Ten Virtues of Structured Graphs

(or why structured graphs can be better than flat ones)

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also based on join work with F. Gadducci, A. Lluch-Lafuente, U. Montanari, E. Tuosto

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Outline

GT-VMT 2009

- Graph-based techniques
  - formal semantics, concurrency, logics, verification, tools
- Visual modelling
  - project planning, network management, traffic control, business processes, software architectures, www site design, and many more...
- Modern software and Sensoria project (service-oriented computing)
  - key issues such as scalability, representation distance, open-endedness, dynamicity, distribution
  - within specification, design, validation and verification phases

Sensoria Poster Collage (http://www.sensoria-ist.eu)

Software Engineering for Service-Oriented Overlay Computers www.sensoria-ist.eu

develops

semantically well-founded languages, novel theories, methods and tools for constructing and analysing the new generation of high-quality service-oriented systems

integrates

foundational theories, techniques, and methods with pragmatic software engineering

researches

- linguistic primitives for modelling and programming service-oriented systems
- qualitative and quantitative analysis methods for global services
- development and deployment techniques for systems services

offers

- model-driven approach for service-oriented software engineering
- modelling of service-oriented systems
- analysis of behaviour, security and quality of service properties
- suite of tools and techniques for
  - deploying service-oriented systems
  - reengineering legacy software into services

case studies

in automotive, finance, telecommunications and and e-learning domains

SIXTH FRAMEWORK PROGRAMME

Information Society Technologies

List of partners

Coordinator: Prof. Dr. Martin Wirsing, Ludwig-Maximilians-Universität München, Germany
Università di Trento | University of Leicester | Warsaw University | TU Denmark at Lyngby | Università di Pisa
Università di Firenze | Università di Bologna | ISTI Pisa | Universidade de Lisboa | University of Edinburgh | ATX S
Telecom Italia Lab | Imperial College London | FAST GmbH | Budapest University of Technology and Economics
S&N AG | University College London | Politecnico di Milano

All That Graphs

	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9
n_0	a_1		a_2			a_3		a_4		
n_1		a_5	a_6	a_7			a_8		a_9	a_{10}
n_2	a_{11}				a_{12}		a_{13}		a_{14}	a_{15}
n_3	a_{16}	a_{17}	a_{18}	a_{19}	a_{20}	a_{21}	a_{22}	a_{23}		a_{24}
n_4				a_{25}			a_{26}		a_{27}	a_{28}
n_5		a_{29}	a_{30}			a_{31}	a_{32}			a_{33}
n_6			a_{34}		a_{35}					
n_7					a_{36}	a_{37}			a_{38}	
n_8			a_{39}		a_{40}		a_{41}	a_{42}		a_{43}
n_9	a_{44}	a_{45}	a_{46}		a_{47}	a_{48}				

All That Graphs


```
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            <y:Shape type="roundrectangle"/>
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        <data key="d1"/>
      </node>
    - <node id="n1">
```
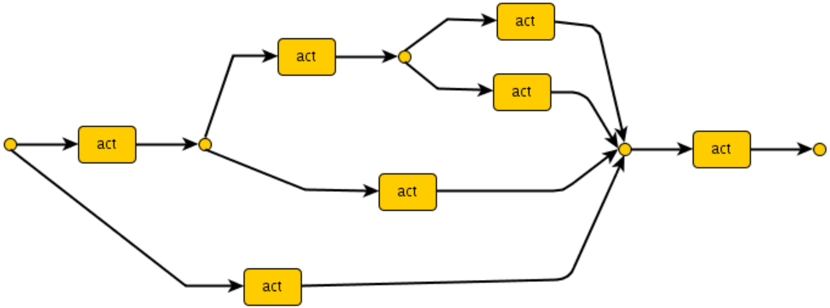

All That Graphs

Our choice



A Scenario: Software Architectures as Graphs

- D. Garlan & D. Perry, 1995
  - "*... the structure of the components of a program / system, their interrelationship, and principles and guidelines governing their design and evolution over time.*"
- components (and connectors) as hyper-edges
  - (here represented as boxes of various shapes)
- ports (and roles) as tentacles
  - (here represented as arrows)
- attachments as nodes
  - (here represented as smaller circles)
- connectors and attachments are sometimes omitted

Why "Spaghetti" Graphs are Considered Harmful

- When GT applied to large case studies, graphs better be structured in order to be comprehensible
- Analogies with structured programming and type theory
  - it is helpful to use graphs that are conveniently formatted and annotated
  - discard / ignore non-conformant graphs
- Analogies with process calculi
  - containment and links (as in bigraphs)
  - dynamics and reconfiguration via inductive, conditional rewrite rules

Our proposal

- From graphs to hierarchical hypergraphs
  - certain hyperdeges can contain hypergraphs that can be hierarchical themselves
  - arbitrary depth of nesting
- ADR (Architectural Design Rewriting)
  - graphs + their blueprint (like binaries + source templates)
  - exploit blueprint for applying formal methods
  - please visit `http://www.albertolluch.com/research/adr` to know more

Outline

Introduction

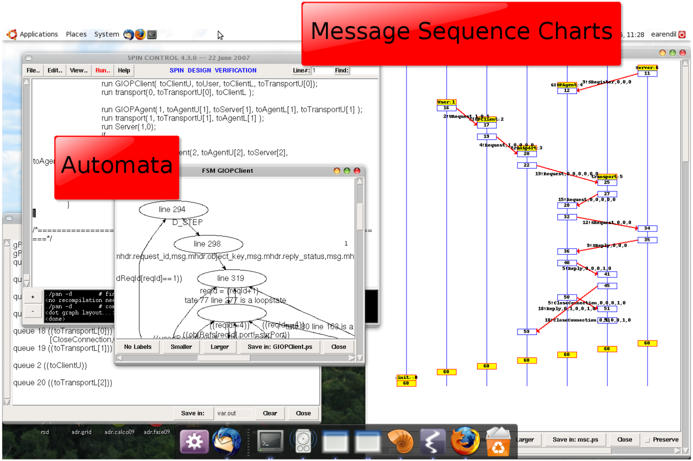
Styles for Visual Support

Dynamics

ADR

Concluding Remarks

Visualization can Support Formal Methods

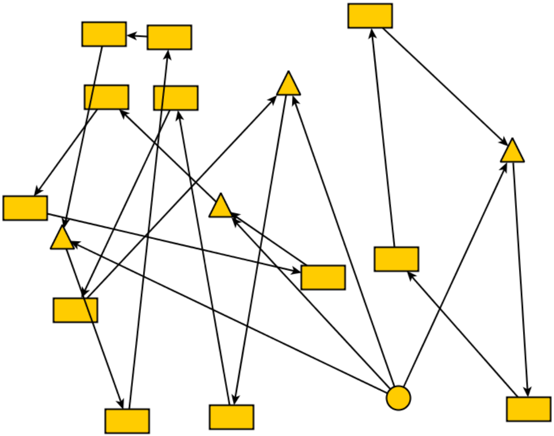


Architectural Styles

- IEEE standard 1471
  - *"... a set of patterns or rules for creating one or more architectures in a consistent fashion."*
- Style = Vocabulary + Rules
  - Used to construct and document
  - Used to describe / explain
  - Used to understand
  - Used to validate
  - Used for conformance check
  - Used to reason about
  - To be reused

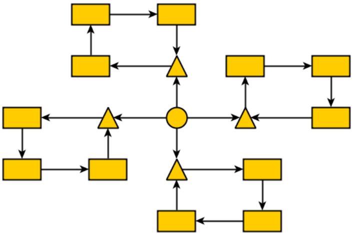
Uhm...

