Non sequential Behaviour of Dynamic Nets

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Adding Dynamicity to PN

- Coloured nets

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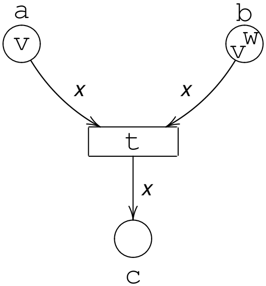

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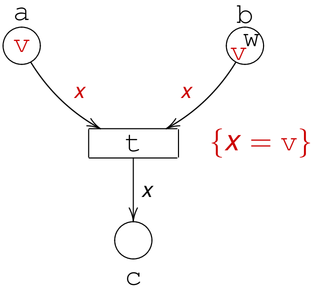

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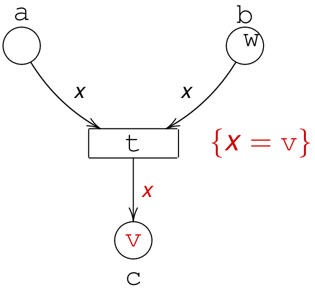

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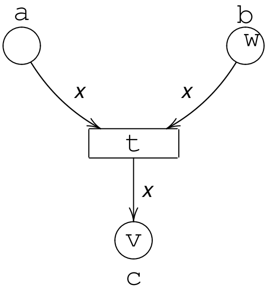

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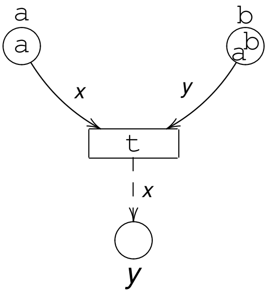

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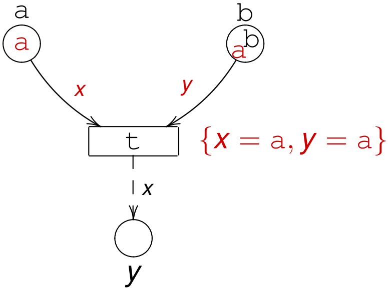

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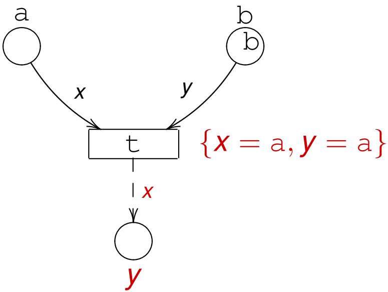

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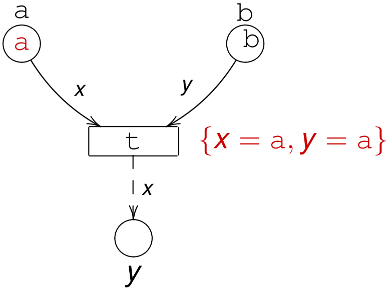

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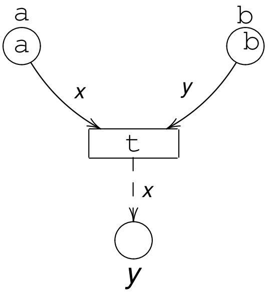

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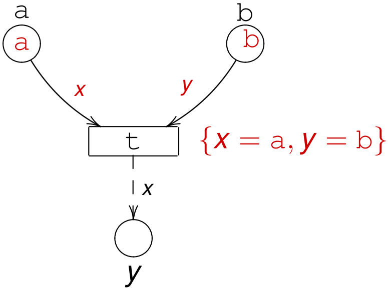

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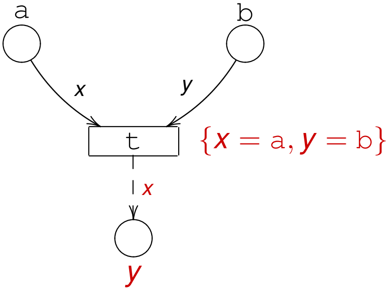

