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Tile Bisimilarity Congruences for Open Systems

Roberto Bruni — Univ. Pisa

joint work with

David de Frutos Escrig — Univ. Complutense de Madrid

Narciso Martí-Oliet — Univ. Complutense de Madrid

Ugo Montanari — Univ. Pisa

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Open Systems

1. **Incomplete Systems:** e.g. repetitive, parametrized.

   $C[x]|D[x]$: same protocol used in C and D to provide a certain service.

   The two instances are instantiated with the same pattern, but can progress independently.

2. **Coordinators:** e.g. black box processes.

   $C[x]|D[x]$: agent that regulates the execution of its argument.

   The x indicates a single computational entity and replication must be avoided.

   $\mathbf{let}\ x = q\ \mathbf{in}\ C[x]|D[x]$

3. **Open Ended Systems:** e.g. dynamically reconfig.

   Dynamic bisimulation (**MS92**)

   $p_1 \sim_{\mathrm{d}} p_2$: $\forall C[_].\ C[p_1] \xrightarrow{a} q_1 \ \Rightarrow\ C[p_2] \xrightarrow{a} q_2 \ \wedge\ q_1 \sim_{\mathrm{d}} q_2$ (and vice versa).

In This Talk...

- OPEN ENDED (3) in (**BMS00**), not here.
- Here PARAMETRIZED (1) vs. COORDINATORS (2).
- BISIMILARITY via 'good formats.'

  related work: *context systems* (**LX90**); *tile model* (**GM97**); *conditional transition systems* (**Ren97**).

(pre)example 1 *Take* $P[x] = x\backslash\alpha|x\backslash\alpha$ *and* $Q[x] = (x|x)\backslash\alpha$.

$P \stackrel{?}{\sim} Q$ *equivalent as coordinators (cannot react to* x*)*

$P \stackrel{?}{\sim} Q$ *but if* instance closure *is expected, they cannot be equivalent as parametrized systems:*
$P[p]$ *is not equivalent to* $Q[p]$ *for* $p = \alpha.nil + \bar{\alpha}.nil$.

Operational semantics

LTS: $p \xrightarrow{a} q$ states are closed terms; labels are actions.

Abstract semantics

Testing: $p \sim_1 q \Leftrightarrow \forall C[_].\, C[p] \equiv C[q].$

Bisimilarity: $p \sim_2 q : p \xrightarrow{a} p' \Rightarrow q \xrightarrow{a} q' \wedge p' \sim_2 q'$
(and vice versa).

Congruence? $p \sim_2 q \stackrel{?}{\Rightarrow} \forall C[_].\, C[p] \sim_2 C[q].$

For free if specification formats employed (e.g. De Simone, `GSOS`, `tyft/tyxt`)

Mixed: e.g. barbed bisimilarity.

Usually $\sim_2$ finer than $\sim_1$, but easier to prove by coinductive methods.

Operational semantics

$$\frac{x_1 \xrightarrow{a_1} y_1 \;\cdots\; x_n \xrightarrow{a_n} y_n}{C[x_1,\ldots,x_n] \xrightarrow{b} D[y_1,\ldots,y_n]} \quad \text{or} \quad C \xrightarrow[b]{\vec{a}} D \quad \text{or} \quad \begin{array}{ccc} \underline{n} & \xrightarrow{C} & \underline{1} \\ {\scriptstyle \vec{a}}\downarrow & & \downarrow{\scriptstyle b} \\ \underline{n} & \xrightarrow[D]{} & \underline{1} \end{array}$$

