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Dynamic Connectors for Concurrency

Roberto Bruni Univ. Pisa

joint work with

Ugo Montanari Univ. Pisa

Research supported

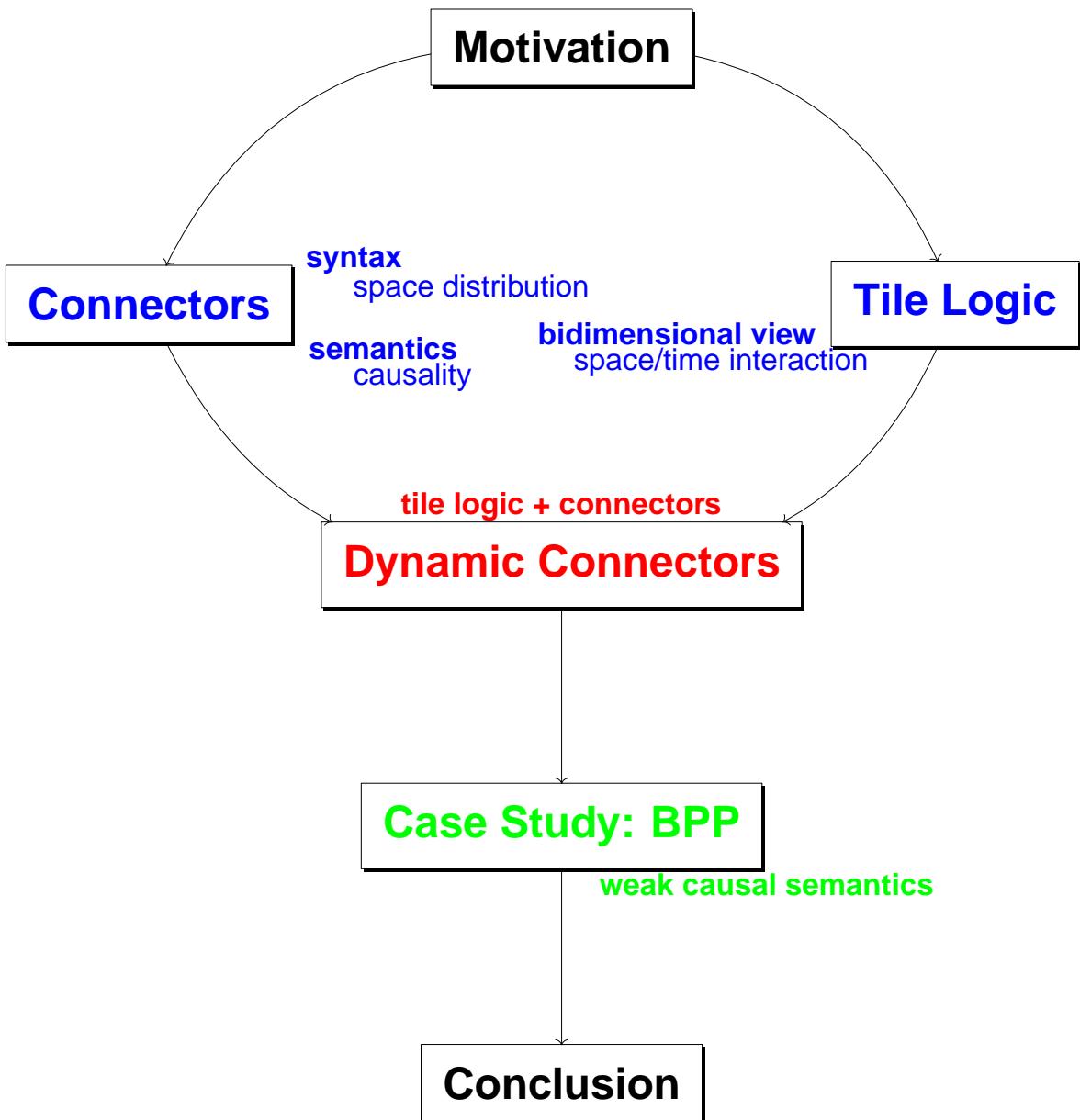
by EC TMR Network *GETGRATS*;

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Submitted draft available at the URL

<http://www.di.unipi.it/~bruni/publications/gsgs.ps.gz>

Sketch of the Talk



Motivation

Keywords: Distribution, Interaction, Reactivity, Orchestration, Mobility, Secrecy, ...