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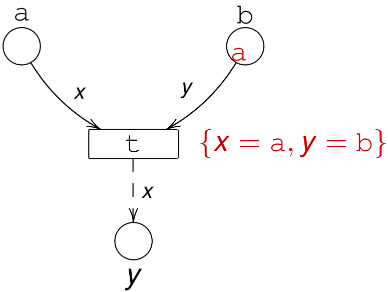

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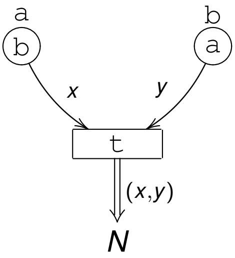

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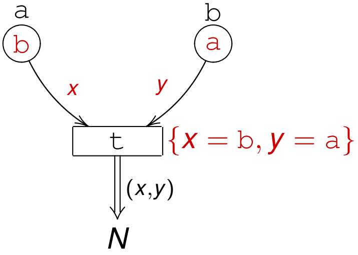

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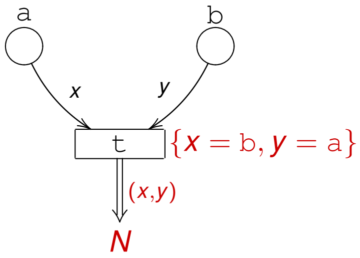

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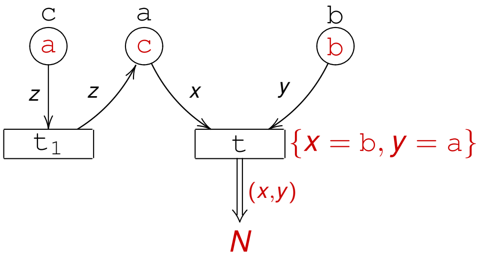

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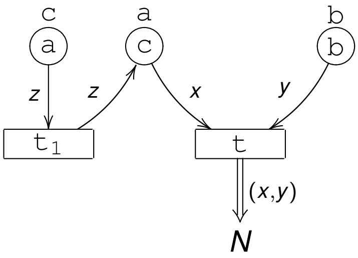

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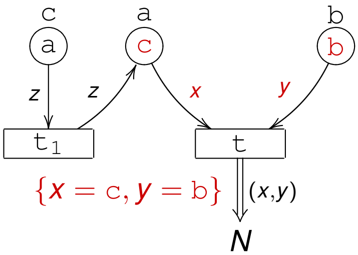

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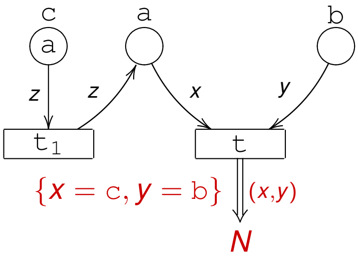

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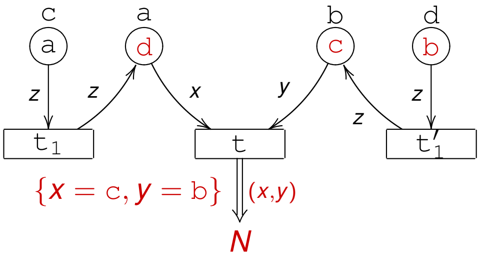

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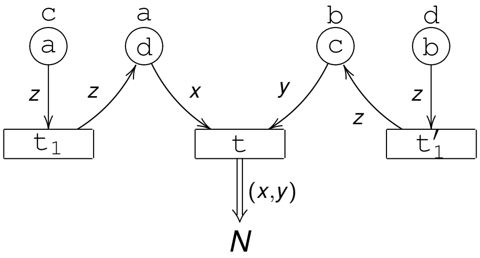