Open transitions derived by contracting proof terms

e.g. $$\frac{\dfrac{x \xrightarrow{a} x' \; y \xrightarrow{\bar{a}} y'}{x|y \xrightarrow{\tau} x'|y'}}{(x|y)\backslash\alpha \xrightarrow{\tau} (x'|y')\backslash\alpha}$$ yields $$\frac{x \xrightarrow{a} x' \; y \xrightarrow{\bar{a}} y'}{(x|y)\backslash\alpha \xrightarrow{\tau} (x'|y')\backslash\alpha}$$

using tiles: $$\begin{array}{ccccc} \underline{2} & \xrightarrow{-1\,|\,-2} & \underline{1} & \xrightarrow{-1\backslash\alpha} & \underline{1} \\ {\scriptstyle a\otimes\bar{a}}\downarrow & & \downarrow{\scriptstyle \tau} & & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[-1\,|\,-2]{} & \underline{1} & \xrightarrow[-1\backslash\alpha]{} & \underline{1} \end{array} = \begin{array}{ccc} \underline{2} & \xrightarrow{(-1\,|\,-2)\backslash\alpha} & \underline{1} \\ {\scriptstyle a\otimes\bar{a}}\downarrow & & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[(-1\,|\,-2)\backslash\alpha]{} & \underline{1} \end{array}$$

structural LTS specifications apt to deal with open terms

Abstract semantics

Via Ground Closure: $C \approx_1 D \;\Leftrightarrow\; \forall \sigma.\, \sigma(C) \sim \sigma(D)$

for σ ground substitution.

Bisimilarity: $C \approx_2 D : C \xrightarrow[b]{\vec{a}} C' \;\Rightarrow\; D \xrightarrow[b]{\vec{a}} D' \;\wedge\; C' \approx_2 D'$

(and vice versa).

'Instance congr.'? $C \approx_2 D \;\wedge\; p \approx_2 q \;\stackrel{?}{\Rightarrow}\; C[p] \approx_2 D[q]$.

On closed terms we have $\approx_2 = \approx_1 = \sim$.

In general $\approx_2$ finer than $\approx_1$.

e.g. CCS with actions $\{a, b\}$

let $p = \mathbf{rec}\; z.\; az + bz + \bar{a}z + \bar{b}z + \tau z$

then $\forall q.\; q|p \approx_1 q\backslash_a|p \approx_1 p$

but $x|p \xrightarrow[a]{a} y|p$, while $x\backslash_a|p$ cannot perform $\xrightarrow[a]{a}$.

Problems and Solutions

What are the good formats?

The original tile model (De Simone extension) in (**GM97**) cannot guarantee that tile bisimilarity is an instance congruence (IC). The problem arises from shared variables in states vs. linear observations.

Proposed solutions:

1. Only **linear open systems**

2. **Variable splitting:** $f(x,x) \xrightarrow{\nabla} f(x_1,x_2)$

3. **Term graphs:** $C[p] \neq \mathbf{let}\ x = p\ \mathbf{in}\ C[x]$ preserves sharing

We define corresponding tile formats and prove the congruence result for each of them.

Categories of Substitutions

Signature Σ (one sorted) and **Variables** in X

Term algebras $\mathbb{T}_\Sigma(X)$ and $\mathbb{T}_\Sigma = \mathbb{T}_\Sigma(\varnothing)$

Substitutions $\sigma: X \to \mathbb{T}_\Sigma(X)$ (extend to $\sigma: \mathbb{T}_\Sigma(X) \to \mathbb{T}_\Sigma(X)$)

Canonical variables: $\underline{n} = \{x_1, x_2, \ldots, x_n\}$

Finite substitution $\sigma = [t_1/x_1, \ldots, t_n/x_n]$

σ' can be applied elementwise to σ yielding $[\sigma'(t_1)/x_1, \ldots, \sigma'(t_n)/x_n]$

Context $t = C[x_1, ., x_n]$ (using at most $x_1, \ldots, x_n$)

$C[t_1, \ldots, t_n]$ given by applying $[t_1/x_1, \ldots, t_n/x_n]$ to $C[x_1, ., x_n]$