Languages: π -calculus, HO π , spi, join, ambients, mobile and dynamic nets, action calculi, ...

Metamodel:

1. uniform mathematical treatment of kernel features;
2. reusable techniques.

Our Approach: 1. connectors 2. tile logic

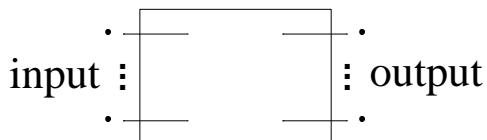
(general approach that can be instantiated, general purpose techniques/results, visual formalism)

Recipe: formats + methodology + proof schemes =

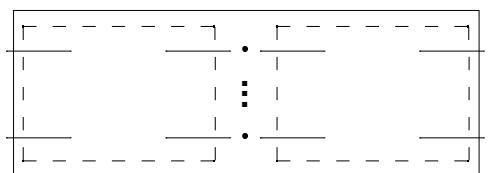
= WIDELY APPLICABLE THEORY

Principles of Connectors

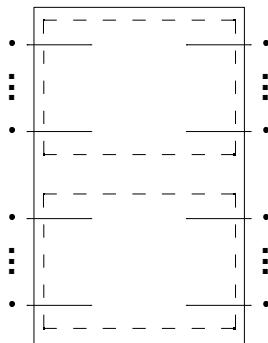
Modules/Boxes



(;)



(\otimes)



Connectors/Links/Wires

ordinary

$$\gamma_{x,y} = \begin{array}{c} x \diagup \diagdown \\ \times \\ y \diagdown \diagup \\ x \end{array}$$

$$\nabla_x = \begin{array}{c} x \\ \diagup \diagdown \\ x \end{array}$$

$$!_x = x - \boxed{!}$$

dual

$$\gamma_{y,x} = \begin{array}{c} y \diagup \diagdown \\ \times \\ x \diagdown \diagup \\ y \end{array}$$

$$\Delta_x = \begin{array}{c} x \\ \diagup \diagdown \\ x \end{array}$$

$$i_x = \boxed{i} - x$$

Symmetries

coherence

$$\begin{array}{ccc} x & \xrightarrow{x} & z \\ & \diagdown & \diagup \\ y & \xrightarrow{z} & x \\ & \diagup & \diagdown \\ z & \xrightarrow{y} & y \end{array} = \begin{array}{ccc} x & \xrightarrow{z} & z \\ & \diagdown & \diagup \\ y & \xrightarrow{x} & x \\ & \diagup & \diagdown \\ z & \xrightarrow{y} & y \end{array}$$

$$\begin{array}{ccc} x & \xrightarrow{y} & x \\ & \diagdown & \diagup \\ y & \xrightarrow{x} & y \\ & \diagup & \diagdown \\ y & \xrightarrow{y} & y \end{array} = \begin{array}{cc} x & x \\ & \diagdown \\ & y & y \end{array}$$

naturality

$$\begin{array}{ccc} x & \xrightarrow{\boxed{f}} & y \\ & \diagdown & \diagup \\ z & \xrightarrow{\boxed{g}} & w \end{array} = \begin{array}{ccc} x & \xrightarrow{z} & \boxed{g} & w \\ & \diagdown & \diagup \\ z & \xrightarrow{x} & \boxed{f} & y \end{array}$$