Dynamic Nets

Names for

- Places: $\mathcal{P} = \{\mathrm{a}, \mathrm{b}, \ldots\}$.
- Variables: $\mathcal{X} = \{x, y, \ldots\}$. Moreover $\mathcal{X} \cap \mathcal{P} = \emptyset$.
- Basic Colours: $\mathcal{C} = \mathcal{P} \cup \mathcal{X}$ ranged over by $c_1, c_2, \ldots$..
- Colours: $C^* = \{(c_1, \ldots, c_n) \mid \forall i \ s.t. \ 0 \leq i \leq n : c_i \in C\}$

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Definition (Coloured Multiset over S and C)

- Coloured Multiset: $m : S \rightarrow C \rightarrow \mathbb{N}$.
- Set of all finite (coloured) multisets over S and C^*: $\mathcal{M}_{S,C}$.

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Definition (DN)

DN is the least set satisfying:

$$\mathcal{N} = \{(S_N, T_N, \delta_{0N}, \delta_{1N}, m_{0N}) \mid S_N \subseteq \mathcal{P} \;\wedge\; \delta_{0N} : T_N \rightarrow \mathcal{M}_{S_N,\mathcal{C}} \;\wedge\; \delta_{1N} : T_N \rightarrow \mathcal{N} \;\wedge\; m_{0N} \in \mathcal{M}_{\mathcal{C},\mathcal{C}}\}$$

Elements of DN

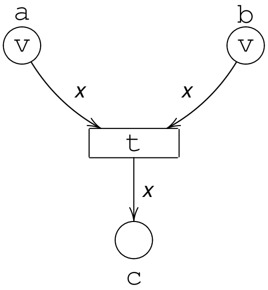
A C-P/T net

- $N = (\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, \{\mathtt{t}\}, \delta_{0N}, \delta_{1N}, \mathtt{a}(\mathtt{v}) \oplus \mathtt{b}(\mathtt{v}))$

  with

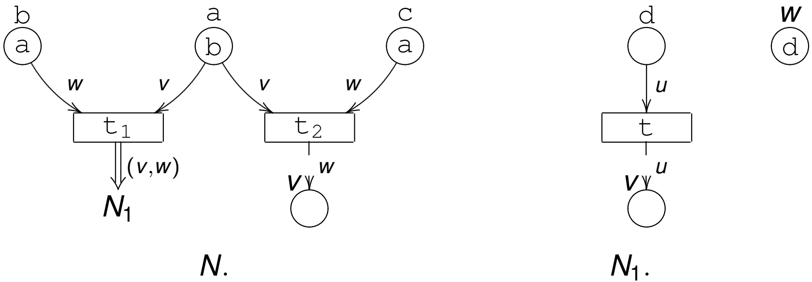
  - $\delta_{N_0}(\mathtt{t}) = \mathtt{a}(x) \oplus \mathtt{b}(x)$, and
  - $\delta_{N_1}(\mathtt{t}) = (\emptyset, \emptyset, \emptyset, \emptyset, \mathtt{c}(x))$

    (Written also $\mathtt{t} = \mathtt{a}(x) \oplus \mathtt{b}(x)[\rangle \mathtt{c}(x)$)



Elements of DN

A Dynamic Net



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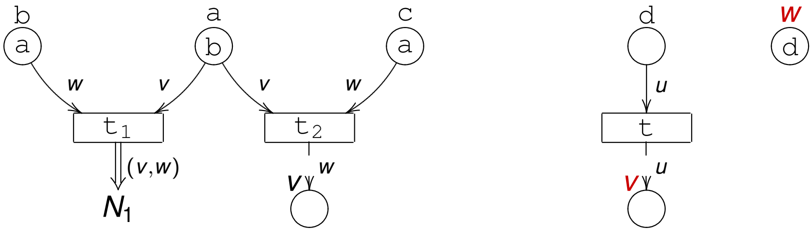

N. N_1.

- Names v and w are free in N_1 !!!