Linear terms, contexts, substitutions

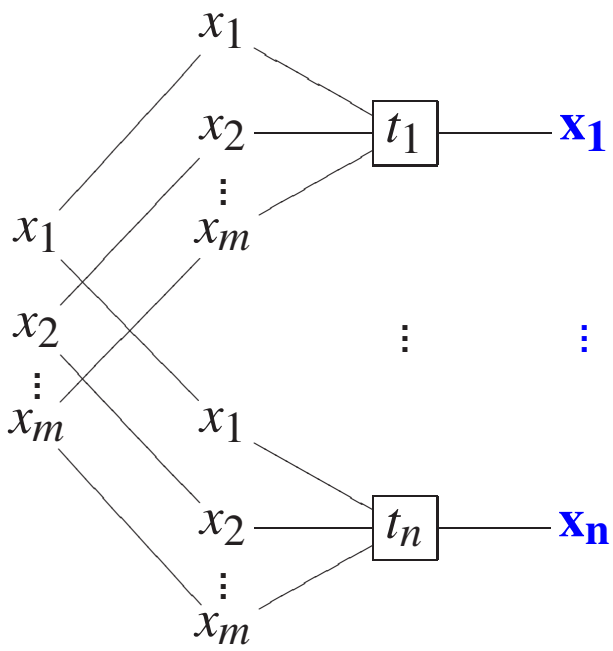
Categories of Substitutions (II)

Context as substitution from n terms to 1: $[C[x_1,\ldots,x_n]/x_1]$.

More generally: For $t_i \in \mathbb{T}_\Sigma(\underline{m})$, $\sigma\colon \underline{m} \to \underline{n}$.

given m terms as inputs, return n terms as output

$[t_1/x_1,\ldots,t_n/x_n]$ written just as $\langle t_1,\ldots,t_n\rangle$



Sequential composition is substitution application.

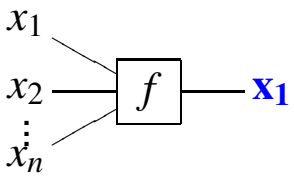
Brackets can be omitted **ex. a, b const. and f binary**

$\langle \mathbf{a},\mathbf{b}\rangle;\langle \mathbf{f}(\mathbf{x_1},\mathbf{x_2}),\mathbf{x_1}\rangle;\mathbf{f}(\mathbf{x_2},\mathbf{x_1}) = \langle \mathbf{f}(\mathbf{a},\mathbf{b}),\mathbf{a}\rangle;\mathbf{f}(\mathbf{x_2},\mathbf{x_1}) = \mathbf{f}(\mathbf{a},\mathbf{f}(\mathbf{a},\mathbf{b}))$

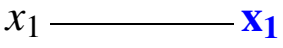
Categories of Substitutions (III)

Algebraic Theories **(Law63)**: **finite subs on Σ generated by**

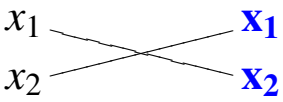
$\langle \mathbf{f} \rangle : \underline{n} \to \underline{1}$ **for** $f \in \Sigma$ **with arity** n



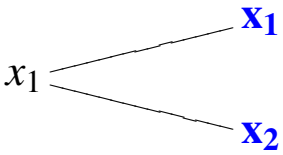
$\mathbf{id}_{\underline{1}} = \langle \mathbf{x_1} \rangle : \underline{1} \to \underline{1}$



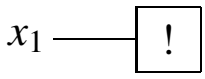
$\gamma_{\underline{1},\underline{1}} = \langle \mathbf{x_2}, \mathbf{x_1} \rangle : \underline{2} \to \underline{2}$



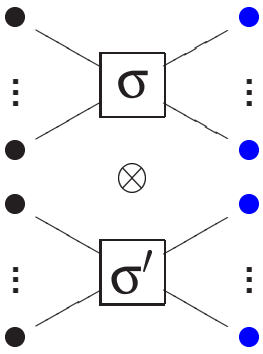
$\nabla_{\underline{1}} = \langle \mathbf{x_1}, \mathbf{x_1} \rangle : \underline{1} \to \underline{2}$