(co)Duplicators and (co)Dischargers

coherence

$$\begin{array}{c}
 x \otimes y \quad x \otimes y \\
 \diagup \quad \diagdown \quad \diagup \quad \diagdown \\
 x \otimes y = x \quad x \quad y \quad x \\
 \diagdown \quad \diagup \quad \diagdown \quad \diagup \\
 x \otimes y \quad y \otimes y
 \end{array}
 \qquad
 \begin{array}{c}
 x \otimes y \quad x \quad ! \\
 \diagup \quad \diagdown \quad \boxed{!} \\
 x \otimes y = x \quad y \quad !
 \end{array}$$

$$\begin{array}{c}
 x \quad x \quad x \\
 \diagup \quad \diagdown \quad \diagup \\
 x \quad x \quad x \\
 \diagdown \quad \diagup \quad \diagdown \\
 x \quad x \quad x
 \end{array}
 \qquad
 \begin{array}{c}
 x \quad x \quad ! \\
 \diagup \quad \diagdown \quad \boxed{!} \\
 x \quad x \quad x
 \end{array}$$

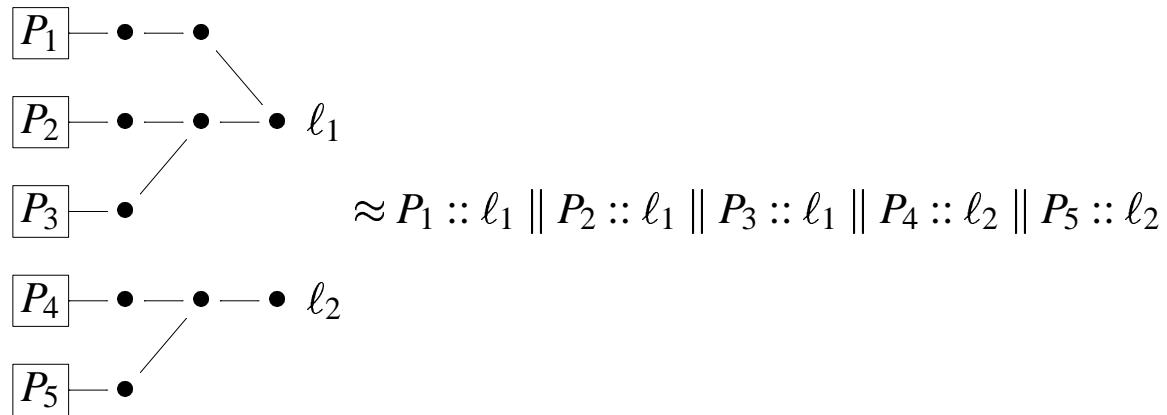
$$\begin{array}{c}
 x \quad x \\
 \diagup \quad \diagdown \quad \diagup \\
 x \quad x \quad x \\
 \diagdown \quad \diagup \quad \diagdown \\
 x \quad x
 \end{array}
 \qquad
 \begin{array}{c}
 x \\
 \diagup \quad \diagdown \\
 x \quad x
 \end{array}$$

naturality

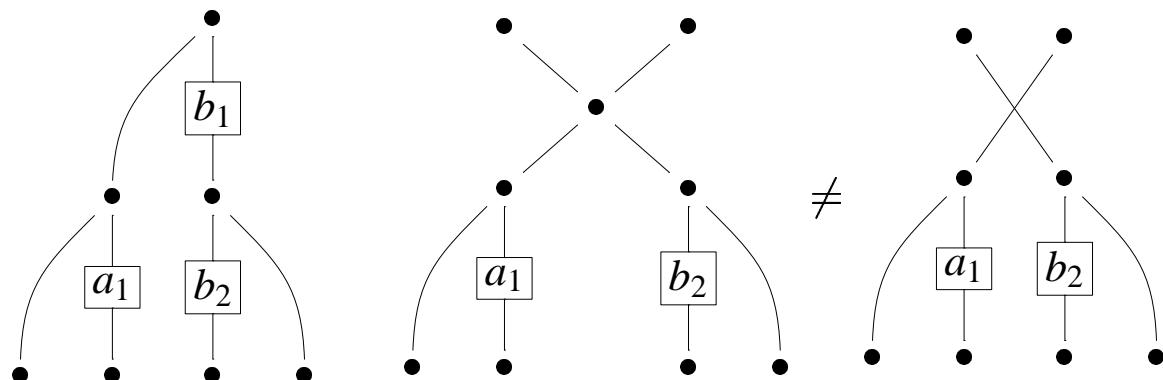
$$\begin{array}{c}
 x \quad y \\
 \diagup \quad \diagdown \quad \boxed{f} \\
 x \quad y
 \end{array}
 \qquad
 \begin{array}{c}
 x \quad f \quad y \\
 \diagup \quad \diagdown \quad \diagup \\
 x \quad x \quad y
 \end{array}
 \qquad
 \begin{array}{c}
 x \quad y \quad ! \\
 \diagup \quad \diagdown \quad \boxed{!} \\
 x \quad x \quad !
 \end{array}$$