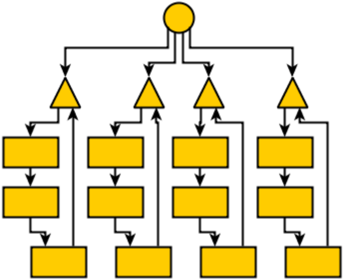
Can you spot some "regularity"?



Graph Re-drawing

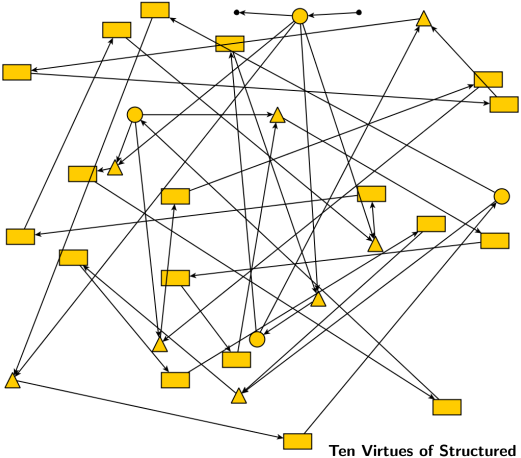
And now?





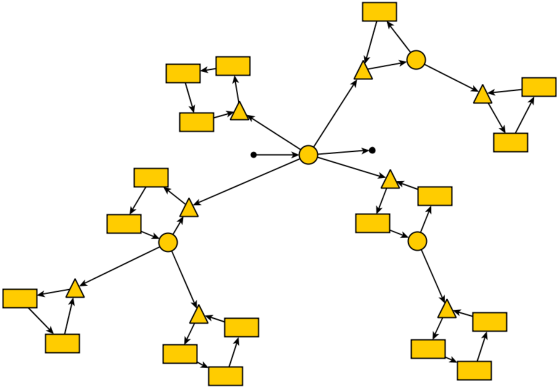
Well...

Another try?



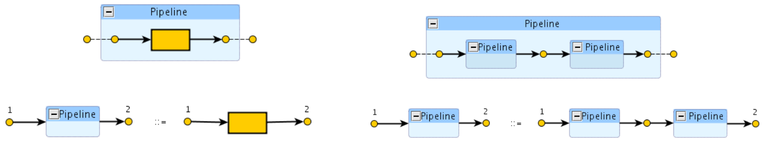
Another Graph Re-drawing

Can you describe its “shape” (or style)?



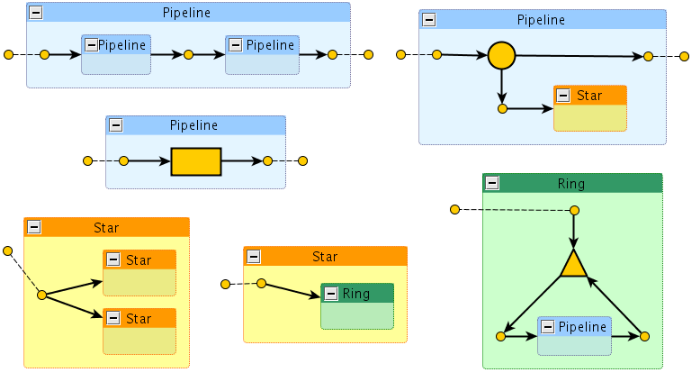
Styles from Productions

- Legenda: titled boxes as non-terminals, ordinary boxes as terminals

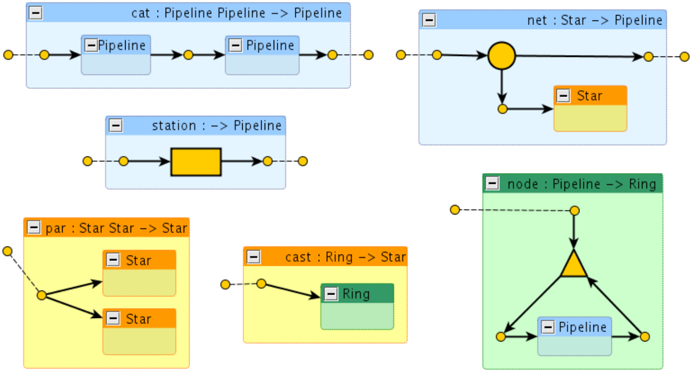


- Several readings are possible:
  - Refinement
  - Types (`Pipeline`) and ops (*station* and $cat(\cdot)$, based on hyperedge replacement)
    - $\mathit{station} :\longrightarrow \texttt{Pipeline}$
    - $\mathit{cat} : \texttt{Pipeline} \times \texttt{Pipeline} \longrightarrow \texttt{Pipeline}$
  - Abstraction

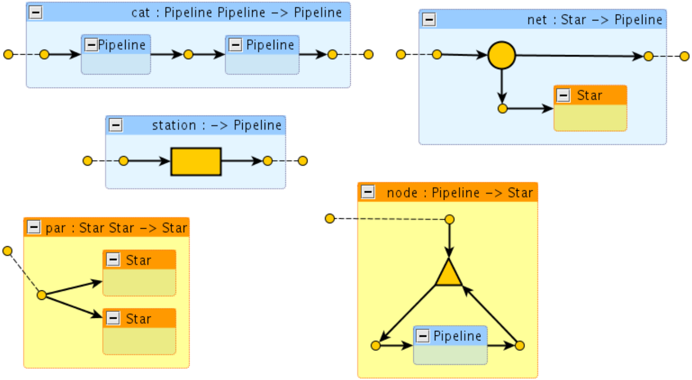
Types for Pipelines, Rings and Stars



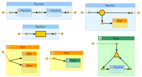
Types and Ops for Pipelines, Rings and Stars

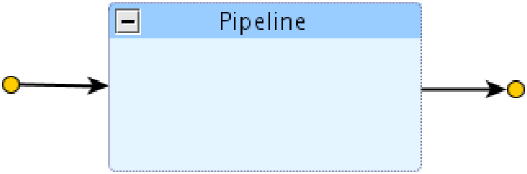


Simplified Memberships (for Pipelines and Stars)



An Example of Derivation (with “Blueprint”)





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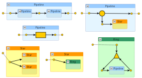

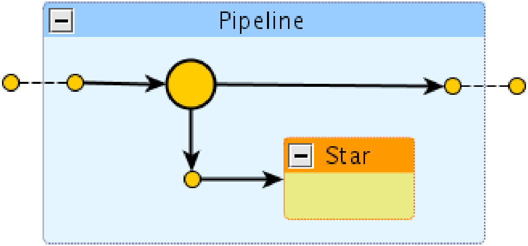

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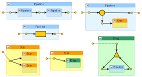

