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Dynamic Connectors for Concurrency

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joint work with
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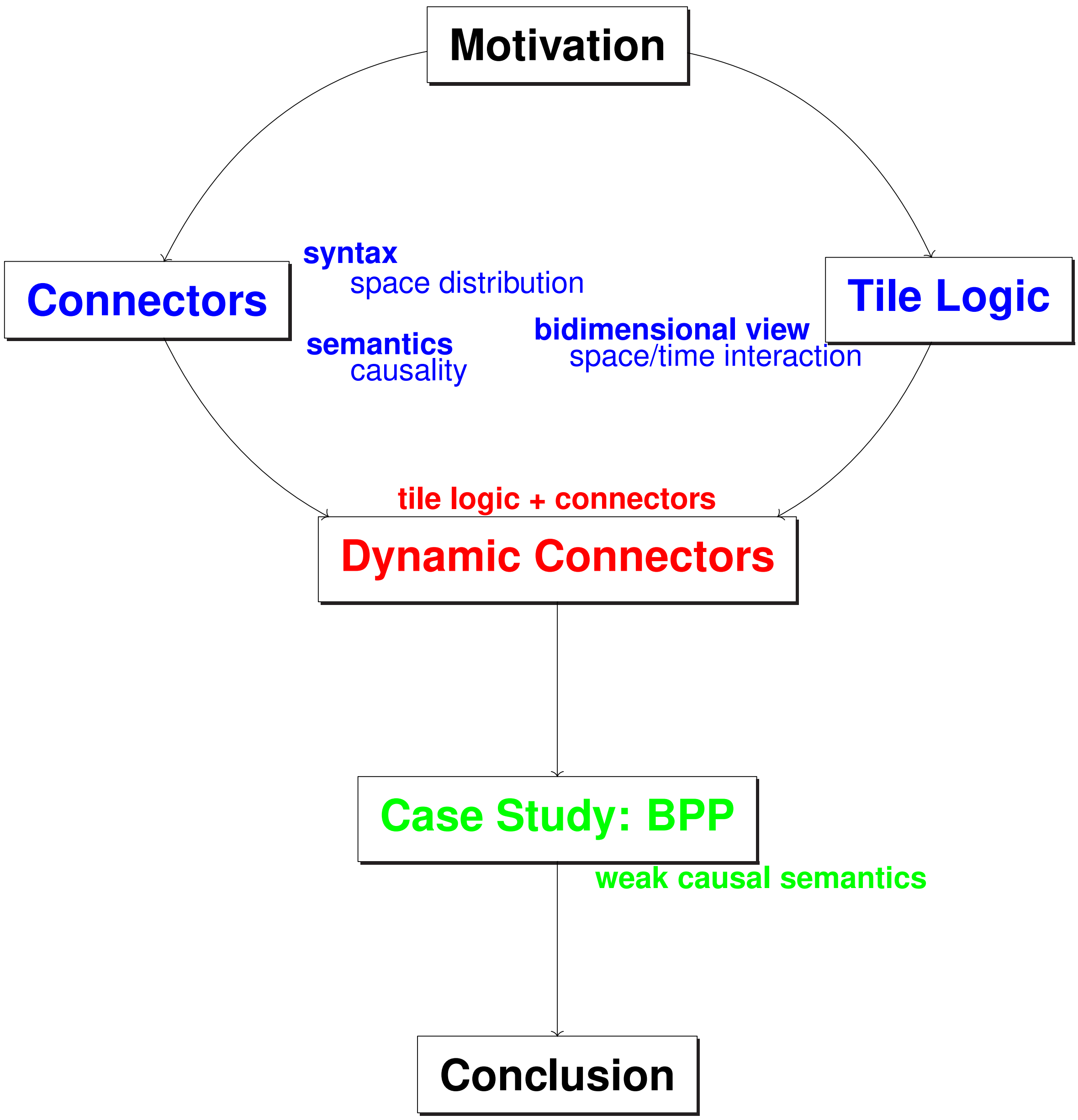
Submitted draft available at the URL

`http://www.di.unipi.it/~bruni/publications/gsgs.ps.gz`

Sketch of the Talk



Motivation

Keywords: Distribution, Interaction, Reactivity, Orchestration, Mobility, Secrecy, ...

Languages: π-calculus, HOπ, spi, join, ambients, mobile and dynamic nets, action calculi, ...

Metamodel:

1. uniform mathematical treatment of kernel features;
2. reusable techniques.

