Esercizio 1

(a) Poniamo $k=1$ e verifichiamo che $\forall x = a^n b^m$ tale che $|x| \geq k$ $\exists\, uwz = x$ che soddisfa il lemma (iii)
Abbiamo due casi $\forall$ $|a^n b^m| \geq 1$:

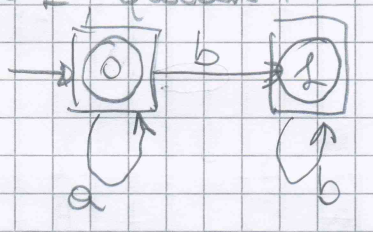
caso 1: $n > 0$ ovvero $n \geq k$
$a^n b^m = a^{k+h} b^m$ (per $n > 0 \wedge m, h \geq 0$)
Allora
$$u = a^{k_1},\quad w = a^{k_2},\quad z = a^{k_3+h} b^m$$
con $k_1 + k_2 + k_3 = 1 \wedge k_2 \neq 0$
$$\Downarrow$$
$$k_1 = k_3 = 0,\quad k_2 = 1$$

ed infatti, $\forall i \in \mathbb{N}$:
$$u w^i z = a^i a^h b^m \in \{a^n b^m \mid n, m \geq 0\}$$

caso 2: $n = 0$ ovvero $m \geq k$
$b^m = b^{k+h}$ (per $m > 0 \wedge h \geq 0$)
allora
$$u = b^{k_1},\quad w = b^{k_2},\quad z = b^{k_3+h}$$
ma $k_1 + k_2 + k_3 = 1 \wedge k_2 \neq 0 \Rightarrow$
$\Rightarrow k_1 = 0 = k_3 \wedge k_2 = 1$

e verifichiamo il lemma, infatti:
$\forall i \in \mathbb{N}$, $u w^i z = b^i b^h \in \{a^n b^m \mid n, m \geq 0\}$

(b) la prova ci dice che l'automa ha cicli di lunghezza minima $k = 1$ quindi:



$\langle \{0,1\}, \{\langle\langle 0,a\rangle,0\rangle\ \langle\langle 0,b\rangle,1\rangle\ \langle\langle 1,b\rangle,1\rangle\}, 0, \{0,1\}\rangle$

Esercizio 2

(a) Sia $k>0$ la costante che soddisfa il pumping lemma sul linguaggio. Consideriamo la stringa $a^k b^m$ che appartiene al linguaggio essendo $k \leq m$, quindi:

$$a^k b^m \quad \text{con } k \leq m$$

scomponibile in:

$$u = a^{k_1} \quad w = a^{k_2}, \quad z = a^{k_3} b^m$$
$$\text{con } k_1 + k_2 + k_3 = k, \quad k_2 \neq 0$$

e in accordo al lemma:

$$\forall i \in \mathbb{N}, \quad a^{k_1} a^{i k_2} a^{k_3} b^m \in L$$

ma per $i = m$

$$a^{k_1} (a^{k_2})^m a^{k_3} b^m \notin L \quad \text{poiché } \underline{k_1 + m k_2 + k_3 > m}$$

(b) la grammatica è:

0 $S ::= AB$
1|2 $A ::= aAb \mid \varepsilon$
3|4 $B ::= bB \mid \varepsilon$

(c) produzioni per S: niente da verificare
produzioni per A: × $FS(aAb) \cap FS(\varepsilon) = \{a\} \cap \{\varepsilon\} = \{\}$
× $FS(aAb) \cap FW(A) = \{a\} \cap \{b\} = \{\}$
produzioni per B: − $FS(bB) \cap FS(\varepsilon) = \{b\} \cap \{\varepsilon\} = \{\}$
− $FS(bB) \cap FW(B) = \{b\} \cap \{\$\} = \{\}$

(d)

	FS	FW
AB	{a, b, $}	
aAb	{a}	
bB	{b}	
S		{$}
A		{b, $}
B		{$}

⟹

	a	b	$
S	0	0	0
A	1	2	2
B		3	4

(e)

$\begin{array}{c} \underset{\wedge}{a}bbb\$ \\ S \\ \$ \end{array} \xRightarrow{0} \begin{array}{c} \underset{\wedge}{a}bbb\$ \\ A \\ B \\ \$ \end{array} \xRightarrow{1} \begin{array}{c} \underset{\wedge}{\cancel{a}}bbb\$ \\ \cancel{a} \\ A \\ b \\ B \\ \$ \end{array} \xRightarrow{\text{match}} \begin{array}{c} a\underset{\wedge}{b}bb\$ \\ A \\ b \\ B \\ \$ \end{array} \xRightarrow{2} \begin{array}{c} a\underset{\wedge}{\cancel{b}}bb\$ \\ \cancel{b} \\ B \\ \$ \end{array} \xRightarrow{\text{match}}$

