

# SLR Parsing

## The Table M (Action-Goto) for SLR

Let  $G = \langle S, V, \Pi, s \rangle$  and  $I$  be the set of LR(0) Collection of  $G$ .

Table  $M$  (also, called, Action-Goto) consists of two Tables:

**Action:** is  $|I|$ -rows by  $|V \cup \{\$\}|$ -columns

**Goto:** is  $|I|$ -rows by  $|S|$ -columns

For each state, of  $I$ :

**Action:** The operation to apply (shift or reduce)

**Goto:** The state to go after reduction

The definition of  $M$  requires the computation of:

- LR(0) Collection
- Follow( $s$ ), for each  $s \in S$

# SLR Parsing

## The Table Action for SLR(1)

Component, Action, of table M for SLR(1)

**ACTION(i,a)=s/j**

if goto(I<sub>i</sub>,a)=I<sub>j</sub> and a ∈ Σ

**ACTION(i,a)= r/p**

if A ::= α . ∈ I<sub>i</sub> and p ≡ A ::= α and  
a ∈ follow(A)

**ACTION(i,\$)= <accept,->**

if S' ::= S . ∈ I<sub>i</sub>

Where: s/j = shortening for Shift(I<sub>j</sub>);  
r/p = shortening for Reduce(p)

# SLR Parsing

## The Table Goto for SLR(1)

Let  $G = \langle S, V, \Pi, s \rangle$

**GOTO(i,A)=j**  
if goto(Ii,A)=Ij and  $A \in S$

# Example

## Table Action-Goto SLR(1)

	a	b	c	d	e	\$
0	S/2					
1						ACC
2		S/4				
3		S/6		S/7		
4		R/2		R/2		
5					S/8	
6			S/9			
7					R/3	
8						R/0
9		R/1		R/1		

	S	A	B
0	1		
1			
2		3	
3			5
4			
5			
6			
7			
8			
9			

### LR(0) Collection

- I0: Closure( $\{S' ::= S\}$ ) =  $\{S' ::= S, S ::= aABe\}$
- I1: Goto(I0, S) =  $\{S' ::= S.\}$
- I2: Goto(I0, a) =  $\{S ::= a.Abe, A ::= .Abc, A ::= .b\}$
- I3: Goto(I2, A) =  $\{S ::= aA.Be, A ::= A.bc, B ::= .d\}$
- I4: Goto(I2, b) =  $\{A ::= b.\}$
- I5: Goto(I3, B) =  $\{S ::= aAB.e\}$
- I6: Goto(I3, b) =  $\{A ::= Ab.c\}$
- I7: Goto(I3, d) =  $\{B ::= d.\}$
- I8: Goto(I5, e) =  $\{S ::= aABe.\}$
- I9: Goto(I6, c) =  $\{A ::= Abc.\}$