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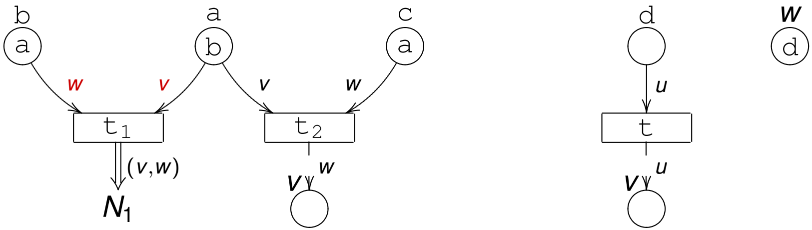

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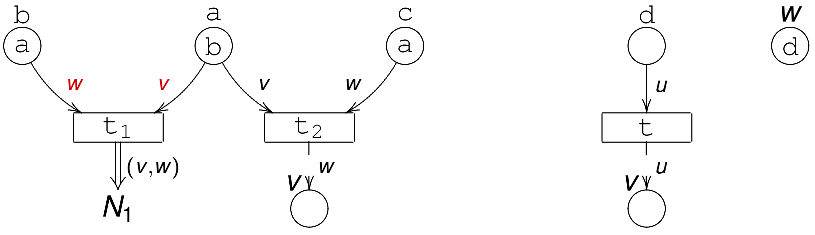

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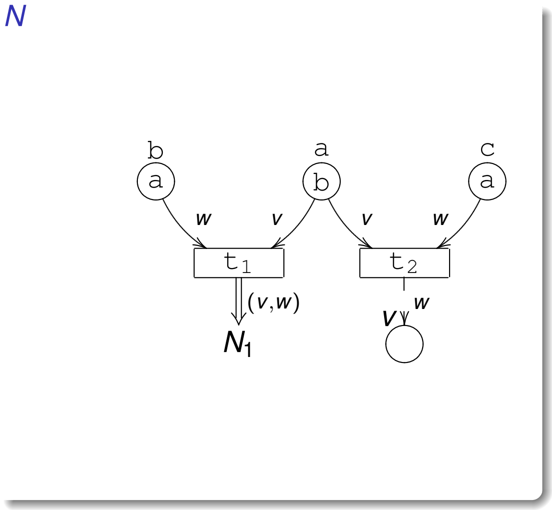
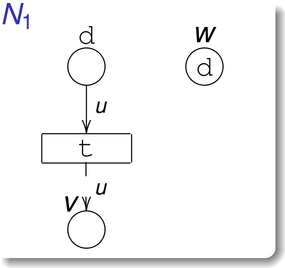
Definition (Dynamic Net)

$N \in$ DN is a *dynamic net* if $fn(N) = \emptyset$.

Firing: Example





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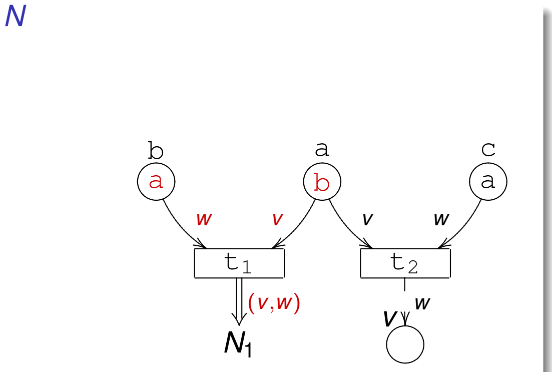
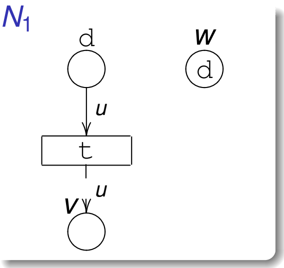