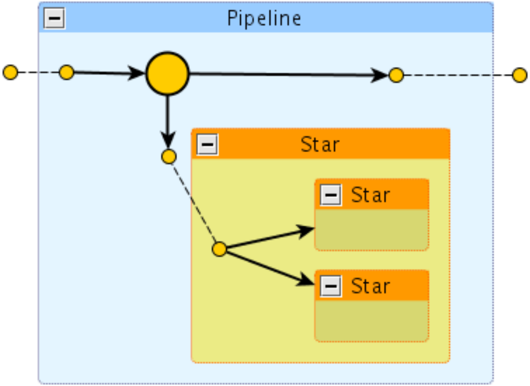

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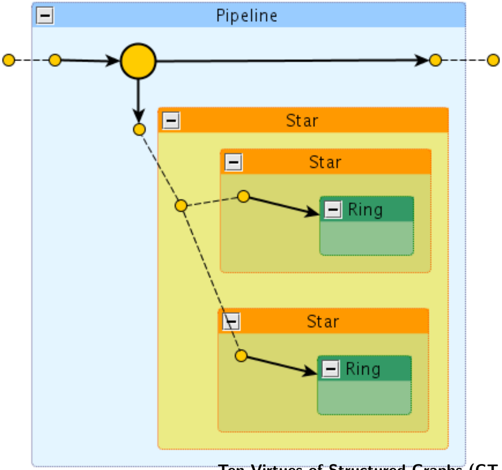

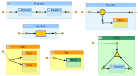

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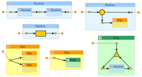

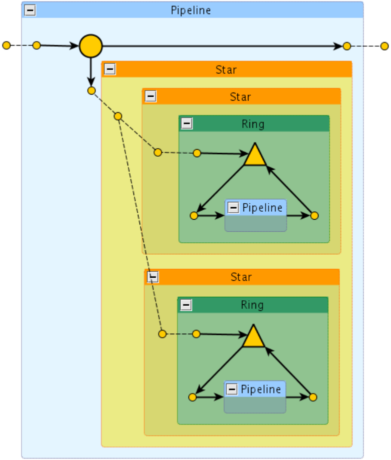

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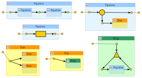

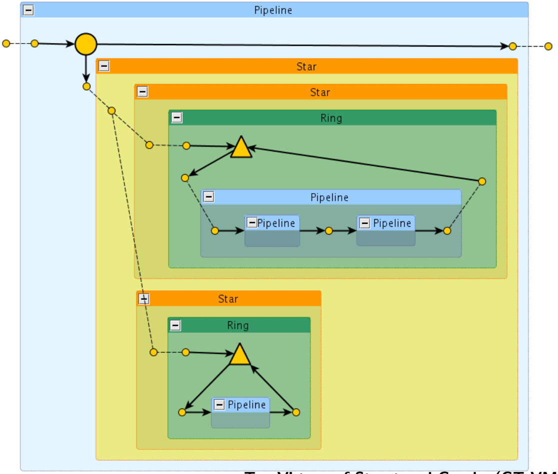

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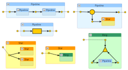

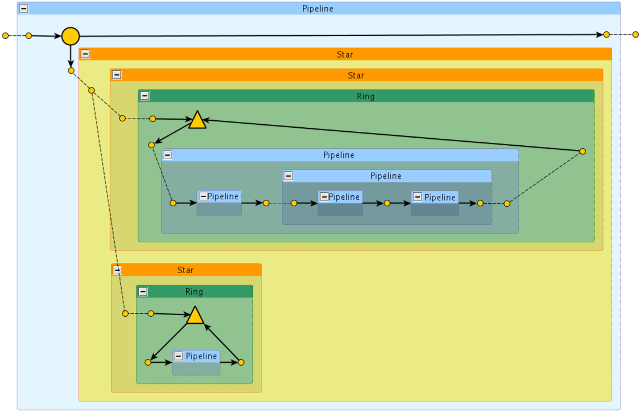

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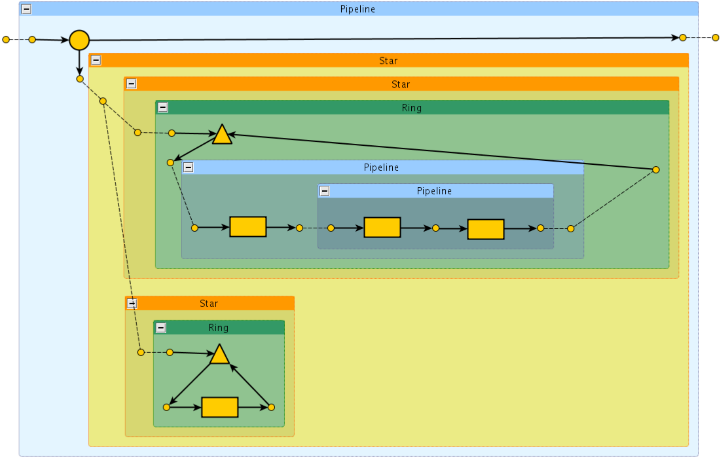
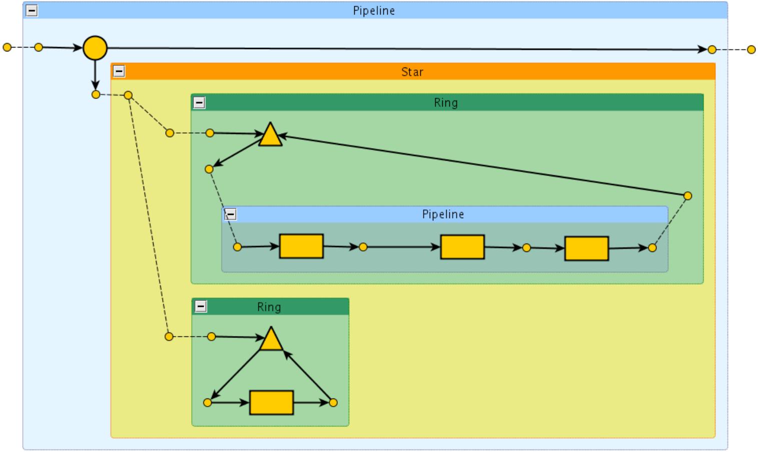


Simplified Typing and Drawing (“Flattening”)



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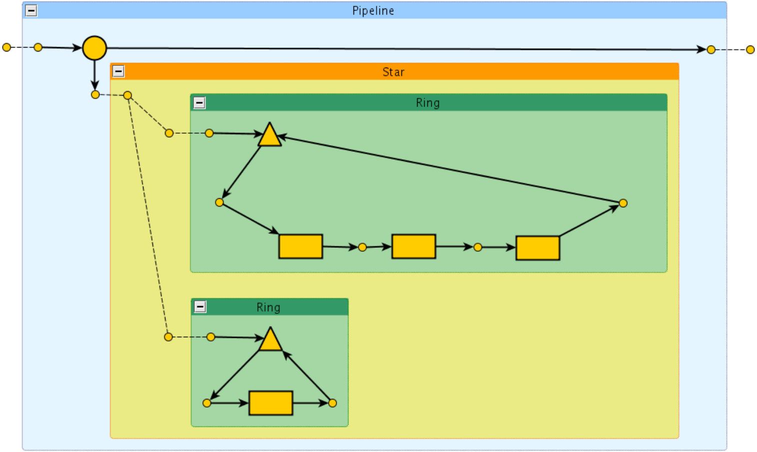