### Augmented G'

- $S' ::= S$
- 0  $S ::= aABe$
- 1|2  $A ::= Abc | b$
- 3  $B ::= d$

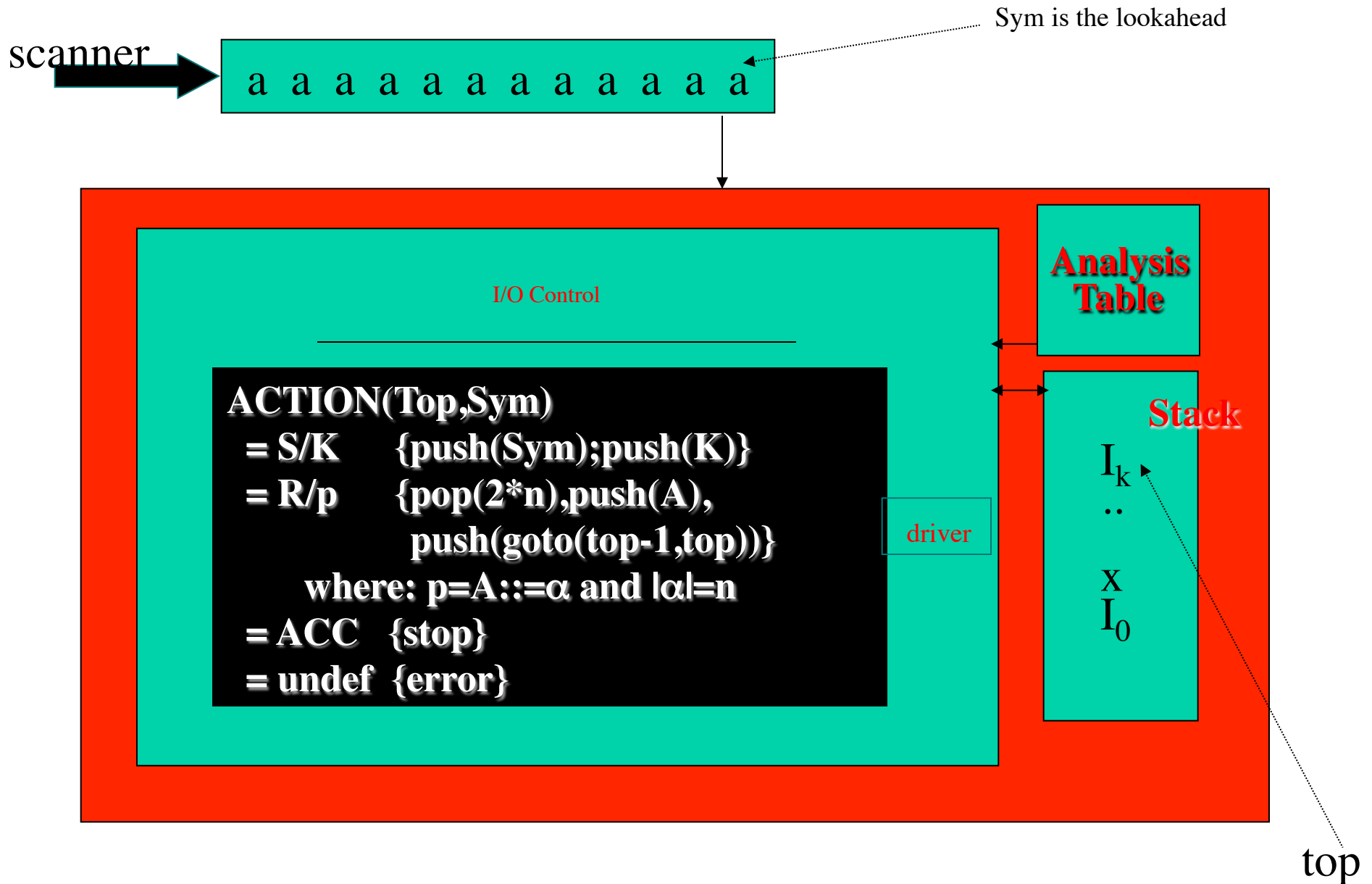
### Source Grammar

- $S' ::= S$
- 0  $S ::= aABe$
- 1|2  $A ::= Abc | b$
- 3  $B ::= d$

### Table of Follow

- follow(S) =  $\{\$ \}$
- follow(A) =  $\{b, d\}$
- follow(B) =  $\{e\}$

# DRIVER: shift-reduce



# SLR(1) Parsing

## Properties of Applicability

**Prop1: (No shift-reduce)**

$(\forall I_k \in \text{Coll}(0)), (\forall A ::= \alpha.a\gamma, B ::= \beta. \in I_k)$   
 $a \notin \text{follow}(B)$

**Prop2: (No reduce-reduce)**

$(\forall I_k \in \text{Coll}(0)), (\forall A ::= \alpha., B ::= \beta. \in I_k)$   
 $\text{follow}(A) \cap \text{follow}(B) = \{\}$

**Theorem. Let  $G$  be a grammar.**

- $G \in \text{SLR}(1)$  if and only if both Prop1 and Prop2 hold.
- $G$  has shift-reduce, 1 symbol of lookahead, SLR parser if and only if both Prop1 and Prop2 hold.

# SLR

## Concluding Remarks

1. In what sense is "bottom-up" non predictive ?

Bottom-up decides the derivation on the basis of the maximum VP to the handle and this VP entirely, includes the string to derive.