Connectors in Space and Time

Spatial distribution



Causal dependencies



Axioms for Connections

$$\begin{array}{c} x \\ \diagup \quad \diagdown \\ x & x \\ \diagdown \quad \diagup \\ x \end{array} = x - x$$

$$\boxed{i} - x - \boxed{!} = e$$

$$\begin{array}{c} x \\ \diagup \quad \diagdown \\ x & x \\ \diagup \quad \diagdown \\ x & x \end{array} = \begin{array}{c} x - x - x - x \\ \diagup \quad \diagdown \quad \diagup \quad \diagdown \\ x & x & x & x \\ \diagdown \quad \diagup \quad \diagdown \quad \diagup \\ x - x - x - x \end{array}$$

$$\boxed{i} - x \begin{array}{c} x \\ \diagup \quad \diagdown \\ x & x \\ \diagdown \quad \diagup \\ x \end{array} = \begin{array}{c} \boxed{i} - x \\ \diagup \quad \diagdown \\ \boxed{i} - x \end{array}$$

$$\begin{array}{c} x \\ \diagup \quad \diagdown \\ x & x \\ \diagup \quad \diagdown \\ x & x \end{array} = \begin{array}{c} x - x - x \\ \diagup \quad \diagdown \\ x & x \\ \diagup \quad \diagdown \\ x - x - x \end{array}$$

$$\begin{array}{c} x \\ \diagup \quad \diagdown \\ x & x \\ \diagup \quad \diagdown \\ x \end{array} - \boxed{!} = \begin{array}{c} x - \boxed{!} \\ \diagup \quad \diagdown \\ x - \boxed{!} \end{array}$$

Investigated by Căzănescu, Ștefănescu, Carboni, Walters, Power, Robinson, Milner, Gardner, Hasegawa, Mowbray, Ferrari, Montanari, Corradini, Gadducci, Bruni, ...

A Taxonomy of Connectors

	ShCat	GSCat	coGSCat	RMCat	MShCat	NBCat	PMCat	DGS Cat	TRMCat
∇	+	+	-	+	+	-	+	+	+
Δ	-	-	+	+	+	-	+	+	+
!	-	+	-	+	-	+	+	-	-
-	-	-	+	+	-	+	+	+	+
$\diamond = -$	-	-	-	+	+	-	+	+	+
$X = \Sigma$	-	-	-	+	+	-	+	+	+
$X = Z$	-	-	-	-	+	-	+	+	-
$H = e$	-	-	-	-	+	-	+	-	-
$\vdash = \models$	-	-	-	-	+	-	-	+	-
$\dashv = \dashv$	-	-	-	-	+	-	-	-	-

Tiles

A **tiles system** is a tuple $\mathcal{R} = (\mathcal{H}, \mathcal{V}, N, R)$ where:

\mathcal{H} monoidal category of **configurations**

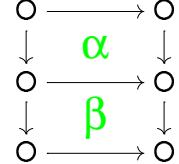
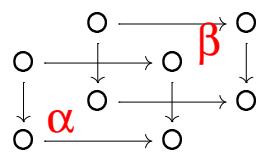
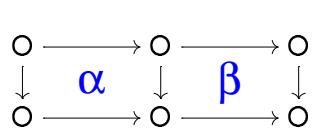
\mathcal{V} monoidal category of **observations**