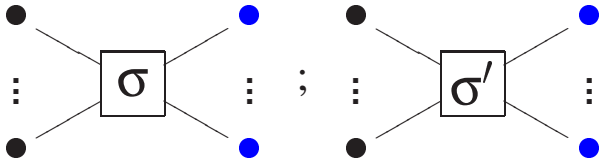


$!_{\underline{1}} = \langle \rangle : \underline{1} \to \underline{0}$



composing with tensor $_ \otimes _$ **and application** $_ ; _$





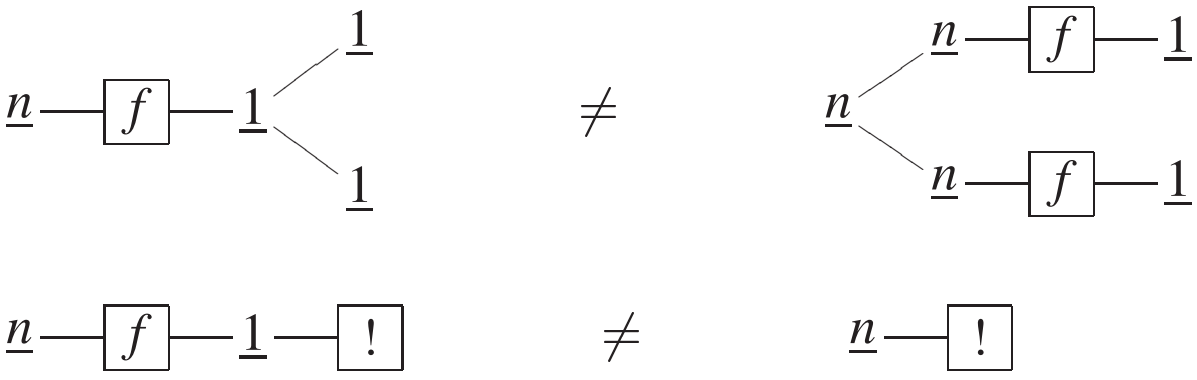
Terms

$$f;\nabla = \nabla;(f \otimes f)$$

$$f;! = !$$

Term Graphs

Abandon the naturality of ∇ and $!$ (two axioms above) (**CG99**)



Bisimulation and SOS

Bisimulation:

$t_1 \sim t_2$ and $t_1 \xrightarrow{a} s_1$ imply $t_2 \xrightarrow{a} s_2$ with $s_1 \sim s_2$

(and vice versa)

Bisimilarity $\simeq$ is the largest bisimulation

Is $\simeq$ a congruence? $t_i \simeq s_i \stackrel{?}{\Rightarrow} f(t_1,\ldots,t_n) \simeq f(s_1,\ldots,s_n)$

De Simone, positive GSOS, `tyft/tyxt` formats

$$\frac{\{x_i \xrightarrow{a_i} y_i \mid i \in I\}}{f(x_1,\ldots,x_n) \xrightarrow{a} t}$$

Conclusions $C[x_1,\ldots,x_n] \xrightarrow{a} t$ never allowed!

Main Features of Tile Logic

TL is a logic for modular descriptions of the dynamic evolution of concurrent systems

- Suited to deal with **synchronization issues**
- General notion of **configuration** and of **observation**

  horizontal structure different than terms **(linear terms, term graphs, graphs, higher-order terms)**

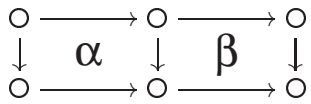
  vertical structure different than actions **(causality and locality dependencies, name creation, name abstraction)**
- Defines compositional models of computation in **space** and in **time**
- Allows to reason about **contexts**
- Admits suggestive **graphical presentation**
- Can model **heterogeneous systems**
- Algebra of **concurrent computation proofs**
- **auxiliary tiles** to model built-in operations

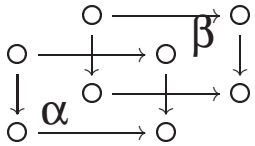
Tile Model

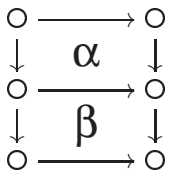
A ***tiles system*** is a tuple $\mathcal{R} = (\mathcal{H}, \mathcal{V}, N, R)$ where:
$\mathcal{H}$ monoidal category of ***configurations***
$\mathcal{V}$ monoidal category of ***observations***
$\mathcal{H}$ and $\mathcal{V}$ have the same objects (called ***interfaces***)
N is the set of *tile names*
$R: N \to \mathcal{H} \times \mathcal{V} \times \mathcal{V} \times \mathcal{H}$

if $R(\alpha) = \langle s, a, b, t \rangle$, written α: $s \overset{a}{\underset{b}{\to}} t$, then $\begin{array}{ccc} x & \xrightarrow{s} & y \\ a\downarrow & \alpha & \downarrow b \\ z & \xrightarrow[t]{} & w \end{array}$