2. Let  $G$  be a SLR(1) grammar.

- Can its ACTION contain more than one action per entry ?
- Can its GOTO contain more than one state for a same entry ?

3. What means that  $G \in \text{SLR}(1)$  has Table:  
ACTION such that: for each  $I$ , for each pair  $a \neq b$ ,  
 $\text{ACTION}(I,a) = \text{ACTION}(I,b)$   
when  $\text{ACTION}(I,a) \neq \perp \neq \text{ACTION}(I,b)$  ?

To answer 3, let us consider the **SLR(0)** grammar below:

```
E ::= E+T | T
T ::= num
```

# SLR

## Concluding Remarks - 2

4. Is  $G \in \text{SLR}(1)$  decidable for all Context Free grammars ?

5. Is  $L \in \text{SLR}(1)$  decidable for all Context free languages ?

Recall that:  $L \in \text{SLR}(1) \equiv \exists G \in \text{LL}(1): L(G) = L$

6. Is the class of  $\text{SLR}(1)$  grammars including that of  $\text{LL}(1)$  grammars ?

To answer 6, let us consider the grammar below:

$S ::= Bc \mid b \mid A$

$A ::= aBb$

$B ::= \epsilon$



# SLR

## Concluding Remarks - 3

7. Is the class of SLR(1) languages, including that of LL(1) languages ?

To answer 7, let us consider the role of non-terminal B:

$S ::= Bc \mid b \mid A$

$A ::= aBb$

$B ::= \epsilon$

8. Consider the condition C:

$C \equiv A ::= \alpha \cdot \in I_k \in \text{Coll}(0) \text{ iff } A ::= \alpha \text{ is the handle for all } a \in \text{follow}(A).$

C states a necessary and sufficient for reduction in SLR(1) Grammars.

- Is C again necessary and sufficient for reduction in bottom-up (non SLR grammars) ?
- Is C, only a necessary, condition for reduction in bottom-up (non SLR grammars) ?
- Is C, only a sufficient, condition for reduction in bottom-up (non SLR grammars) ?

To answer to 8, let us consider the grammar above and the existence of a rightmost derivation for string b. Such a derivation (in reverse order) is  $b \leq S$ , but SLR(1) fails in doing it. Why?

# Bottom-Up Analysis:

$$First(\beta) \in Follow(A)$$

Let:  $S \Rightarrow_r^* \gamma\alpha\beta$  for

$\gamma\alpha \in$  viable prefix - is maximum w.p.  
 $First(\beta) \in follow(A)$   
 $A ::= \alpha$  is Valid Item for  $\gamma\alpha$



Can you conclude that  
 $A ::= \alpha$  is the handle of  $\gamma\alpha\beta$  ?

Use this grammar for a proof

$S ::= Bc \mid b \mid A$   
 $A ::= aBb$   
 $B ::= \epsilon$

$First(\beta) \in follow(A)$  is not a sufficient condition

**Proposition.** (Inclusion is not a sufficient condition)

$\forall G, \forall \gamma\alpha \in VP_G$  with Valid Item  $A \rightarrow \alpha$ .

$First(\beta) \in follow(A)$  does not imply that  $A ::= \alpha$  is the handle of  $\gamma\alpha\beta$

Use this grammar for a proof: (proof)

Let  $\gamma\alpha\beta = b$ . Then,  $S \Rightarrow b$

$\gamma\alpha = \lambda \in VP$  is maximum prefix;  $First(\beta) \in Follow(B)$ ;  $B ::= \epsilon$  is valid for  $\gamma\alpha$  BUT  $B ::= \epsilon$  is not the handle of  $b$

# A New Class of More Powerful Items

Let  $X ::= \alpha \cdot A \beta \in I_k$  and  $A ::= \gamma$  be a grammar (normal) production

Then

$A ::= \cdot \gamma \in I_k$  only for the symbols  
in the set  $U = \text{First}(\beta)$  (for  $B \neq \lambda$ )

$\text{First}(\beta \text{ look}(X ::= \alpha A \beta))$

Pairing each LR(0) item with the set  $U$  of all the symbols that can follow the handle string associated to the item

$A ::= \cdot \gamma, U \in I_k$

Item LR(1)