Our Approach: 1. connectors 2. tile logic

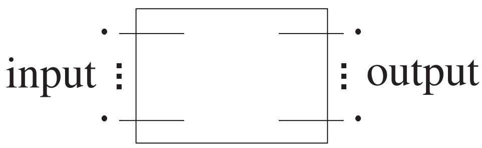
(general approach that can be instantiated, general purpose techniques/results, visual formalism)

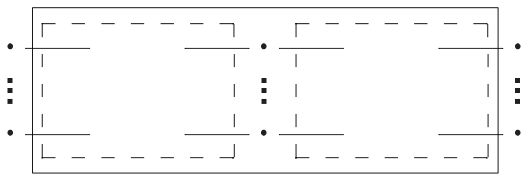
Recipe: formats + methodology + proof schemes =

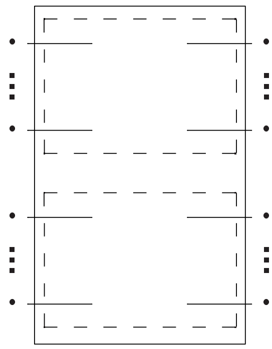
= WIDELY APPLICABLE THEORY

Principles of Connectors

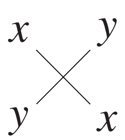
Modules/Boxes

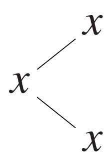


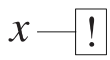
(;) 

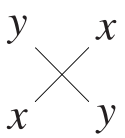
(⊗) 