$$\sigma = \{v = \mathrm{b}, w = \mathrm{a}\}$$



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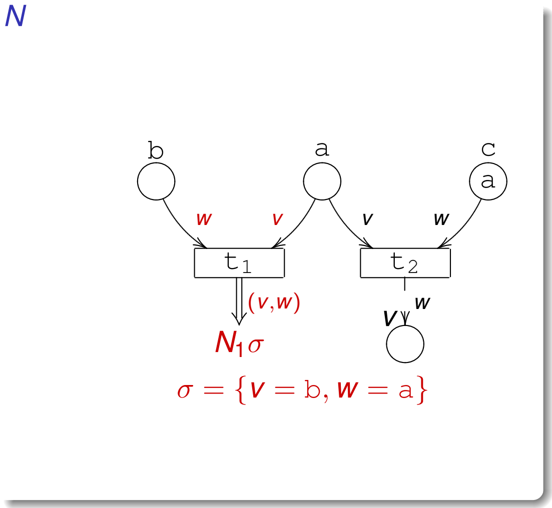

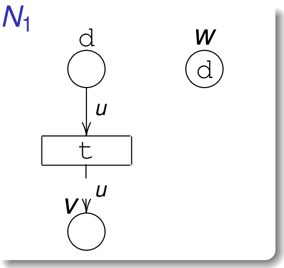

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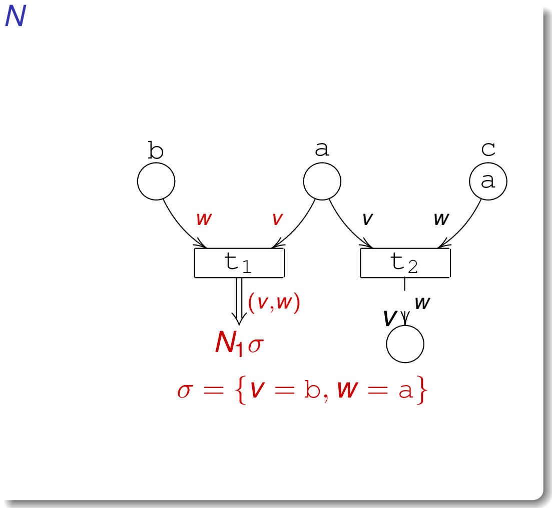
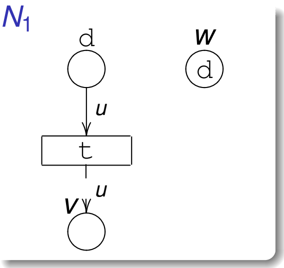
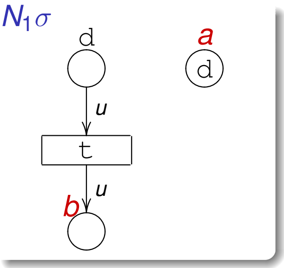
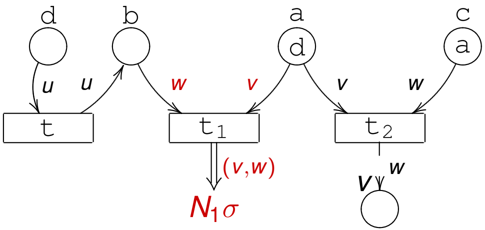






Firing: Example

$N \oslash N_1\sigma$



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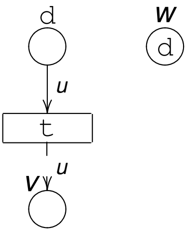

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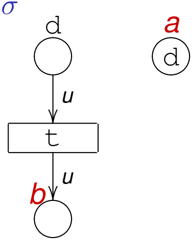

Firing

Operational semantics

(DYN-FIRING)

$$\frac{\mathtt{t} = m \,[\rangle\, N_1 \in T \qquad m'' \in \mathcal{M}_{S,\mathcal{C}}}{(S, T, m\sigma \oplus m'') \rightarrow (S, T, m'') \otimes N_1\sigma}$$

$rn(\mathtt{t}) \subseteq dom(\sigma)$ and $range(\sigma) \subseteq S$

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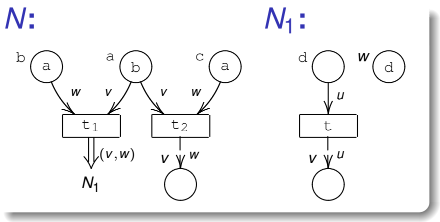
(DYN-SEQ)

$$\frac{N_1 \rightarrow N_1'' \quad N_1'' \rightarrow N_1'}{N_1 \rightarrow N_1'}$$