$U$  is called  $\text{look}(A ::= \cdot \gamma)$

# Canonical collection Coll(1)

A New Class of Items that Remember Prefix-Follow Combinations

Let  $G = \langle S, V, \Pi, s \rangle$

$S' ::= \cdot S / \$ \in I_0$  by definition

$$\text{Clos}(I) =_{\min} I \cup \text{Clos}\{B ::= \cdot \gamma / U \mid A ::= \alpha \cdot B \beta / V \in \text{Clos}(I), B ::= \gamma \in \Pi, \\ U = \{\text{first}(\beta x) \mid x \in V\}\}$$

Let B a nonterminal

let  $u \in U$ , then:

if  $u \in V$ ,  $A ::= \alpha \cdot B \beta / V$  *propagates* on  $B ::= \cdot \gamma / U$

if  $u \notin V$ ,  $A ::= \alpha \cdot B \beta / V$  *spontaneously generates* on  $B ::= \cdot \gamma / U$

$$\text{Goto}(I, x) = \text{Closure}\{A ::= \alpha x \cdot \beta / U \mid A ::= \alpha \cdot x \beta / U \in I\}$$

# LR(1) Grammars

Prop1: (No shift-reduce)

$$(\forall I_k \in \text{Coll}(1)), (\forall A ::= \alpha.a\gamma/V, B ::= \beta./U \in I_k) \\ a \notin U$$

Prop2: (No reduce-reduce)

$$(\forall I_k \in \text{Coll}(1)), (\forall A ::= \alpha./V, B ::= \beta./U \in I_k) \\ V \cap U = \{\}$$

**Theorem. Let  $G$  be a grammar.**

- **$G \in \text{LR}(1)$  if and only if both Prop1 and Prop2 hold.**
- **$G$  has shift-reduce, 1 symbol of lookahead, LR parser if and only if both Prop1 and Prop2 hold.**

# LR Parsing

## The Table Action for LR(1)

**ACTION(i,a)=<shift,j>**

if goto(I<sub>i</sub>,a)=I<sub>j</sub> and a ∈ Σ

**ACTION(i,a)= <reduce,p>**

if A ::= α . / U ∈ I<sub>i</sub> and

p ≡ A ::= α and a ∈ U

**ACTION(i,\$)= <accept,->**

if S' ::= S . / \$ ∈ I<sub>i</sub>

**GOTO(i,A)=j**

if goto(I<sub>i</sub>,A)=I<sub>j</sub> and A ∈ N

**N=Nonterminal Set**

# Apply LR(1) to $G \notin \text{SLR}(1)$

I0: Closure( $\{S' ::= .S\}$ ) =  $\{S' ::= .S, \$$

$S ::= .Au, \$, S ::= .av, \$, S ::= .Bv, \$,$

$A ::= .a, u, B ::= .xA, v\}$

I1: Goto(I0, S) =  $\{S' ::= S., \$\}$

I2: Goto(I0, A) =  $\{S ::= A.u, \$\}$

I3: Goto(I0, a) =  $\{S ::= a.v, \$$

$A ::= a., u\}$  **No  
conflict**

I4: Goto(I0, B) =  $\{S ::= B.v, \$\}$

I5: Goto(I0, x) =  $\{S ::= x.A, v$

$A ::= .a, v\}$

I6: Goto(I2, u) =  $\{S ::= Au., \$\}$

I7: Goto(I3, v) =  $\{S ::= av., \$\}$

I8: Goto(I4, v) =  $\{S ::= Bv., \$\}$

I9: Goto(I5, A) =  $\{S ::= xA., v\}$

I10: Goto(I5, a) =  $\{A ::= a., v\}$

$S ::= Au \mid av \mid Bv$

$A ::= a$

$B ::= xA$

# Space Complexity

**Table Size**

**$G = \langle S, V, \Pi, s \rangle$**

<b>LL(1):</b>	$O(N * T)$	(with $N =  S $ , $T =  V $ )
<b>SLR:</b>	$O(N * P * T)$	(with $P =$ production right hand size)
<b>LR(1):</b>	$O(N * P * T * T)$	
<b>LALR(1):</b>	the same of SLR	