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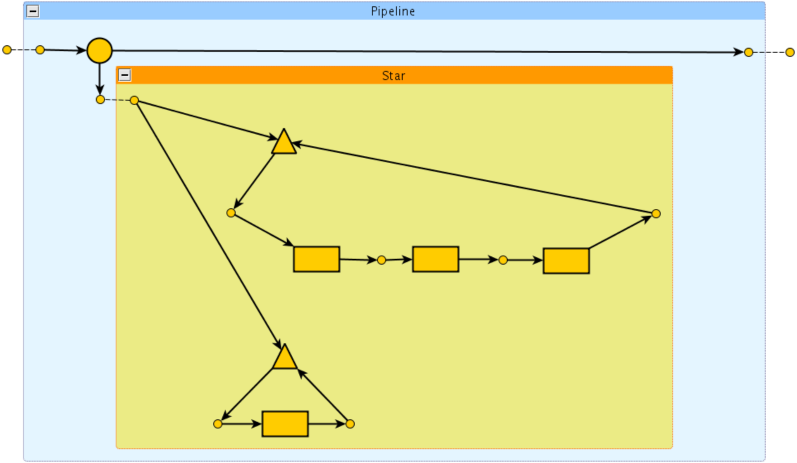

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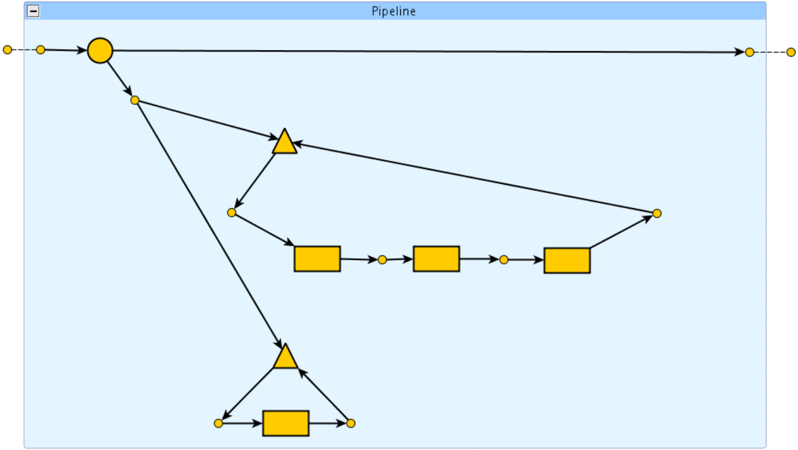

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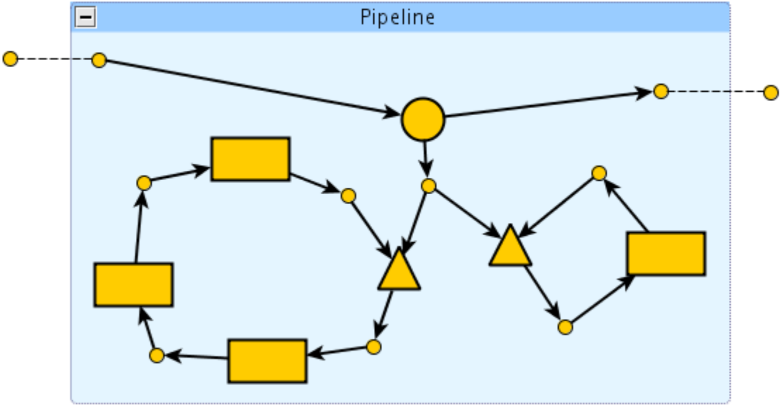

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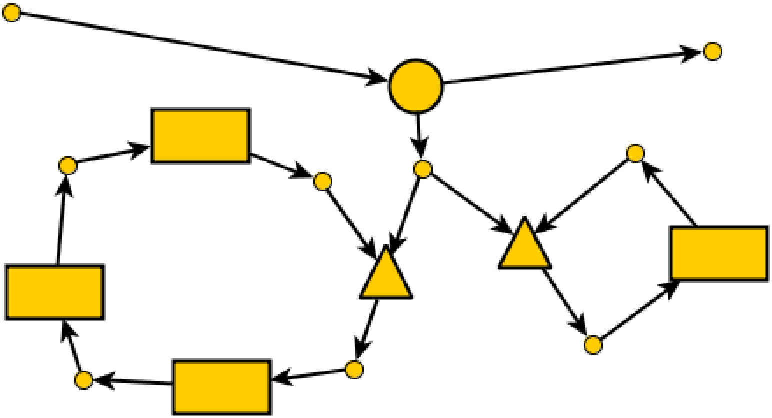

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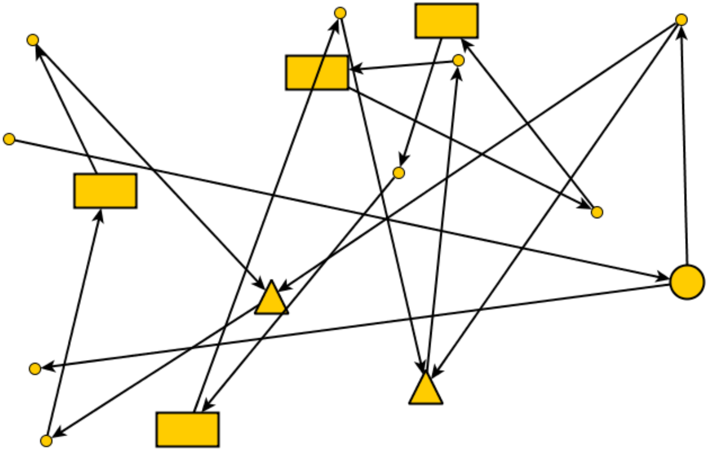


Simplified Typing and Drawing (“Flattening”)

The corresponding proof term is

```
net(par(cast(node(cat(station,
                          cat(station, station)))),
        cast(node(station))
      )
    )
```

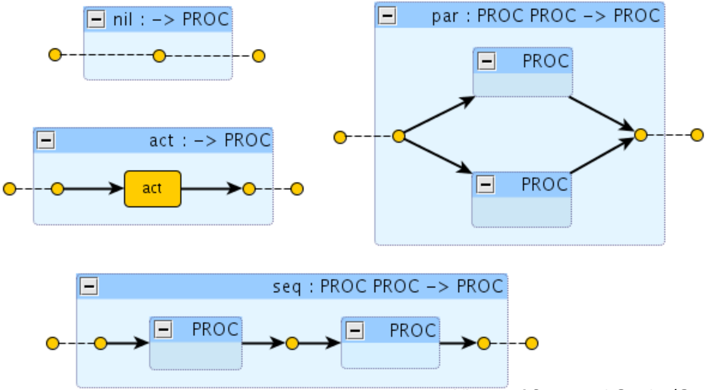
Or just

```
net(par(node(cat(station,station,station)),
        node(station)))
```

Note that nodes need not be mentioned

Another Example: Workflows

Activities composable in series and in parallel (fork & join): disconnected activity and cyclic parts are not allowed



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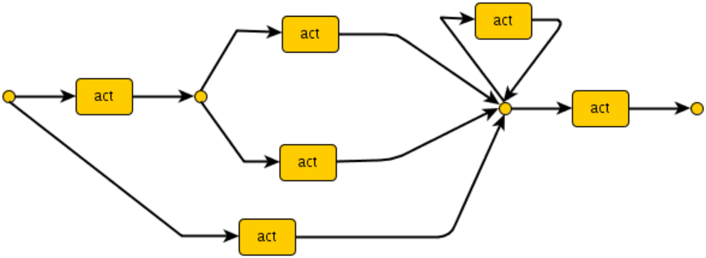
Is this a well-formed workflow?