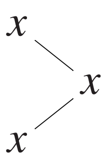
Connectors/Links/Wires

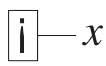
ordinary $\gamma_{x,y} =$ 

$\nabla_x =$ 

$!_x =$ 

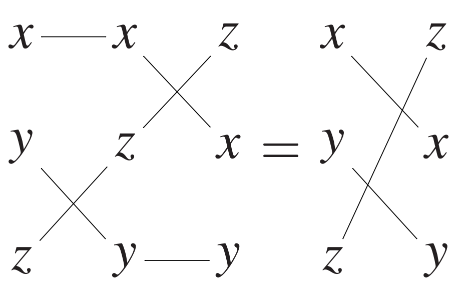
dual $\gamma_{y,x} =$ 

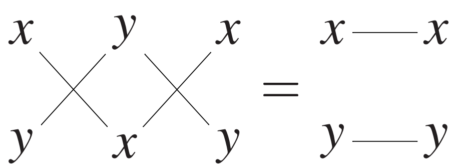
$\Delta_x =$ 

$¡_x =$ 

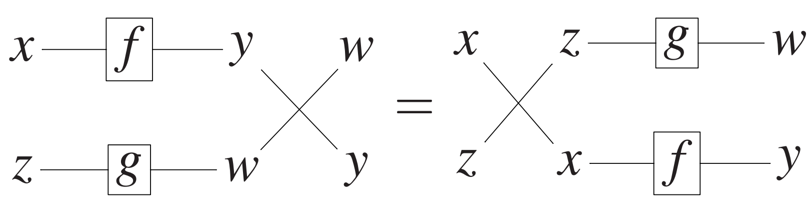
Symmetries

coherence



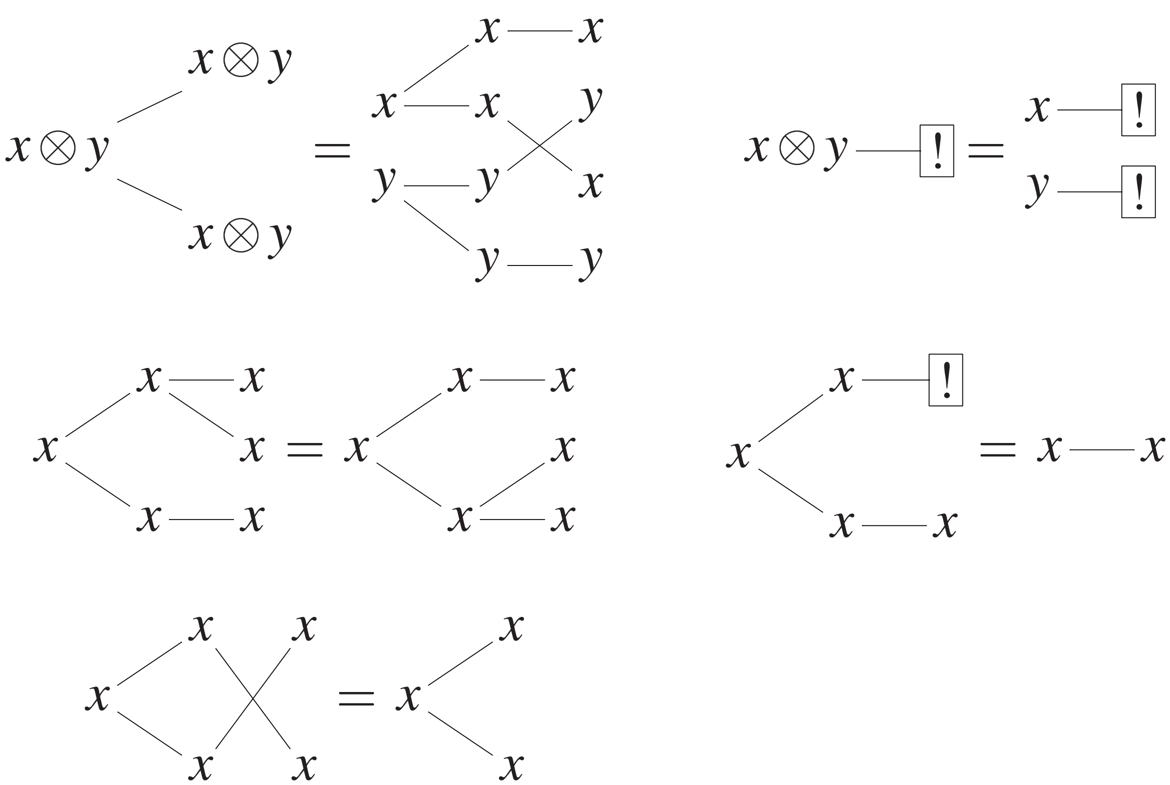


naturality

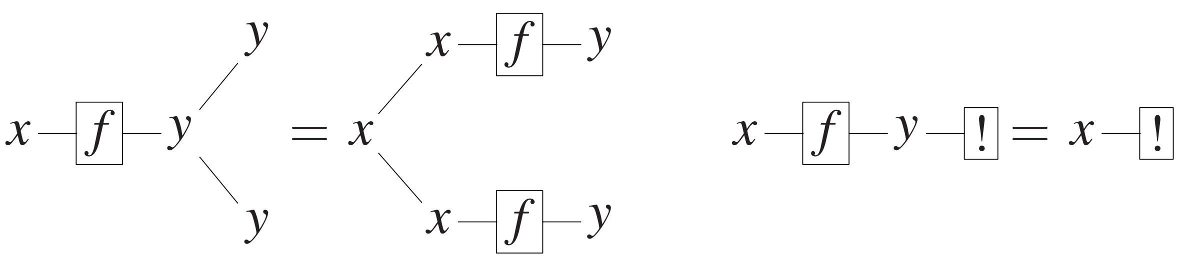


(co)Duplicators and (co)Dischargers