Unfolding a Dynamic Nets



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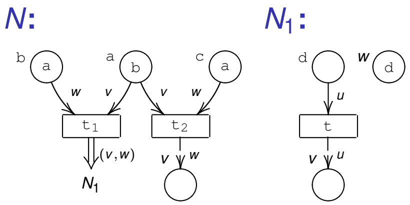

Dynamic Part:

Markings:

Unfolding a Dynamic Nets

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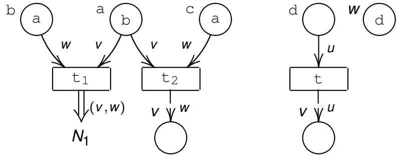

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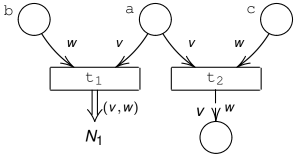

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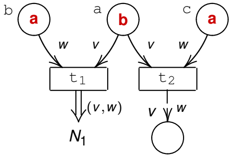

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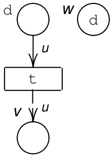

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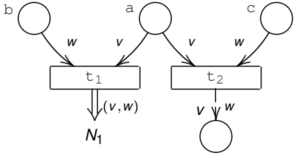

Markings:

$s_1 = (\emptyset, \mathbf{b}(\mathbf{a}), \mathbf{1})$ $s_2 = (\emptyset, \mathbf{a}(\mathbf{b}), \mathbf{1})$ $s_3 = (\emptyset, \mathbf{c}(\mathbf{a}), \mathbf{1})$

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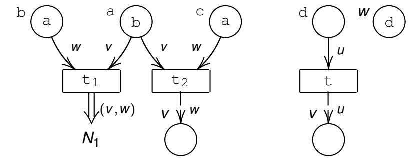

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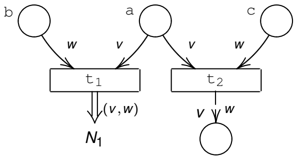

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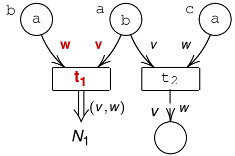

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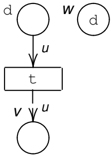

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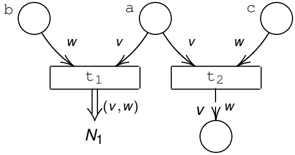

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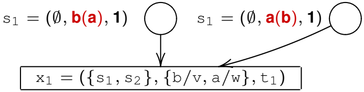

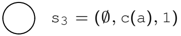

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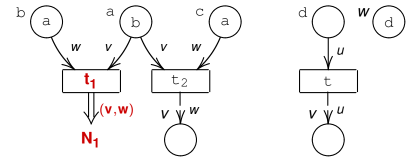

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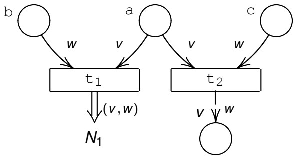
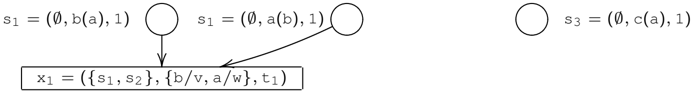
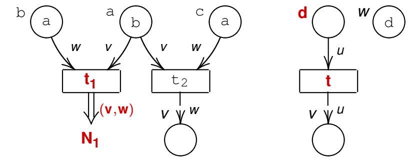


Markings:

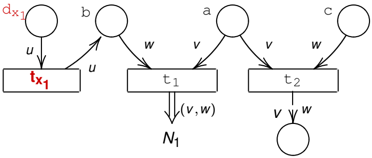


Unfolding a Dynamic Nets

N: N_1:



Dynamic Part:



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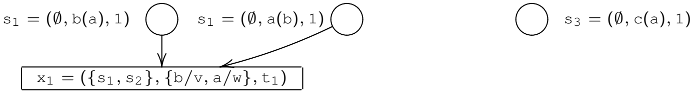

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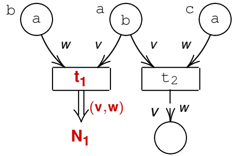

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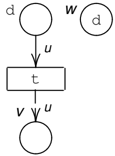

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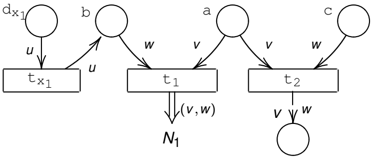

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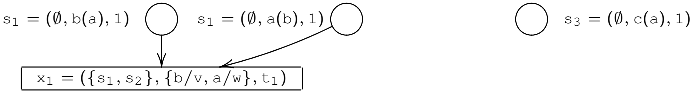

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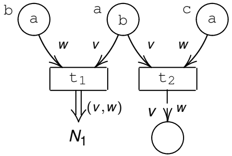

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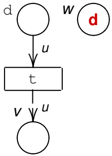

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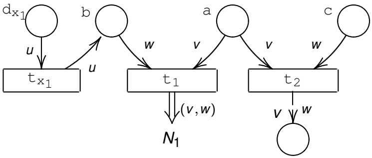

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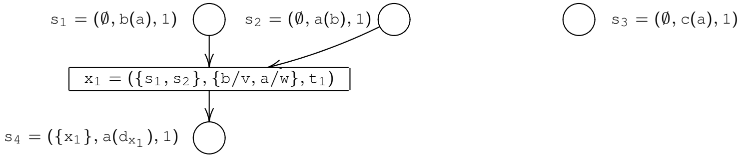

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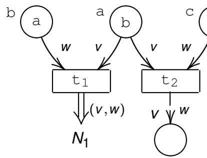

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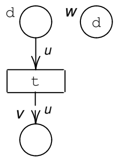

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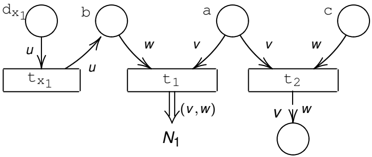

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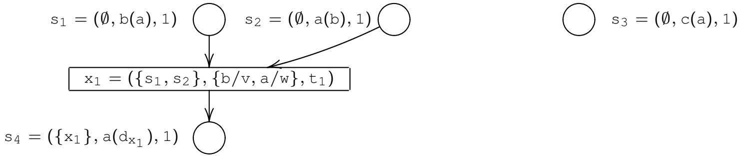

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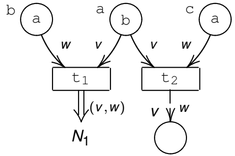

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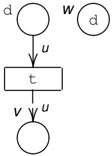

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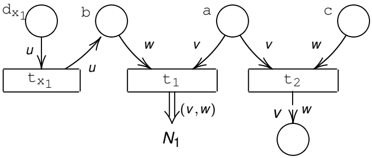

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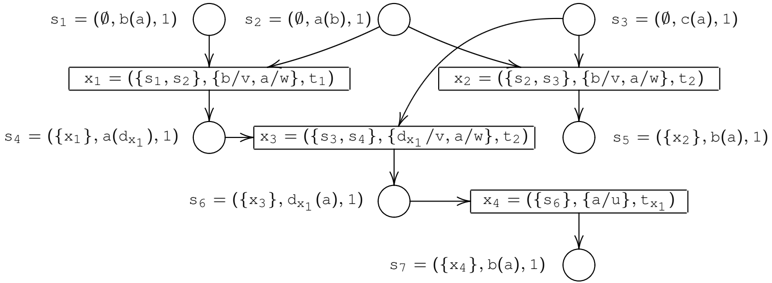