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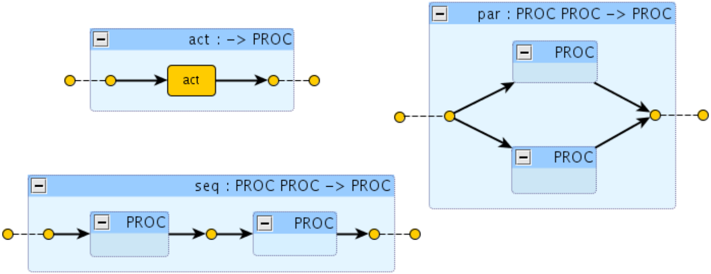

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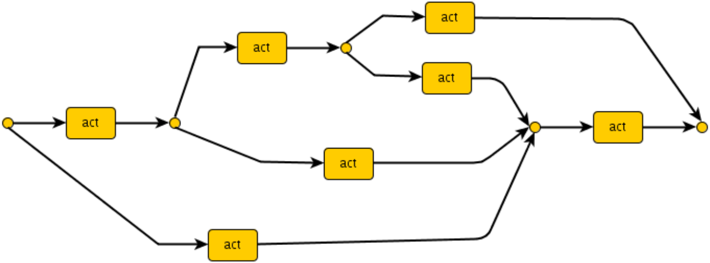

Six Virtues of Structured Graphs

- Requirements
  - Type graphs are ok (and synergic to our approach) but limited
  - Additional logic languages often needed
  - We can account for many patterns in a natural way
- Parsing and browsing
  - Large graphs are hard to "understand" and navigate
  - Their blueprint (if any available) helps quite a lot
- Model Construction and Model conformance
  - Conformance is guaranteed by construction
  - Otherwise hard to recover from scratch (proof-carrying graphs)
- Compositionality and Abstraction & Refinement
  - Interfaces are needed to constrain composition, but hard to recover in flat graphs
  - The hierarchical approach makes them available at any level
  - Different levels of granularity can be considered

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Outline

Style-Preserving Reconfiguration

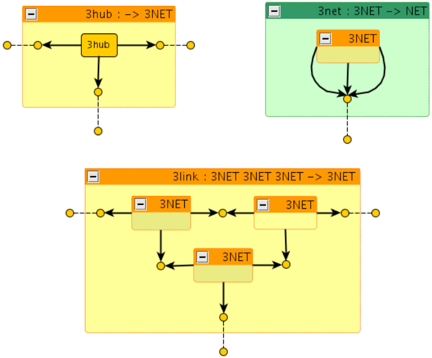
- A reconfiguration is a change in an architecture
  - static? e.g. for deployment on different platforms, improvements, updates, upgrades, model-driven transformation
  - partially specified? e.g. some components are not known at design time, except for their types
  - run-time? e.g. triggered by security policies, load balancing, mobility, QoS assurance, components joining and leaving the system, dynamic binding, wrapping, self-* architectures
- Style-preservation is relevant
  - from well-formed graphs to well-formed graphs (but possibly with different shapes)
- Examples
  - reverse all actions in a pipeline, serialize a workflow, star to ring transformation, migrate all clients of a server, close all sub-sessions upon termination of their parents

How to Write Reconfiguration Rules

- Using graph transformation
  - direct manipulation of flat graphs
  - applicable in non well-formed graphs
  - well-formedness of results must be proved
  - in the flat case: rules manipulate components (many steps required)
  - in the hierarchical case: rules manipulate groups of components (one step can suffice)
- Exploiting structured graphs
  - rules manipulate well-formedness proofs
  - inductive localization of the least part of the proof where the change is needed
  - style-preserving by construction

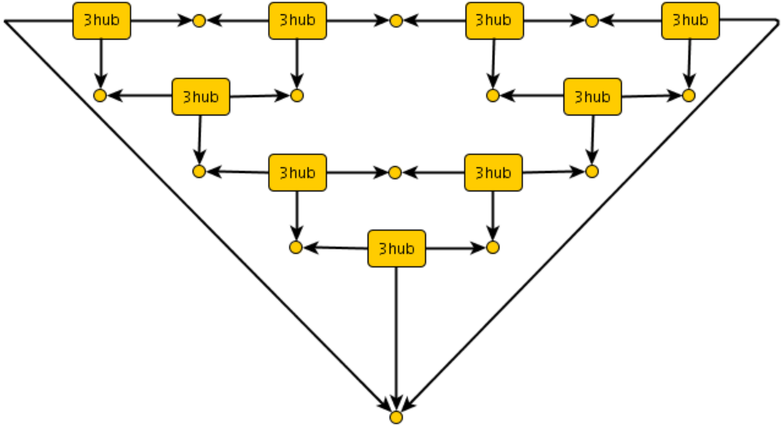
An Example: 3hub Network

Network hubs have three degrees of connectivity and connections are driven by the style (only allowed: some sort of reversed pyramids)



An Example: 3hub Network

A valid 3hubs network



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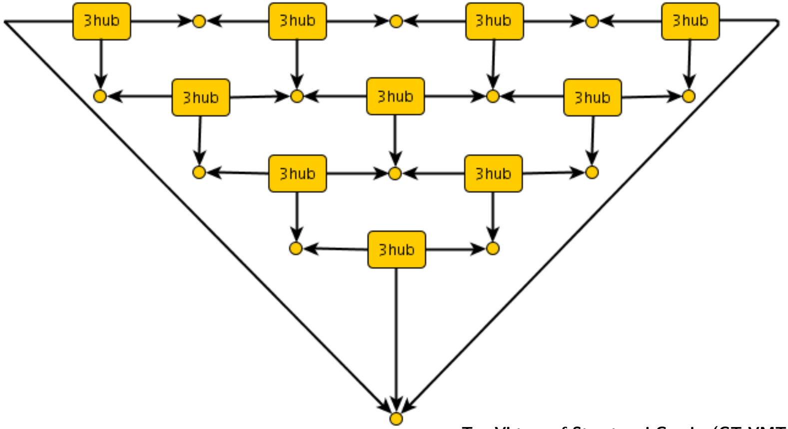
A valid 3hubs network? or maybe not?