\mathcal{H} and \mathcal{V} have the same objects (called **interfaces**)

N is the set of *tile names*, $R: N \rightarrow \mathcal{H} \times \mathcal{V} \times \mathcal{V} \times \mathcal{H}$

$$R(\alpha) = \langle s, a, b, t \rangle \quad \text{is written} \quad \alpha: s \xrightarrow[b]{a} t = \begin{array}{c} x \xrightarrow{s} y \\ a \downarrow \quad \alpha \quad \downarrow b \\ z \xrightarrow[t]{a} w \end{array}$$

horizontal, parallel, and **vertical** compositions



identities

$$\begin{array}{ccc} x & \xrightarrow{s} & y \\ id_x \downarrow & id_s & \downarrow id_x \\ x & \xrightarrow[s]{s} & y \end{array}$$

$$\begin{array}{ccc} x & \xrightarrow{id_x} & x \\ a \downarrow & id_a & \downarrow a \\ z & \xrightarrow[id_z]{id_z} & z \end{array}$$

the **tile logic** associated with \mathcal{R} is obtained by adding suitable ‘auxiliary’ tiles and then closing by all kinds of composition.

$$\mathcal{R} \vdash s \xrightarrow[b]{a} t$$

Auxiliary Structure

idea: auxiliary structure shared between \mathcal{H} and \mathcal{V} ('commuting' squares)



$\mathcal{V} \setminus \mathcal{H}$	mon	sym	gs	cart	ccc
mon	MM,BFMM	GM	GM,BFMM	GM	-
sym	(BMM)	BMM	-	-	-
gs	-	-	(FM)	-	-
cart	-	-	-	BMM,BFMM	-
ccc	-	-	-	-	BM

methodology: Given a theory of connectors \mathcal{T} , add

$$\begin{array}{ccc} \cdot & \xrightarrow{\chi_x} & \cdot \\ \chi_x \downarrow & & \downarrow id \\ \cdot & \xrightarrow{id} & \cdot \end{array} \quad \begin{array}{ccc} \cdot & \xrightarrow{id} & \cdot \\ id \downarrow & & \downarrow \chi_x \\ \cdot & \xrightarrow{\chi_x^\perp} & \cdot \end{array}$$

for any connector χ in \mathcal{T} and object x
(plus suitable axioms)

Basic Parallel Processes

action labels: $Act = \{a, b, \dots\}$

process constants: $Const = \{X, Y, \dots\}$

processes: $t ::= \varepsilon \mid X \mid t \parallel t$

(ε distinguished inactive constant, \parallel AC1)

BPP(1,P): $\Omega = \{X_1 \xrightarrow{a_1} t_1, \dots, X_n \xrightarrow{a_n} t_n\}$

$$\frac{X \xrightarrow{a} t \in \Omega}{X \xrightarrow{a} t \in T_\Omega} \qquad \frac{t_1 \xrightarrow{a} t'_1 \in T_\Omega}{t_1 \parallel t_2 \xrightarrow{a} t'_1 \parallel t_2 \in T_\Omega}$$

Ordinary bisimilarity cannot distinguish $X \parallel Y$ from Z in

$$\Omega = \{ \quad X \xrightarrow{a} \varepsilon, Y \xrightarrow{b} \varepsilon, \\ Z \xrightarrow{a} Z_1, Z \xrightarrow{b} Z_2, Z_1 \xrightarrow{b} \varepsilon, Z_2 \xrightarrow{a} \varepsilon \quad \}$$

$(a|b = ab + ba)$

Concurrent Semantics for BPP(1,P)

configurations

$$[[X]] : \underline{0} \rightarrow \underline{1} \quad [[\varepsilon]] = i_1 \quad [[t_1 \parallel t_2]] = ([[t_1]] \otimes [[t_2]]); \Delta_1$$