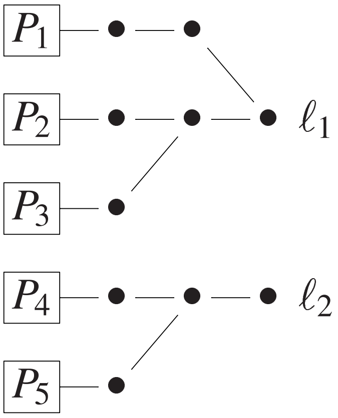
coherence



naturality

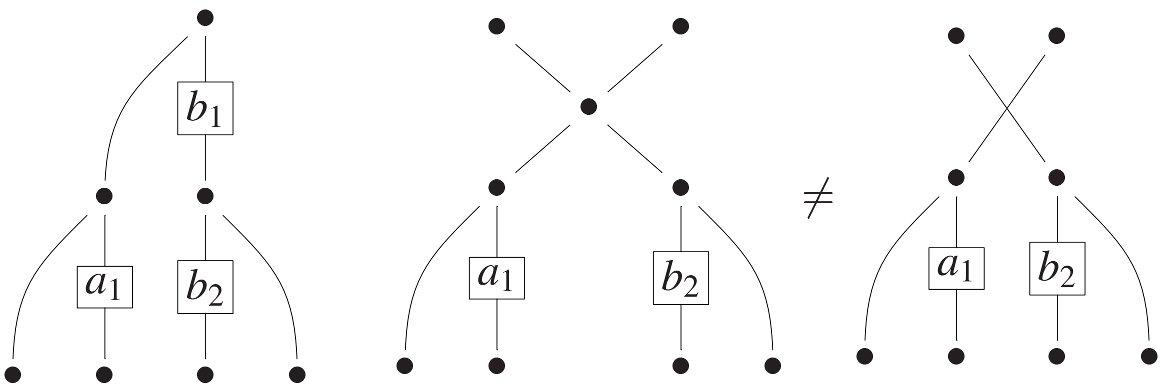


Spatial distribution

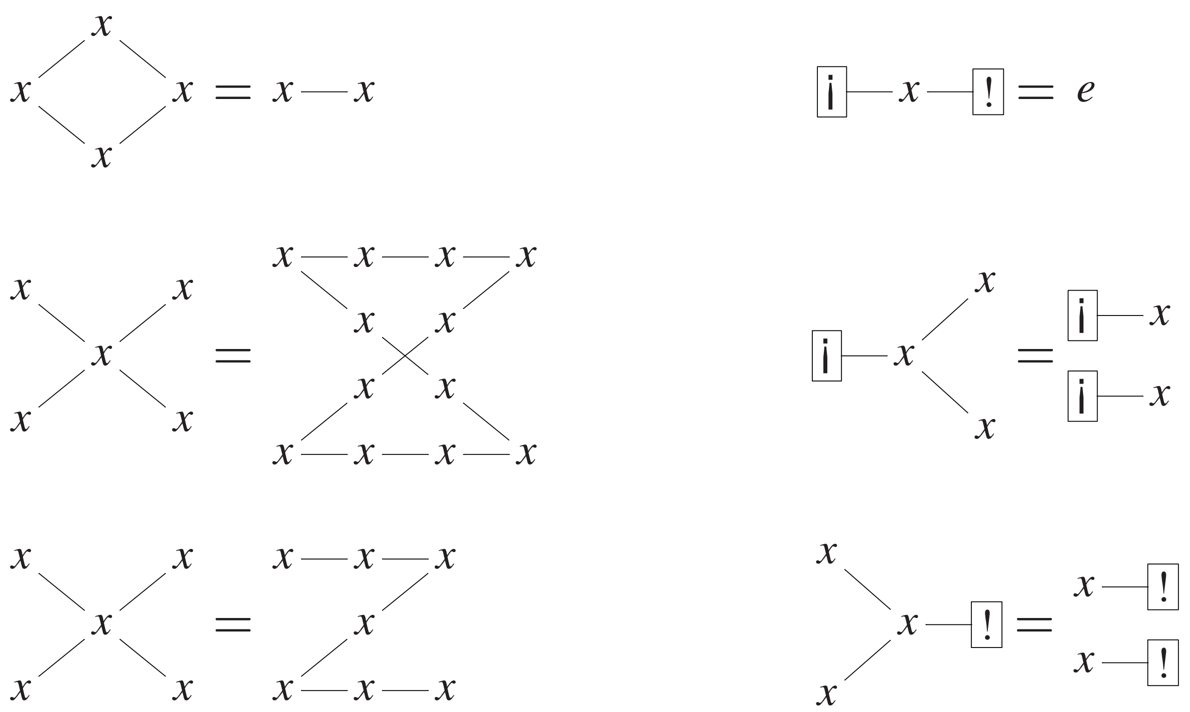


$$\approx P_1 :: \ell_1 \parallel P_2 :: \ell_1 \parallel P_3 :: \ell_1 \parallel P_4 :: \ell_2 \parallel P_5 :: \ell_2$$

Causal dependencies



Axioms for Connections



Investigated by Căzănescu, Ştefănescu, Carboni, Walters, Power, Robinson, Milner, Gardner, Hasegawa, Mowbray, Ferrari, Montanari, Corradini, Gadducci, Bruni, ...