Tiles can be composed







Identities

$$\begin{array}{ccc} x & \xrightarrow{s} & y \\ id_x\downarrow & id_s & \downarrow id_x \\ x & \xrightarrow[s]{} & y \end{array} \qquad \begin{array}{ccc} x & \xrightarrow{id_x} & x \\ a\downarrow & id_a & \downarrow a \\ z & \xrightarrow[id_z]{} & z \end{array}$$

(possibly auxiliary tiles)

Tile Logic and Auxiliary Tiles

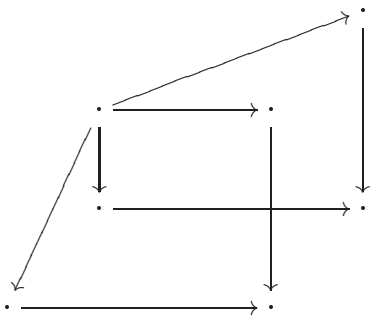
Given $\mathcal{R}$, the associated *tile logic* is obtained by adding some canonical 'auxiliary' tiles and then closing by composition both auxiliary and basic tiles.

$$\mathcal{R} \vdash s \frac{a}{b} t$$

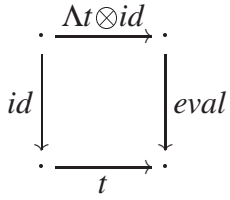
aux. tiles can e.g. model isomorphic transf. of interfaces

e.g. identity tiles allow for reduction in any context $\dfrac{x \xrightarrow{id} y}{f(x) \xrightarrow{id} f(y)}$

e.g. variable splitting



e.g. function application



Tile Bisimulation and Tile Decomposition

Tiles allow for a very general notion of bisimulation (note that t_1 and t_2 can be open terms)

$\mathbf{t_1} \sim_{\mathrm{t}} \mathbf{t_2}$ and $\mathcal{R} \vdash t_1 \xrightarrow[b]{a} s_1$ imply $\mathcal{R} \vdash t_2 \xrightarrow[b]{a} s_2$ with $\mathbf{s_1} \sim_{\mathrm{t}} \mathbf{s_2}$ (and vice versa)

Tile bisimilarity $\simeq_{\mathrm{t}}$ is the largest tile bisimulation

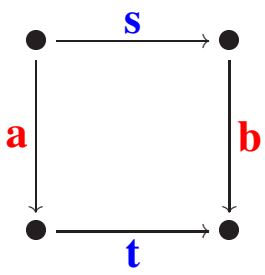
Definition 1 *A tile system $\mathcal{R}$ enjoys the* **decomposition property** *if for all $\mathcal{R} \vdash s \xrightarrow[b]{a} t$*

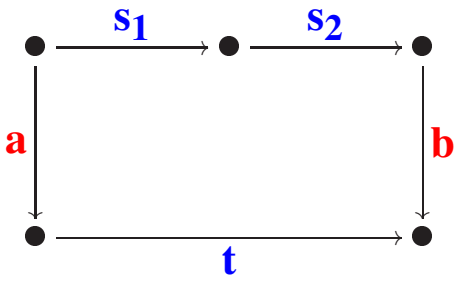
1. *if $s = s_1; s_2$ then*
   $\mathcal{R} \vdash s_1 \xrightarrow[c]{a} t_1$, $\mathcal{R} \vdash s_2 \xrightarrow[b]{c} t_2$ *with* $t = t_1; t_2$

2. *if $s = s_1 \otimes s_2$ then*
   $\mathcal{R} \vdash s_1 \xrightarrow[b_1]{a_1} t_1$, $\mathcal{R} \vdash s_2 \xrightarrow[b_2]{a_2} t_2$,
   with $a = a_1 \otimes a_2$, $b = b_1 \otimes b_2$ *and* $t = t_1 \otimes t_2$