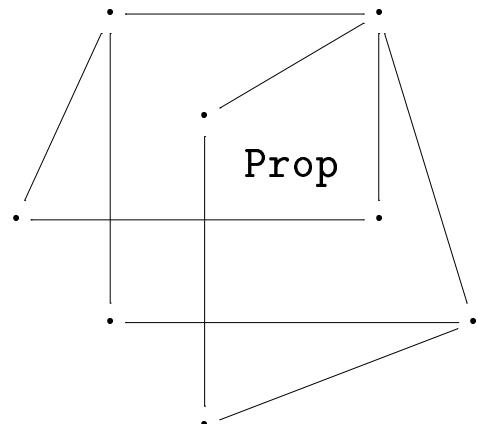
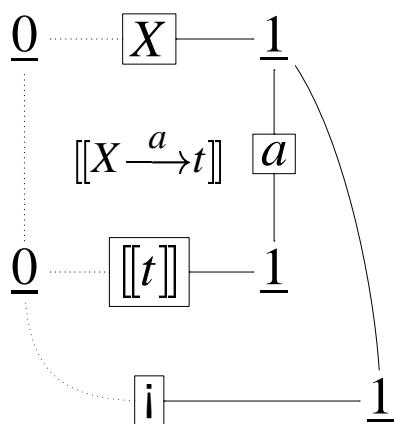
observations

$$[[a]] : \underline{1} \rightarrow \underline{1}$$

basic tiles

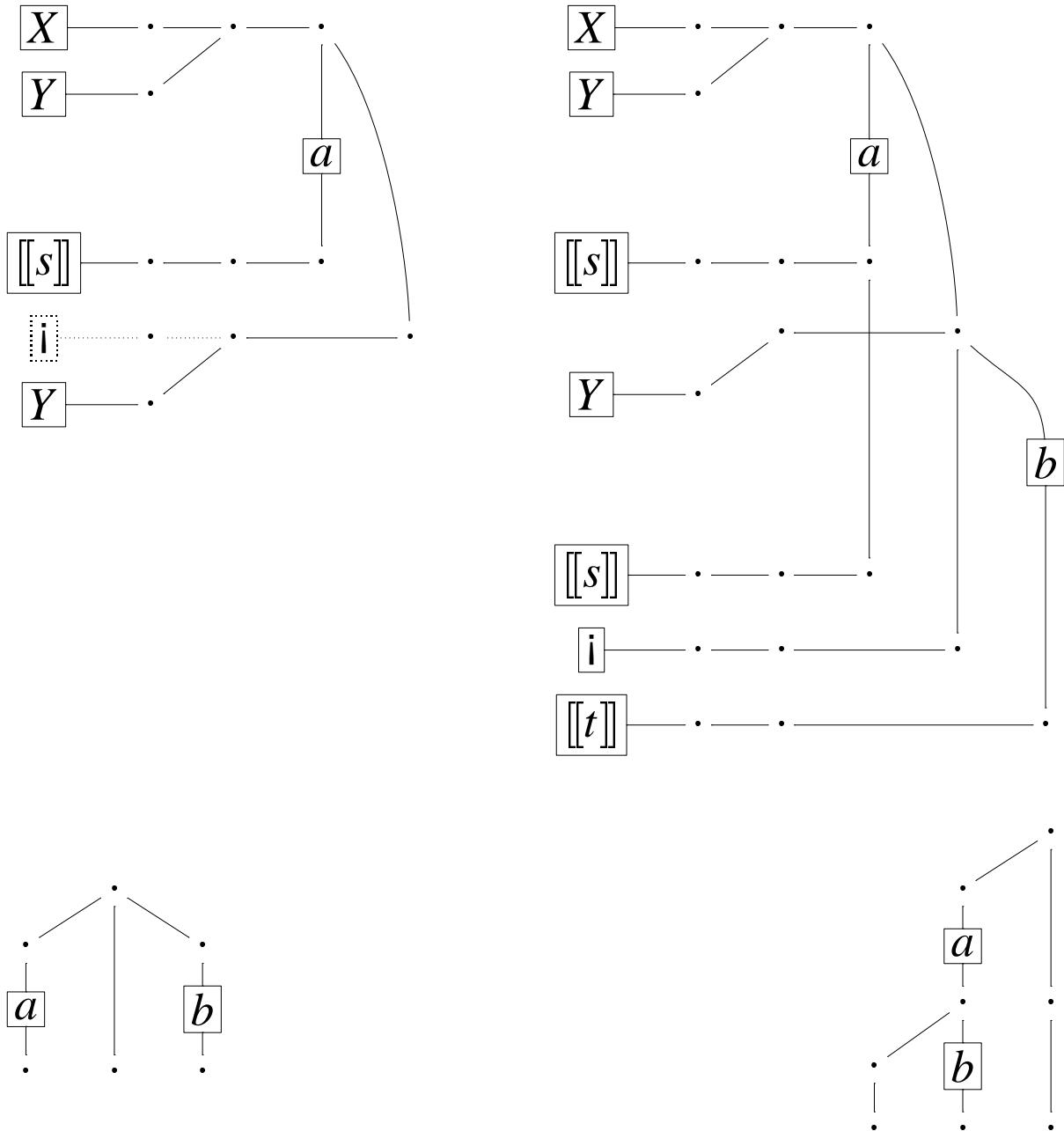
(op vertical category)