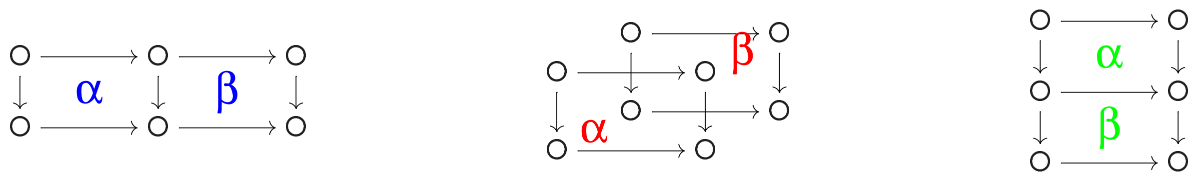
A Taxonomy of Connectors

	ShCat	GSCat	coGSCat	RMCat	MShCat	NBCat	PMCat	DGSCat	TRMCat
∇	+	+	-	+	+	-	+	+	+
Δ	-	-	+	+	+	-	+	+	+
!	-	+	-	+	-	+	+	+	-
¡	-	-	+	+	-	+	+	+	+
$\diamond$ = —	-	-	-	+	+	-	+	+	+
X = ⧖	-	-	-	+	+	-	+	+	+
X = Z	-	-	-	-	+	-	+	+	-
⊢⊣ = e	-	-	-	+	-	+	+	-	-
⊢≺ = ⊨	-	-	-	+	-	-	-	-	+
≻⊣ = ⫤	-	-	-	+	-	-	-	-	-

A ***tiles system*** is a tuple $\mathcal{R} = (\mathcal{H}, \mathcal{V}, N, R)$ where:
$\mathcal{H}$ monoidal category of ***configurations***
$\mathcal{V}$ monoidal category of ***observations***
$\mathcal{H}$ and $\mathcal{V}$ have the same objects (called ***interfaces***)
N is the set of *tile names*, $R\colon N \to \mathcal{H} \times \mathcal{V} \times \mathcal{V} \times \mathcal{H}$

$R(\alpha) = \langle s, a, b, t \rangle$ is written $\alpha\colon s \xrightarrow[b]{a} t \; = \; \begin{array}{ccc} x & \xrightarrow{s} & y \\ a\downarrow & \alpha & \downarrow b \\ z & \xrightarrow[t]{} & w \end{array}$

horizontal, **parallel**, and **vertical** compositions



identities $\quad \begin{array}{ccc} x & \xrightarrow{s} & y \\ id_x\downarrow & id_s & \downarrow id_x \\ x & \xrightarrow[s]{} & y \end{array} \qquad \begin{array}{ccc} x & \xrightarrow{id_x} & x \\ a\downarrow & id_a & \downarrow a \\ z & \xrightarrow[id_z]{} & z \end{array}$

the **tile logic** associated with $\mathcal{R}$ is obtained by adding suitable 'auxiliary' tiles and then closing by all kinds of composition.

$$\mathcal{R} \vdash s \xrightarrow[b]{a} t$$

Auxiliary Structure

idea: auxiliary structure shared between $\mathcal{H}$ and $\mathcal{V}$ (*'commuting' squares*)

$$\begin{tikzcd} x \arrow[r, "s"] \arrow[d, "a"'] \arrow[dr, phantom, "\mathtt{aux}"] & y \arrow[d, "b"] \\ z \arrow[r, "t"'] & w \end{tikzcd} \qquad \begin{tikzcd} & y \arrow[dr, bend left, "b"] \arrow[d, phantom, "="] & \\ x \arrow[ur, bend left, "s"] \arrow[dr, bend right, "a"'] & {} & w \\ & z \arrow[ur, bend right, "t"'] & \end{tikzcd}$$

$\mathcal{V}\backslash\mathcal{H}$	mon	sym	gs	cart	ccc
mon	MM,BFMM	GM	GM,BFMM	GM	-
sym	(BMM)	BMM	-	-	-
gs	-	-	(FM)	-	-
cart	-	-	-	BMM,BFMM	-
ccc	-	-	-	-	BM

methodology: Given a theory of connectors $\mathcal{T}$, add

$$\begin{tikzcd} \cdot \arrow[r, "\chi_x"] \arrow[d, "\chi_x"'] \arrow[dr, phantom, "\chi_x^{\top}"] & \cdot \arrow[d, "id"] \\ \cdot \arrow[r, "id"'] & \cdot \end{tikzcd} \qquad \begin{tikzcd} \cdot \arrow[r, "id"] \arrow[d, "id"'] \arrow[dr, phantom, "\chi_x^{\perp}"] & \cdot \arrow[d, "\chi_x"] \\ \cdot \arrow[r, "\chi_x"'] & \cdot \end{tikzcd}$$

for any connector χ in $\mathcal{T}$ and object x
(plus suitable axioms)

action labels: $Act = \{a, b, \ldots\}$

process constants: $Const = \{X, Y, \ldots\}$

processes: $t ::= \varepsilon \mid X \mid t \parallel t$

(ε distinguished inactive constant, $\parallel$ AC1)