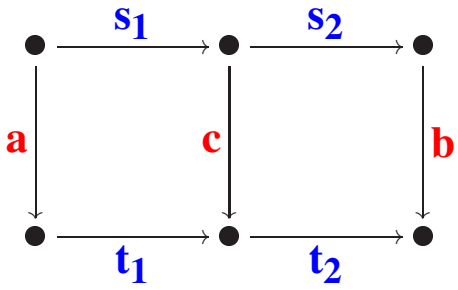
Proposition 1 *If $\mathcal{R}$ enjoys the decomposition property, then tile bisimilarity is a congruence.*

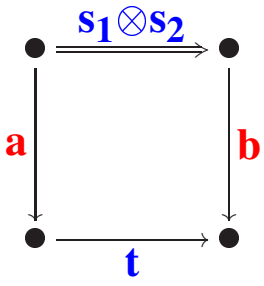
Decomposition Illustrated



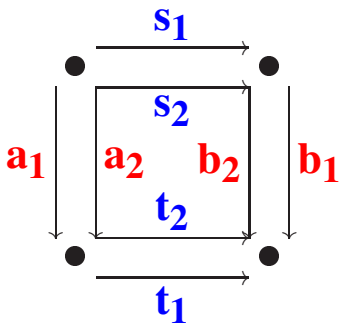


implies





implies



Tile Formats

Starting from signatures (generating $\mathcal{H}$ and $\mathcal{V}$)

$$\frac{\{x_i \xrightarrow{a_i} y_i\}}{C[x_1,\ldots,x_n] \xrightarrow{a} D[y_1,\ldots,y_n]}$$

$$\begin{array}{ccc} \underline{n} & \xrightarrow{C[x_1,\ldots,x_n]} & \underline{1} \\ {\scriptstyle a_1\otimes\cdots\otimes a_n}\downarrow & & \downarrow{\scriptstyle a} \\ \underline{n} & \xrightarrow[D[x_1,\ldots,x_n]]{} & \underline{1} \end{array}$$

note the canonical renaming of the y's

Basic Source (BS): C consists of one operator.

Main Results

Format	hor.	vert.	BS $\Rightarrow$ IC?
Algebraic (GM97)	term	linear	no
monoidal (MM97)	linear	linear	yes
term (BMM98)	term	term	yes
term graph	term graph	linear	yes

Tiles for CCS

$$\frac{}{\mu.x_1 \xrightarrow{\mu} x_1}$$

$$\begin{array}{ccc} \underline{1} & \xrightarrow{\mu} & \underline{1} \\ {\scriptstyle id}\downarrow & & \downarrow{\scriptstyle \mu} \\ \underline{1} & \xrightarrow[id]{} & \underline{1} \end{array}$$

$$\frac{x_1 \xrightarrow{\mu} y_1}{x_1\backslash\alpha \xrightarrow{\mu} y_1\backslash\alpha} \qquad (\mu \neq \alpha, \bar{\alpha})$$

$$\begin{array}{ccc} \underline{1} & \xrightarrow{\backslash\alpha} & \underline{1} \\ {\scriptstyle \mu}\downarrow & & \downarrow{\scriptstyle \mu} \\ \underline{1} & \xrightarrow[\backslash\alpha]{} & \underline{1} \end{array}$$

$$\frac{x_1 \xrightarrow{\mu} y_1}{x_1 + x_2 \xrightarrow{\mu} y_1} \qquad \text{(and symmetric)}$$

$$\begin{array}{ccc} \underline{2} & \xrightarrow{+} & \underline{1} \\ {\scriptstyle \mu\otimes id}\downarrow & & \downarrow{\scriptstyle \mu} \\ \underline{2} & \xrightarrow[id\otimes !]{} & \underline{1} \end{array}$$

$$\frac{x_1 \xrightarrow{\mu} y_1}{x_1|x_2 \xrightarrow{\mu} y_1|x_2} \qquad \text{(and symmetric)}$$