auxiliary tiles



Example

$$\Omega = \{ \dots, X \xrightarrow{a} s, Y \xrightarrow{b} t, \dots \}$$

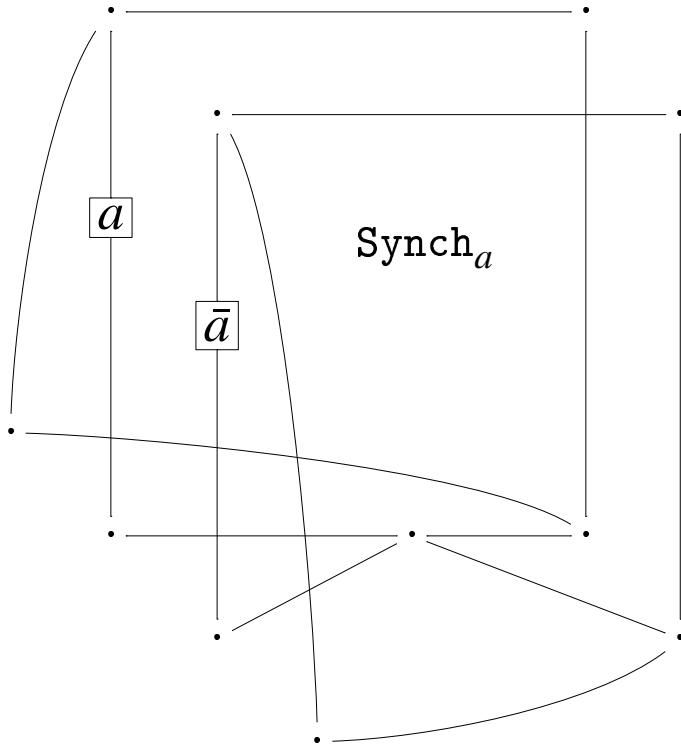


Weak Causal Semantics for ws-BPP(1,P)

Synchronization:

$$\frac{t_1 \xrightarrow{a} t'_1 \in T_\Omega, \quad t_2 \xrightarrow{\bar{a}} t'_2 \in T_\Omega}{t_1 \parallel t_2 \xrightarrow{\tau} t'_1 \parallel t'_2 \in T_\Omega}$$

Basic tiles



Some results: adequacy, complete concurrency, recover causal weak bisimilarity via causal trees [DD], truly concurrent operational machinery

Concluding Remarks

connectors + tiles = concurrent causal semantics

metamodel parametric w.r.t. connectors

tile logic: uniform in space and time

dynamic connectors: fine tuning around issues of interest

methodology:

1. fix configurations and connectors;
2. fix observations, reusing the same connectors;
3. define the basic tiles of the system;
4. dynamic connectors are automatically added.

tile bisimilarity: concurrent abstract semantics

tile formats for congruence result

dynamic tile bisimilarity for open-ended systems