BPP(1,P): $\Omega = \{X_1 \xrightarrow{a_1} t_1, \ldots, X_n \xrightarrow{a_n} t_n\}$

$$\frac{X \xrightarrow{a} t \in \Omega}{X \xrightarrow{a} t \in T_\Omega} \qquad \frac{t_1 \xrightarrow{a} t_1' \in T_\Omega}{t_1 \parallel t_2 \xrightarrow{a} t_1' \parallel t_2 \in T_\Omega}$$

Ordinary bisimilarity cannot distinguish $X \parallel Y$ from Z in

$$\begin{aligned} \Omega \quad = \{ \quad & X \xrightarrow{a} \varepsilon, Y \xrightarrow{b} \varepsilon, \\ & Z \xrightarrow{a} Z_1, Z \xrightarrow{b} Z_2, Z_1 \xrightarrow{b} \varepsilon, Z_2 \xrightarrow{a} \varepsilon \quad \} \end{aligned}$$

($a|b = ab + ba$)

Concurrent Semantics for BPP(1,P)

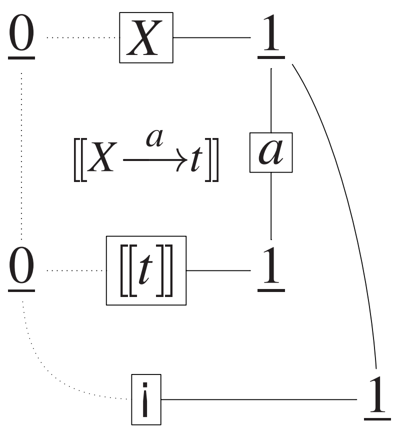
configurations

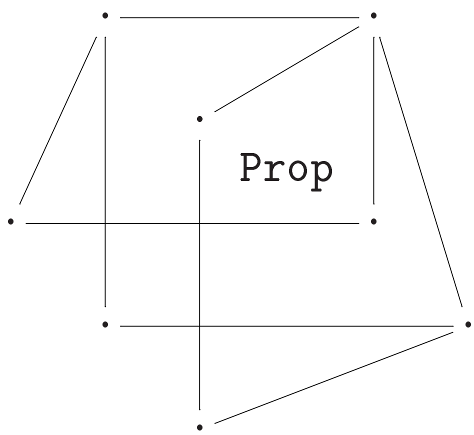
$$[\![X]\!] : \underline{0} \to \underline{1} \qquad [\![\varepsilon]\!] = \mathsf{i}_1 \qquad [\![t_1 \parallel t_2]\!] = ([\![t_1]\!] \otimes [\![t_2]\!]);\Delta_1$$

observations

$$[\![a]\!] : \underline{1} \to \underline{1}$$

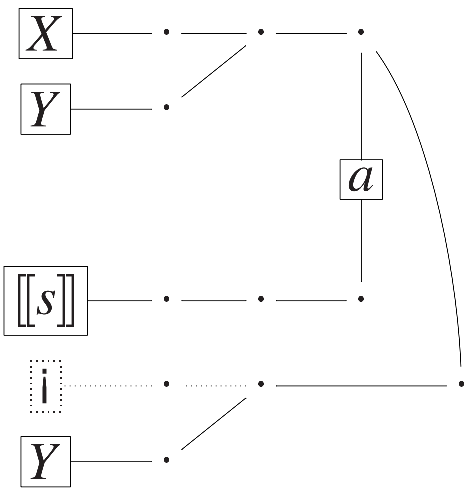
basic tiles (op vertical category) **auxiliary tiles**

