=F.depth+1, F'.depth:=F.depth+1**
F'::= * F [4]	**F'.tree:= mk-tree('F'', mk-leaf('*'), F.tree), *.depth:=F'.depth+1, F.depth:=F'.depth+1**
F'::= ε [5]	**F'.tree:= mk-tree('F'', mk-leaf('ε')) ε.depth:=F'.depth+1**
H::= num [6]	**H.tree:= mk-tree('H', mk-leaf(num)), num.depth:=H.depth+1**
H::= (E) [7]	**H.tree:= mk-tree('H', mk-leaf('('), E.tree, mk-leaf(')')), E.depth:= H.depth+1, (.depth:=H.depth+1,).depth:=H.depth+1,**

1

Two Kinds of Attributes: Syntesized - Inherited



Node 1 occurs, in the tree, in 2 different way:

- **left side grammatical of**
  E::= F E'
- **right side grammatical of**
  F::= H F'

Hence, attributes of node 1 can be defined in actions of:
2 different attribute productions:

This is the case of our grammar:

F.depth *is defined* in E::= F E'
F.tree *is defined* in F::= H F'

Attribute:

F.depth *is called* Inherited
F.tree *is called* Syntesized

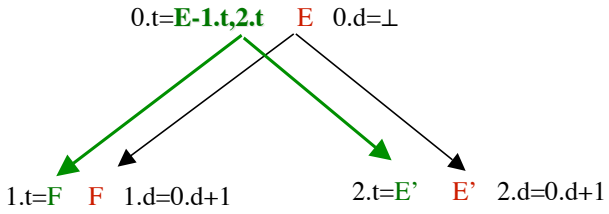
Syntesized Attributes

Let $G^A \equiv \{\sum, V, s, P^A, \{a_i\}\}$ be an attribute grammar.
Let $p \equiv B := \beta \{\alpha\} \in P^A$.
Let X.a be an attribute occurring in $\{\alpha\}$. Then
X.a is a **synthesized attribute** if and only if one the two:

- $\exists$ X.a=e $\in \{\alpha\}$ and $X \equiv B$
- $\exists$ $X_i.a_{ij} = e_{ij} \in \{\alpha\}$ and X.a $\in$ Var(e_{ij}) and X $\in$ Sym(β)

[X.a = e ∉ {α}]

where: Sym(β) is the set of grammatical symbols in β
Var(e) is the set of attributes occurring in e



E::= F E' { E.tree:= mk-tree('E', F.tree, E'.tree)…}

A Pragmatic View:

- **Attribute of the node only depends from attributes of the sons**
- **It expresses Compositional Properties**

Let $G^A \equiv \{\sum, V, s, P^A, \{a_i\}\}$ be an attribute grammar.
Let X be a grammatical. Then
A-Syn(X) is the set of all Syntesized attribute of X in G^A

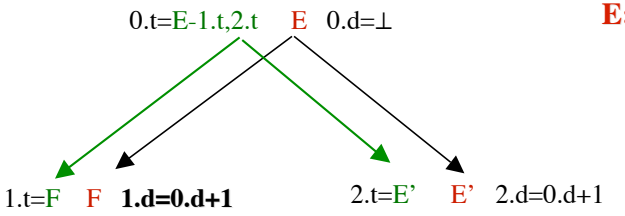
Inherited Attributes

Let $G^A \equiv \{\Sigma, V, s, P^A, \{a_i\}\}$ be an attribute grammar.
Let $p \equiv B := \beta \ \{\alpha\} \in P^A$.
Let X.a be an attribute occurring in $\{\alpha\}$. Then
X.a is an **inherited attribute** if and only if one the two

- $\exists$ X.a=e $\in \{\alpha\}$ and $X \equiv Sym(\beta)$
- $\exists$ $X_i.a_{ij}=e_{ij} \in \{\alpha\}$ and X.a $\in Var(e_{ij})$ and $X \equiv B$

[X.a = a ∉ {α}]

where: Sym(β) is the set of grammatical symbols in β
Var(e) is the set of attributes occurring in e



E::= F E' {F.depth:=E.depth+1)…}

A Pragmatic View:

- **Attribute of the node only depends from attributes of the context (father, brothers)**
- **It expresses Contextual Properties**

Let $G^A \equiv \{\Sigma, V, s, P^A, \{a_i\}\}$ be an attribute grammar.
Let X be a grammatical. Then
A-Inh(X) is the set of all Inherited attribute of X in G^A

Applications of the Attribute Grammars

- **Power: Context Sensitives and Attribute Grammars**
- **Attribute Evaluation: Three Execution Methods**
- **Oblivious and L-Attributed Grammars**
- **Bottom-up Executors for S-Attributed**
- **Top-down Executors for L-Attributed**
- **Bottom-up: Transformations for L-Attributed**

Attribute grammars are greatly powerful

because of the combination with a *meta* that can be a programming language

Consider the language L_2 on the right side. $L_2 \notin$ CF, and a Context Sensitive grammar for L_2 is shown.