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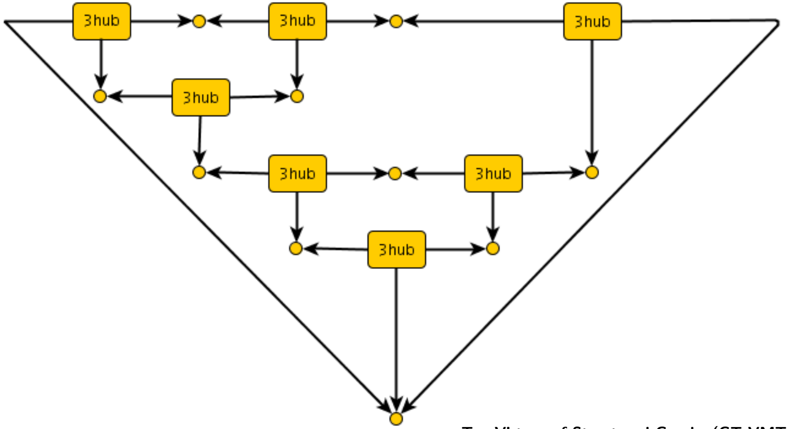
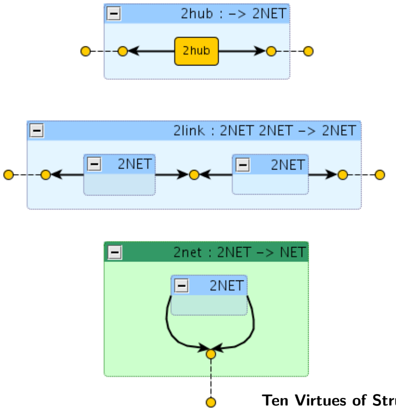


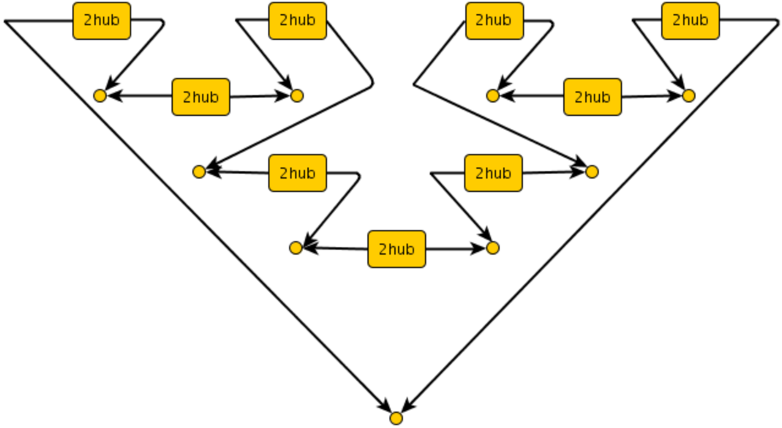
An Example: 2hub Network

Network hubs have just two degrees of connectivity and connections are driven by the style (only allowed: rings)



An Example: 2hub Network

A valid 2hubs network

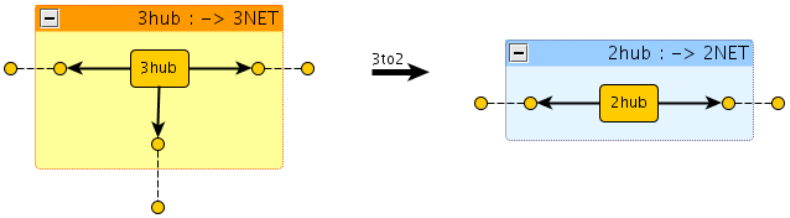


An Example: From 3hub Networks to 2hub Networks

- Under certain circumstances, it is required to reconfigure any valid 3hub network to a valid 2hub network
  - the whole network must be reconfigured (not just part of it)
  - total number of hubs is unchanged
  - 2hubs must form a ring
- Idea:
  - exploit blueprint, not the flat graph
  - reconfiguration is defined inductively on the structure of the network
  - exploit conditional rewrite rules

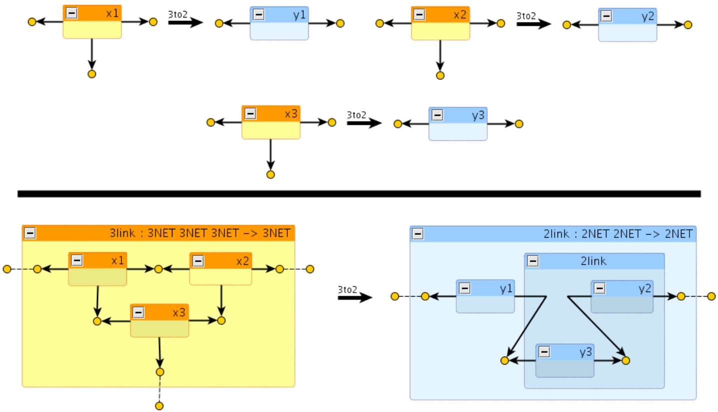
An Example: From 3hub Networks to 2hub Networks

Reconfigure a single 3hub (note that type is changed: some sort of transduction, context must be adapted)



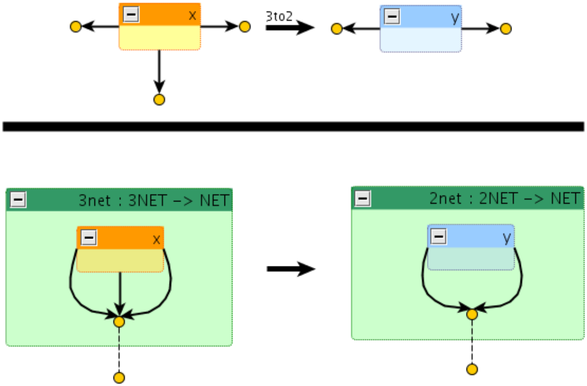
An Example: From 3hub Networks to 2hub Networks

Reconfigure the link structure (a transduction, again)



An Example: From 3hub Networks to 2hub Networks

Reconfigure the whole network (note that type is preserved, rewrite is silent, applicable in any larger context)



An Example: Rewrite Rules for Network Transformation

$$\texttt{3hub} \xrightarrow{\texttt{3to2}} \texttt{2hub}$$

$$\frac{x_1 \xrightarrow{\texttt{3to2}} y_1 \quad x_2 \xrightarrow{\texttt{3to2}} y_2 \quad x_3 \xrightarrow{\texttt{3to2}} y_3}{\texttt{3link}(x_1, x_2, x_3) \xrightarrow{\texttt{3to2}} \texttt{2link}(y_1, \texttt{2link}(y_3, y_2))}$$

$$\frac{x \xrightarrow{\texttt{3to2}} y}{\texttt{3net}(x) \longrightarrow \texttt{2net}(y)}$$

Three More Virtues of Structured Graphs

- Reconfiguration and Evolution
  - (flat) graph transformation requires ad-hoc studies and techniques (e.g., negative application conditions, interfaces, atomicity issues), augmenting the representation distance (high expertize, technology transfer more difficult)
  - structured graph rewrites can be more handy and efficient (e.g. graph matching not necessarily required)
  - style preservation: to be proved vs guaranteed by proofs
  - concurrency? special cases (edge to edge rules)?
- Graphical encoding
  - seamless grouping of item through the hierarchy (e.g. for representing nested sessions, transactions, scopes)
  - in the case of process calculi, facilitated by suitable graph algebras (see next part of the talk)
  - Encoding properties (soundness, completness) shown by structural induction

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Outline

Introduction

Styles for Visual Support

Dynamics

ADR

Concluding Remarks

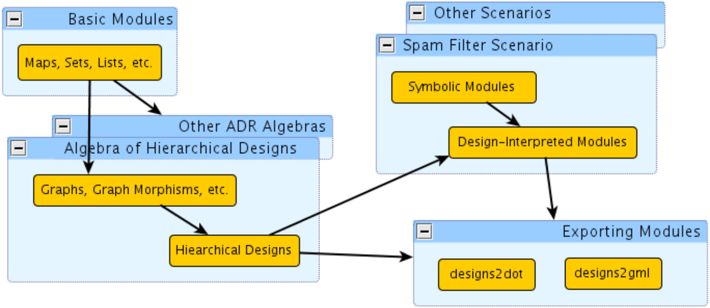
ADR in a Nutshell

- ADR formulas:
  - ADR = Designs + Term Rewriting
  - Designs = Typed Hierarchical Graphs (with Interfaces)
- ADR ingredients:
  - Sorts: Vocabulary, Types (edge and node labels)
  - Values: Designs (hierarchical graphs with interfaces)
  - Operations: Graph-grammar-like rules
  - Terms: proofs of construction
  - Terms (with variables): partial Designs, partial proofs
  - Axioms: properties of operations
  - Membership predicates: additional style rules
  - Rewrite rules: behaviour, reconfigurations
  - Rewrite strategies: style conformance, style analysis, etc.