$$\begin{array}{ccc} \underline{2} & \xrightarrow{|} & \underline{1} \\ {\scriptstyle \mu\otimes id}\downarrow & & \downarrow{\scriptstyle \mu} \\ \underline{2} & \xrightarrow[|]{} & \underline{1} \end{array}$$

$$\frac{x_1 \xrightarrow{\lambda} y_1 \quad x_2 \xrightarrow{\bar{\lambda}} y_2}{x_1|x_2 \xrightarrow{\tau} y_1|y_2}$$

$$\begin{array}{ccc} \underline{2} & \xrightarrow{|} & \underline{1} \\ {\scriptstyle \lambda\otimes\bar{\lambda}}\downarrow & & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[|]{} & \underline{1} \end{array}$$

Counterexample Algebraic Format

Horizontal: **term** and Vertical: **linear**

$$\underline{1} \xrightarrow{(x|x)\backslash a} \underline{1} \qquad \sim_{\mathrm{t}} \qquad \underline{1} \xrightarrow{(x\backslash a)|(x\backslash a)} \underline{1}$$

In fact they can do the same, e.g.

$$\begin{array}{ccc} \underline{1} & \xrightarrow{(x|x)\backslash a} & \underline{1} \\ {\scriptstyle\tau}\downarrow & & \downarrow{\scriptstyle\tau;\tau} \\ \underline{1} & \xrightarrow[(x|x)\backslash a]{} & \underline{1} \end{array} \qquad \begin{array}{ccc} \underline{1} & \xrightarrow{(x\backslash a)|(x\backslash a)} & \underline{1} \\ {\scriptstyle\tau}\downarrow & & \downarrow{\scriptstyle\tau;\tau} \\ \underline{1} & \xrightarrow[(x\backslash a)|(x\backslash a)]{} & \underline{1} \end{array}$$

However

$$\underline{0} \xrightarrow{a.nil+\bar{a}.nil} \underline{1} \xrightarrow{(x|x)\backslash a} \underline{1} \qquad \not\sim_{\mathrm{t}} \qquad \underline{0} \xrightarrow{a.nil+\bar{a}.nil} \underline{1} \xrightarrow{(x\backslash a)|(x\backslash a)} \underline{1}$$

Since

$$\begin{array}{ccc} \underline{0} & \xrightarrow{((a.nil+\bar{a}.nil)|(a.nil+\bar{a}.nil))\backslash_a} & \underline{1} \\ {\scriptstyle id}\downarrow & & \downarrow{\scriptstyle\tau} \\ \underline{0} & \xrightarrow[(nil|nil)\backslash_a]{} & \underline{1} \end{array} \qquad \begin{array}{ccc} \underline{0} & \xrightarrow{((a.nil+\bar{a}.nil)\backslash_a)|((a.nil+\bar{a}.nil)\backslash_a)} & \underline{1} \\ {\scriptstyle id}\downarrow & \mathbf{NO} & \downarrow{\scriptstyle\tau} \\ \underline{0} & \xrightarrow[?]{} & \underline{1} \end{array}$$

Example Monoidal Format

Horizontal: **linear** and Vertical: **linear**

$$\underline{2} \xrightarrow{(x_1|x_2)\backslash a} \underline{1} \qquad \not\prec_{\mathrm{t}} \qquad \underline{2} \xrightarrow{(x_1\backslash a)|(x_2\backslash a)} \underline{1}$$

In fact

$$\begin{array}{ccc} \underline{2} & \xrightarrow{(x_1|x_2)\backslash a} & \underline{1} \\ {\scriptstyle a\otimes\bar{a}}\downarrow & & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[(x_1|x_2)\backslash a]{} & \underline{1} \end{array} \qquad \begin{array}{ccc} \underline{2} & \xrightarrow{(x_1\backslash a)|(x_2\backslash a)} & \underline{1} \\ {\scriptstyle a\otimes\bar{a}}\downarrow & \mathbf{NO} & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[?]{} & \underline{1} \end{array}$$

Example Term Format

Horizontal: **term** and Vertical: **term**

Open systems seen as **parametrized**

$$\underline{1} \xrightarrow{(x|x)\backslash a} \underline{1} \qquad \not\sim_{\mathrm{t}} \qquad \underline{1} \xrightarrow{(x\backslash a)|(x\backslash a)} \underline{1}$$