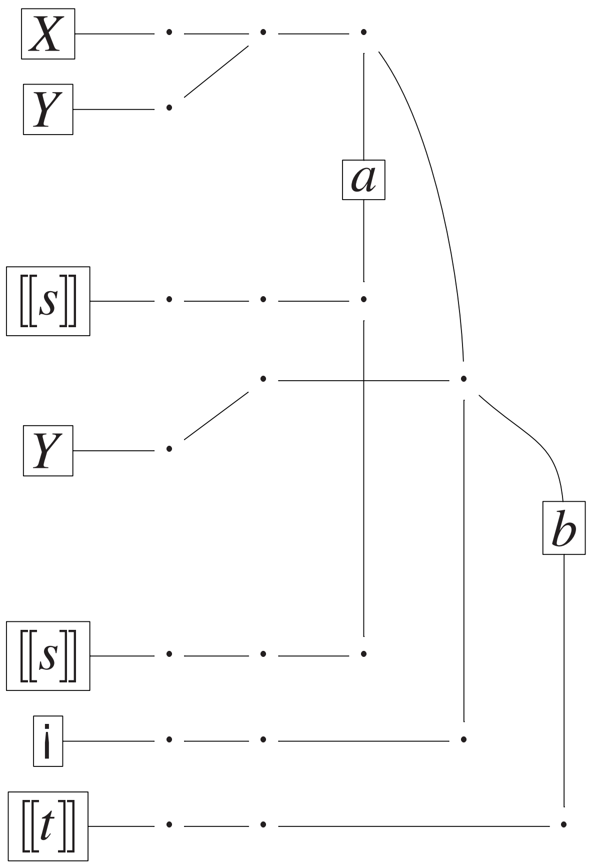


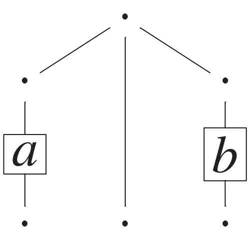


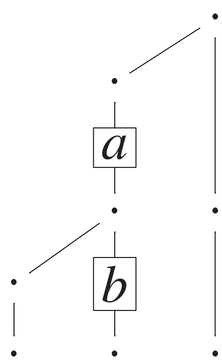
Example

$$\Omega = \{\ldots, X \xrightarrow{a} s, Y \xrightarrow{b} t, \ldots\}$$



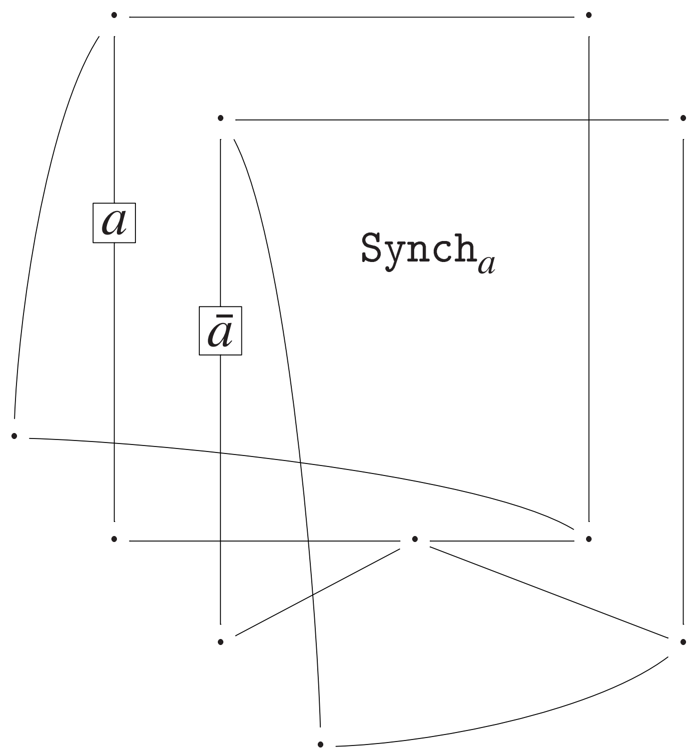






Synchronization:
$$\frac{t_1 \xrightarrow{a} t_1' \in T_\Omega, \;\; t_2 \xrightarrow{\bar{a}} t_2' \in T_\Omega}{t_1 \parallel t_2 \xrightarrow{\tau} t_1' \parallel t_2' \in T_\Omega}$$

Basic tiles



Some results: adequacy, complete concurrency, recover causal weak bisimilarity via causal trees [DD], truly concurrent operational machinery

Concluding Remarks

connectors + tiles = concurrent causal semantics

metamodel parametric w.r.t. connectors

tile logic: uniform in space and time

dynamic connectors: fine tuning around issues of interest

methodology:

1. fix configurations and connectors;

2. fix observations, reusing the same connectors;

3. define the basic tiles of the system;

4. dynamic connectors are automatically added.

tile bisimilarity: concurrent abstract semantics

tile formats for congruence result

dynamic tile bisimilarity for open-ended systems