A Flexible Unifyig Framework for Design, Execution, Reconfiguration

- Not necessarily in the spirit of universal models:
  - node as names + hyper-edge as ops + parallel composition + name fusion + name hiding = any graph can be obtained
  - node as names + hyper-edge as ops + type annotation + tailored constructors = only well-formed designs are described
- Some other ADR features:
  - Membership equational theory (e.g. ACI1, subsorting, overloading)
  - Flattening axioms (e.g. not all operators are hierarchic)
- Some ADR caveats:
  - different proof terms for the same graph are possible
  - constraints not fully integrated yet
  - concurrency aspects not addressed yet

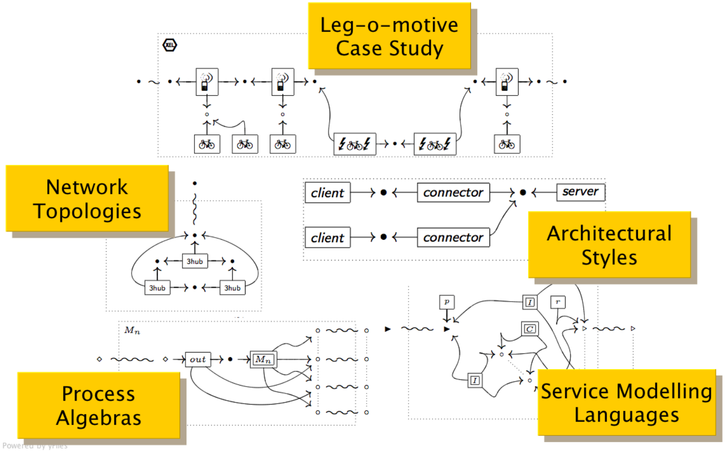
Maude Prototype for ADR



Maude Prototype for ADR

- Why Maude?
  - built-in membership equational theories (e.g. to support style conformance check)
  - conditional rewrite rules supported
  - standard encoding of LTS
  - built-in search strategies (e.g. to support model finding)
  - built-in LTL model-checker
  - defineable logic languages (within the same framework): e.g. graph logics (Courcelle's MSO), modal logics, spatial logics

ADR Case Studies



An Example: From Process Calculi to Graphs

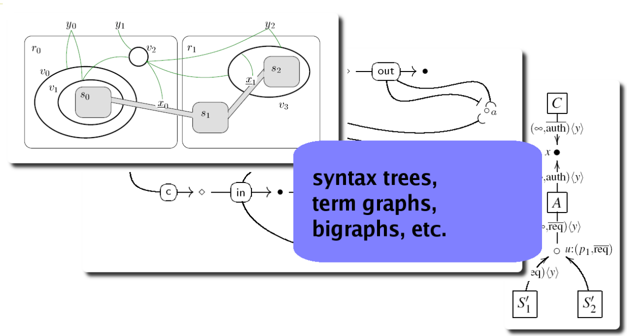
The syntax of process calculi (with name handling)

$P, Q ::=$	$\sum_{i \in I} \pi_i P_i$	Guarded Sum
$\mid$	$s.P$	Service Definition
$\mid$	$\overline{s}.P$	Service Invocation
$\mid$	$r \triangleright P$	Sessio
$\mid$	$P > Q$	Pipeli
$\mid$	$P \vert Q$	Parallel Composition
$\mid$	$(\nu n)P$	Restriction
$\mid$	$!P$	Replication

Algebraic form:
- grammar
- structural congruence

An Example: From Process Calculi to Graphs

Terms as graphs



An Example: From Process Calculi to Graphs

The syntax of graphs

Definition 22 (bigraph) A *bigraph* over the signature $\mathcal{K}$ takes the form $G = (V, ctrl, G^{\mathsf{T}}, G^{\mathsf{M}}) : I \to J$ where: $I = \langle m, X\rangle$ and $J = \langle n, Y\rangle$ are its *inner and outer interfaces*, each combining a *width* (a finite ordin… each … and 3…

Definition 7 (hypergra… …isms). *A hypergraph G is a triple $\langle E_G, N_G, t_G\rangle$ suc…* …es, N_G *is the set of nodes, and $t_G : E_G \to N_G^*$ is the* … of … fun…

Definition 2.1 (Grap… …nt *has form $\Gamma \vdash G$ where:*

1. *$\Gamma \subseteq \mathcal{N}$ is a finite set of names (the free nodes of the graph);*
2. *G is a* graph term *generated by the grammar*

$$G ::= L(\mathbf{x}) \quad | \quad G|G \quad | \quad \nu y\, G \quad | \quad nil$$

where $\mathbf{x}$ is a tuple of names, $L \in \mathcal{L}$, $\mathrm{rank}(L) = |\mathbf{x}|$ and y is a name.

tuples,
sets,
morphisms, etc.

An Example: From Process Calculi to Graphs

Encoding can become cumbersome

$$[\![(\nu a)P]\!]_\Gamma = \begin{cases} [\![P]\!]_\Gamma & \textit{if } a \notin \mathtt{fn}(P) \\ (id_p \otimes \nu_c \otimes id_\Gamma) \circ [\![P\{^c/_a\}]\!]_{\{c\}\uplus\Gamma} & \textit{otherwise} \end{cases}$$

$$[\![P \mid Q]\!]_\Gamma = [\![P]\!]_\Gamma \otimes [\![Q]\!]_\Gamma \qquad [\![a(b).P]\!]_\Gamma = (in_{a,c} \otimes id_\Gamma) \circ [\![P\{^c/_b\}]\!]_{\{c\}\uplus\Gamma}$$

$$[\![0]\!]_\Gamma = 0_p \otimes 0_\Gamma \qquad [\![\overline{a}b.P]\!]_\Gamma = (out \ldots$$

$$[\![\mathbf{0}]\!]_X = 1 \curlywedge X \qquad [\![P|Q]\!]_X = [\![P]\!]_X \curlywedge [\![Q]\!]_X \ldots$$

$$[\![zx.P]\!]_X = \mathsf{get}^{x,z} \circ [\![P]\!]_X \qquad [\![\bar{z}x.P]\!]_X = \mathsf{se}\ldots$$

$$[(\nu a)P]^p_n = \mathtt{hide}_n([P\{\ldots/_a \ldots$$

$$[P \mid Q]^p_n = \mathtt{par}_n([P]^p_n, [Q]^p_n) \qquad \ldots = \mathtt{in}_{i,n}([P\{^{\ldots+1}/_y\}]^p_{n+1})$$

$$[0]^s_n = \mathtt{nil}_n \qquad [M + N]^s_n = \mathtt{choice}_n([M]^s_n, [N]^s_n)$$

operations are parametric, complex, ad hoc

A Re-usable Graph Algebra for Process Calculi

Components as edges $l(\vec{x})$, types as design labels L.