In fact

$$\begin{array}{ccc} \underline{1} & \xrightarrow{(x|x)\backslash a} & \underline{1} \\ {\scriptstyle \nabla_{\underline{1}}}\downarrow & & \downarrow{\scriptstyle id} \\ \underline{2} & \xrightarrow{(x_1|x_2)\backslash a} & \underline{1} \\ {\scriptstyle a\otimes\bar{a}}\downarrow & & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[(x_1|x_2)\backslash a]{} & \underline{1} \end{array} \qquad \begin{array}{ccc} \underline{1} & \xrightarrow{(x\backslash a)|(x\backslash a)} & \underline{1} \\ {\scriptstyle \nabla_{\underline{1}}}\downarrow & & \downarrow{\scriptstyle id} \\ \underline{2} & \xrightarrow{(x_1\backslash a)|(x_2\backslash a)} & \underline{1} \\ {\scriptstyle a\otimes\bar{a}}\downarrow & \mathbf{NO} & \downarrow{\scriptstyle \tau} \\ \underline{2} & \xrightarrow[?]{} & \underline{1} \end{array}$$

We remind that e.g. $(x|x)\backslash a = \nabla_{\underline{1}}; \langle (x_1|x_2)\backslash a \rangle$ and hence we can use the auxiliary tile

$$\begin{array}{ccc} \underline{1} & \xrightarrow{\nabla_{\underline{1}}} & \underline{2} \\ {\scriptstyle \nabla_{\underline{1}}}\downarrow & & \downarrow{\scriptstyle id} \\ \underline{2} & \xrightarrow[id]{} & \underline{2} \end{array}$$

Example Term Graph Format

Horizontal: **term graph** and Vertical: **linear**

Open systems seen as **coordinators**

$$\underline{1} \xrightarrow{(x|x)\backslash a} \underline{1} \qquad \sim_t \qquad \underline{1} \xrightarrow{(x\backslash a)|(x\backslash a)} \underline{1}$$

In fact they can do the same, e.g.

$$\begin{array}{ccc} \underline{1} & \xrightarrow{(x|x)\backslash a} & \underline{1} \\ {\scriptstyle\tau}\downarrow & & \downarrow{\scriptstyle\tau;\tau} \\ \underline{1} & \xrightarrow[(x|x)\backslash a]{} & \underline{1} \end{array} \qquad \begin{array}{ccc} \underline{1} & \xrightarrow{(x\backslash a)|(x\backslash a)} & \underline{1} \\ {\scriptstyle\tau}\downarrow & & \downarrow{\scriptstyle\tau;\tau} \\ \underline{1} & \xrightarrow[(x\backslash a)|(x\backslash a)]{} & \underline{1} \end{array}$$

Furthermore

$$\underline{0} \xrightarrow{a.nil+\bar{a}.nil} \underline{1} \xrightarrow{(x|x)\backslash a} \underline{1} \qquad \sim_t \qquad \underline{0} \xrightarrow{a.nil+\bar{a}.nil} \underline{1} \xrightarrow{(x\backslash a)|(x\backslash a)} \underline{1}$$

Because both

$$\underline{0} \xrightarrow{\textbf{let } x=a.nil+\bar{a}.nil \textbf{ in } (x|x)\backslash_a} \underline{1} \quad \text{and} \quad \underline{0} \xrightarrow{\textbf{let } x=a.nil+\bar{a}.nil \textbf{ in } (x\backslash_a)|(x\backslash_a)} \underline{1}$$

cannot move.

Concluding Remarks

Tile logic as an **abstract, compositional framework** suitable to deal in a uniform way with

- ⋆ parametrized systems
- ⋆ coordinators
- ⋆ open ended systems (**BMS00**)

Part of a research on extending concepts like

actions, synchronization, restriction, transaction

LTS, SOS, formats

bisimulation, reduction

concurrency, causality, locality, mobility

to structures different than terms

graphs, higher-order graphs, UML-like diagrams