$L_2=\{u^n v^n z^n \mid n \geq 0\}$

S::=A E
A::=u A v B | e
B v::= v B
B E::= z
B z::= z z

- **Such a grammar is difficult to write and even worse to analyze**
- **Context Sensitive Analyzers are complicated to build and impractical to use**
- **Attribute Grammars can be profitably used**

Using an LL Attribute Grammar for Analyzing $u^n v^n z^n$

- Select a language $L_1 \in$ LL(1) including the language we are interested in:

  $u^n v^n z^n \subset L_1 = \{u^n v^m z^n \mid n,m \geq 0\}$

- Let G be a LL(1) grammar for L_1

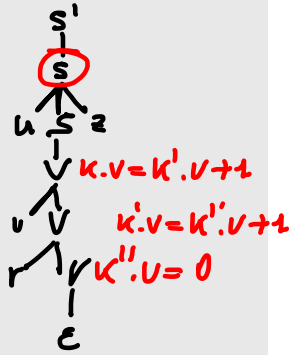
  S' ::= S
  S ::= u S z | V
  V ::= v V | ε

  S ::= u S | V
  V ::= v V | z
  z ::= z z | ε

- Extend G into an Attribute Grammar that computes an attribute of S' to true if and only if the analyzed string belongs to L(G), hence has form $u^n v^m z^k$ and n=m=k.

  S' ::= S **{S.r=(S.u==S.v)&(S.u==S.z)}**
  S_1 ::= u S_2 z **{S_1.u=S.u+1; S_1.v=S_2.v; S_1.z=S_2.z+1}**
  S ::= V **{S.u=0; S.v=V.v; S.z=0}**
  V_1 ::= v V_2 **{V_1.v=V_2.v+1}**
  V ::= ε **{V.v=0}**

The computation of S.z is required if the free grammar is intended, such.

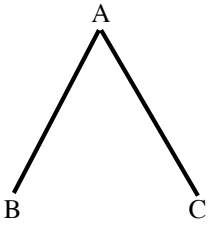


Attribute Evaluation

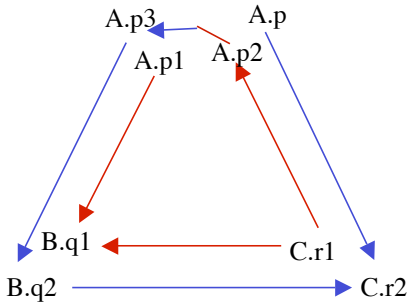
Three different evaluation tools

1) **Parse Tree**:

- Construct the **Parse Tree**, T
- Construct the **Dependency Graph**, T_d of T
- Find, if any, a **Topological Sort** M_{Td} for T_d
- Visit T_d according to M_{Td} and Execute the actions associate to the nodes



T



T_d

A.p, A.p1, C.r1
B.q1, A.p2
A.p3,
B.q2
C.r2

M_{Td}

Node	ActionAttribute
C	C.r1
A	A.p2 - A.p - A.p1
A	A.p3
B	B.q1 - B.q2
C	C.r2

Visit

-Only for Multi-Pass Parser/Compiler
-Method applies at Compile Time

Attribute Evaluation

Three different evaluation tools - 2

2) **Rule-based**:

- For each production:
  - Analyze the meaning of the actions occurring in it
  - State a **proper execution order** for the actions
- Combine such an order with the Parse-Tree constructor:
  - Only one Code for Parse-Tree construction and Action execution
  - **Versus** Distinct, Correlated, Codes

Ad Hoc Construction: The resulting code is hard to modify

- Also for one-Pass Parser/Compiler
- Method applies at Compile Construction Time

Attribute Evaluation

Three different evaluation tools - 3

3) **Oblivious**:

- The **execution order** for the actions is established according to:
  - **same criteria** for all propductions
  - criteria that **ignore the meaning** of the actions
  - but are adequate for **executing actions in the correct way**
- Action Execution is combined with Parsing:
  - **Top-Down Executors**
  - **Bottom-Up Executors**
- **Parser Generators** are extended to **Attribute Grammar Evaluators**

- Only for one-Pass Parser/Compiler
- Method applies at Compile Constrution Time

Attribute Evaluation

Parser Generators as Attribute Grammar Oblivious Evaluators

- **How can Parsing and Action Evaluation be combined ?**
  - At each derivation/reduction, the production actions are evaluated

- **When actions are evaluated in this way, what part of the Parse-Tree has already been traversed and then, known to the actions?**
  - The nodes of a **Depth-First** visit of the Parse-Tree up to the current input:
    + **Top-Down: Preorder Depth First**
    + **Bottom-up: Postorder Depth First**

- Parser Generators can be extended into Oblivious Evaluators of a attribute grammar G if:

Depth-First visit is a Topological Sort of the Dependency Graph of G

L-Attributed Grammars

L-Attributed Grammars is **a class** of Attributed Grammars (or SDD) that has **Depth-First** as a **Topological Sort** of the **Dependency Graph** of the **Parse-Tree attributes** of the grammar.

Let $G^A \equiv \{\sum, V, s, P^A, \{a_i\}\}$ be an attribute grammar.
Let $p \equiv B := B_1 \ldots B_n \{\alpha\} \in P^A$.
G^A is L-attributed if and only if:
$\forall X_i.a_{ij} = e_{ij} \in \{\alpha\}$ for $X_i \in Sym(B_1 \ldots B_n)$:
if $X_k.a_{ik} \in Var(e_{ij})$ then:
- either $X_i \equiv B_{hi}$, $X_k \equiv B_{hk}$ and $1 \le h_k \le h_i \le n$
- or $X_k \equiv B$ and $a_{ik} \in$ A-Inh(B)

- **S-attributed Grammars** are containing only synthesized attributes
- S-attributed are L-attributed.

Theorem. If G has Top-Down/Bottom-up Parser and G^A is L-attribued then **G^A has Top-Down/Bottom-up oblivious evaluator**

2