$$
\begin{array}{llll}
\text{(designs)} & \mathbb{D} & ::= & \mathsf{L}_{\vec{x}}[\mathbb{G}] \\
\text{(graphs)} & \mathbb{G} & ::= & \mathbf{0} \mid x \mid l(\vec{x}) \mid \mathbb{G}|\mathbb{G} \mid (\nu x)\mathbb{G} \mid \mathbb{D}\langle\vec{y}\rangle
\end{array}
$$

- In $\mathsf{L}_{\vec{x}}[\mathbb{G}]$, the nodes $\vec{x}$ in $\mathbb{G}$ are bound by the interface (as arguments), the other free names of $\mathbb{G}$ are global.
- We write $\mathsf{L}_{\langle\vec{y}\rangle}[\mathbb{G}\{^{\vec{y}}/_{\vec{x}}\}]$ as a shorthand for $\mathsf{L}_{\vec{x}}[\mathbb{G}]\langle\vec{y}\rangle$
- A flattening axiom for some inessential design label L takes the form $\mathsf{L}_{\vec{x}}\mathbb{G}\langle\vec{y}\rangle \equiv \mathbb{G}\{^{\vec{y}}/_{\vec{x}}\}$ (but $\mathbb{G}\{^{\vec{y}}/_{\vec{x}}\}$ has still type L)
- Structural equivalence as graph isomorphism

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Some Sketches of Encoding

π-calculus in ADR (**P** process type, **G** guarded sums type)

$$
\begin{aligned}
[\![(\nu x)Q]\!] &= \mathbf{P}_p[\ (\nu x)[\![Q]\!]\langle p\rangle\ ] \\
[\![N + M]\!] &= \mathbf{G}_p[\ [\![N]\!]\langle p\rangle \ \mid\ [\![M]\!]\langle p\rangle\ ] \\
[\![Q \mid R]\!] &= \mathbf{P}_p[\ [\![Q]\!]\langle p\rangle \ \mid\ [\![R]\!]\langle p\rangle\ ]
\end{aligned}
$$

CaSPiS in ADR (**P** process type, **S** session type)

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[\![Q \mid R]\!] &= \mathbf{P}_{p,i,o,r}[\ p \mid i \mid o \mid r \mid [\![Q]\!]\langle p,i,o,r\rangle \ \mid\ [\![R]\!]\langle p,i,o,r\rangle\ ] \\
[\![s^{+} \rhd Q]\!] &= \mathbf{P}_{p,i,o,r}[\ i \mid o \mid \mathbf{S}_{\langle p,r\rangle}[\ [\![Q]\!]\langle p,s^{+},s^{-},r\rangle\ ]\ ] \\
[\![s^{-} \rhd Q]\!] &= \mathbf{P}_{p,i,o,r}[\ i \mid o \mid \mathbf{S}_{\langle p,r\rangle}[\ [\![Q]\!]\langle p,s^{-},s^{+},r\rangle\ ]\ ]
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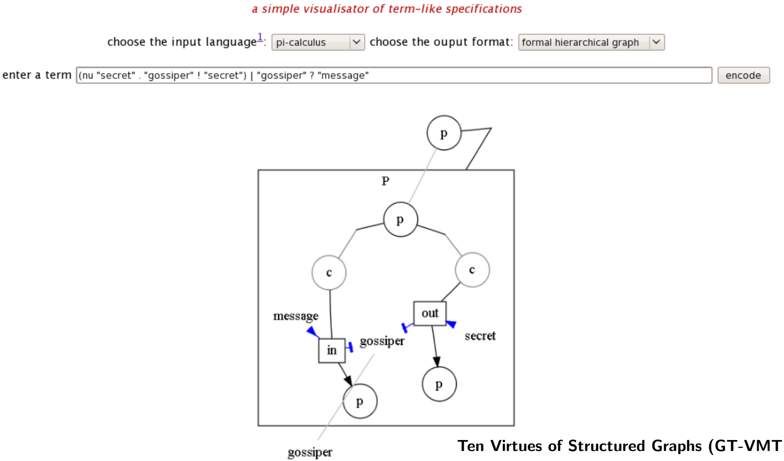
[\![s^- \rhd Q]\!] &= \mathbf{P}_{p,i,o,r}[\ i \mid o \mid \mathbf{S}_{\langle p,r \rangle}[\ [\![Q]\!]\langle p,s^-,s^+,r \rangle\ ]\ ]
\end{aligned}
$$

Visualization: adr2graphs (Early Prototype)

Please have a try at http://www.albertolluch.com/adr2graphs



One Last Virtue of Structured Graphs

- Logical specification and verification
  - ad-hoc spatial logics: from “general” to “derived” modalities
  - formulas closer to visualization (easier to use)
  - types as properties: a property $\mathcal{P}$ demonstrated by structural induction on type T show that all graphs of type T satisfy $\mathcal{P}$.
  - re-use existing (efficient) tools whenever possible

Outline

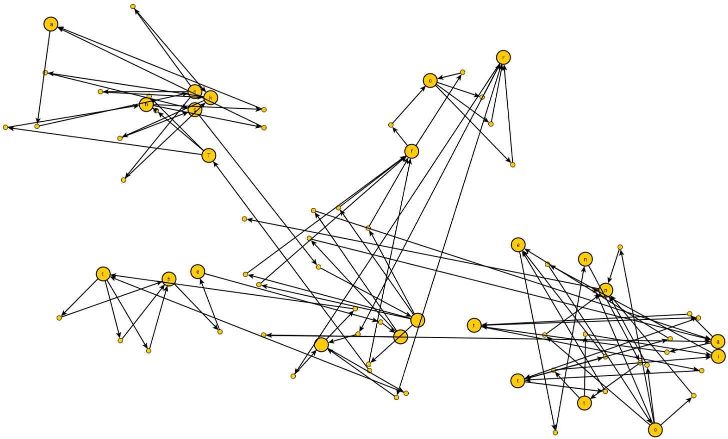
Where ADR can help

- Design of software architectures
  - drop & bind components + check + correct: tedious, error prone
  - bounded FO/SAT (Alloy): performant, but trial & error, no hint, no guidance
- Guaranteed reconfiguration
  - prove theorems on GT: ad-hoc, manual, limited re-use
  - model checking on GT: validate a particular instance, scalability issues, undecidable in general
  - monitor & repair: no guarantees
- Usability
  - other integrated environment require acquaintance with many different languages and theories

Related work

- Ordinary GT:
  - nice theory of concurrency, but structure must be encoded somehow in flat graphs,
  - problems with grouping and atomicity
- Hierarchical graphs:
  - main difference relies on interfaces
- Alloy:
  - highly specialized SAT solver, but Maude is more flexible

End of Talk (Graphs Powered by yEd)



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