$\xRightarrow{\text{match}} \begin{array}{c} ab\underset{\wedge}{b}b\$ \\ B \\ \$ \end{array} \xRightarrow{3} \begin{array}{c} ab\underset{\wedge}{\cancel{b}}b\$ \\ \cancel{b} \\ B \\ \$ \end{array} \xRightarrow{\text{match}} \begin{array}{c} abb\underset{\wedge}{b}\$ \\ B \\ \$ \end{array} \xRightarrow{3} \begin{array}{c} abb\underset{\wedge}{\cancel{b}}\$ \\ \cancel{b} \\ B \\ \$ \end{array} \xRightarrow{\text{match}}$

$\xRightarrow{\text{match}} \begin{array}{c} abbb\underset{\wedge}{\$} \\ B \\ \$ \end{array} \xRightarrow{4} \begin{array}{c} abbb\underset{\wedge}{\$} \\ \$ \end{array}$ **SUCCESSO**

Esercizio 3

(a) - 1

$$S = \{a\} \times S + B$$
$$B = \{a\} \times B \times \{b\} + \{\lambda\}$$

- 2

$$S^0 = \{a\} \times \bot + \bot \equiv \bot$$
$$B^0 = \{a\} \times \bot \times \{b\} + \{\lambda\} \equiv \{\lambda\}$$

$$S^1 = \{a\} \times \bot + \lambda \equiv \{\lambda\}$$
$$B^1 = \{a\} \times \{\lambda\} \times \{b\} + \{\lambda\} \equiv \{ab, \lambda\}$$

$$S^2 = \{a\} \times \{\lambda\} + \{ab, \lambda\} \equiv \{a, ab, \lambda\}$$
$$B^2 = \{a\} \times \{ab, \lambda\} \times b + \{\lambda\} \equiv \{a^2b^2, ab, \lambda\}$$

$$S^3 = \{a\} \times \{a, ab, \lambda\} + \{a^2b^2, ab, \lambda\} \equiv \{a^2, a^2b, a, a^2b^2, ab, \lambda\}$$
$$B^3 = \{a\} \times \{a^2b^2, ab, \lambda\} \times \{b\} + \{\lambda\} \equiv \{a^3b^3, a^2b^2, ab, \lambda\}$$

- 3 : ipotesi induttiva

$$S^i = \{a^n b^m \mid 0 \leq m \leq n < i\}$$
$$B^i = \{a^n b^n \mid 0 \leq n \leq i\}$$

- 4 : Linguaggio è :

$$L(S) = \{a^n b^m \mid n \geq m \geq 0\}$$

(b) $I_0 = \text{Clos}(S' \to .S) = \{S' \to .S,\ S \to .aS,\ S \to .B,\ B \to .aBb,\ B \to .\}$

$I_1 = G(0, S) = \{S' \to S.\}$

$I_2 = G(0, a) = \{S \to a.S,\ B \to a.Bb,\ S \to .aS,\ S \to .B,\ B \to .aBb,\ B \to .\}$

$I_3 = G(0, B) = \{S \to B.\}$

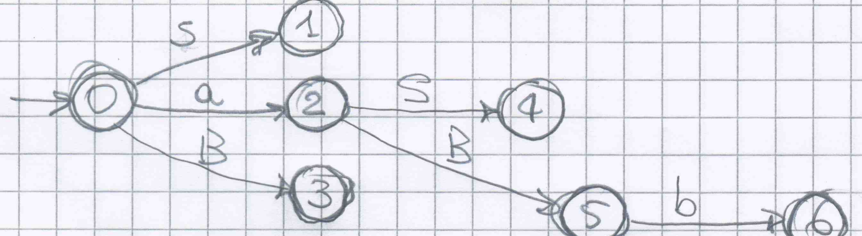
$I_4 = G(2, S) = \{S \to aS.\}$

$I_5 = G(2, B) = \{B \to aB.b,\ S \to B.\}$

$G(2, a) = I_2$

$I_6 = G(5, b) = \{B \to aBb.\}$

(c)



attraversiamo il nello stato $\underline{0}$