Dynamic Net Unfolding

$\mathcal{U}[N] = (S, T, \delta_0, \delta_1, \mathcal{S}, \mathcal{T}, \xi_0, \xi_1)$:

- $(S, T, \delta_0, \delta_1)$ is an occurrence net
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$$\text{(INI-PL)}\ \frac{\mathtt{a} \in S_N}{\mathtt{a} \in \mathcal{S}} \qquad \text{(INI-TR)}\ \frac{\mathtt{t} \in T_N}{\mathtt{t} \in \mathcal{T}, \quad \xi_0(\mathtt{t}) = \delta_{0N}(\mathtt{t}), \quad \xi_1(\mathtt{t}) = \delta_{1N}(\mathtt{t})}$$

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(INI-TR)
$$\frac{\mathtt{t} \in T_N}{\mathtt{t} \in \mathcal{T}, \quad \xi_0(\mathtt{t}) = \delta_{0N}(\mathtt{t}), \quad \xi_1(\mathtt{t}) = \delta_{1N}(\mathtt{t})}$$

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$$\frac{m_N(\mathtt{a})(c) = n}{\{(\emptyset, \mathtt{a}(c))\} \times [n] \subseteq S}$$

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(PRE)
$$\frac{B = \{(\epsilon_j, b_j, i_j) | j \in J\} \subseteq S,\ Co(B),\ \mathtt{t} \in \mathcal{T},\ \xi_0(\mathtt{t})\sigma = \oplus_{j \in J} b_j}{(B, \sigma, \mathtt{t}) \in T, \quad \delta_0(B, \sigma, \mathtt{t}) = B}$$

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(POST)
$$\frac{x = (B, \sigma, \mathtt{t}) \in T,\ \ \xi_1(\mathtt{t}) = N_1}{\begin{array}{c} Q = \{(\{x\}, b(c), i) \mid 0 < i \leq m_{N_1} \rho_x \sigma(\mathtt{b})(c)\} \subseteq S, \quad \delta_1(x) = Q, \quad S_{N_1}\rho_x \subseteq \mathcal{S}, \\ T_{N_1}\rho_x \subseteq \mathcal{T}, \quad \textit{for } \mathtt{t} \in T_{N_1} : \xi_0(\mathtt{t}\rho_x) = \delta_{0N_1}(\mathtt{t})\rho_x, \xi_1(\mathtt{t}\rho_x) = \delta_{1N_1}(\mathtt{t})\rho_x\sigma \end{array}}$$

Process of a Dynamic Net

Process of a dynamic net for N

A net morphism $P : K \rightsquigarrow N$ from a causal net K to $C = (S, T, \delta_0, \delta_1)$ s.t. $P(^\circ K) = {}^\circ C$, where $\mathcal{U}[N] = (S, T, \delta_0, \delta_1, \mathcal{S}, \mathcal{T}, \xi_0, \xi_1)$.

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Theorem (Correspondence)

$N \longrightarrow^* N'$ iff $\exists P : K \rightsquigarrow N$ s.t.:

(i) $pre(P) = m_{0N}$ *and* $post(P) = m_{0N'}$;

(ii) $(S_{N'}, T_{N'}, \delta_{0N'}, \delta_{1N'}, m) = N \oplus \bigoplus_{x=(B,\sigma,\mathtt{t})\in P(T_K)} \mathtt{t}^{\bullet}(\rho_x, \sigma)$;

Unfolding pattern

The unfolding depends on the colours carried on by tokens.
Nevertheless, some colours are irrelevant.

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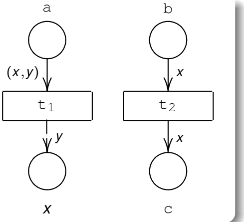

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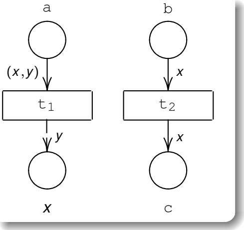

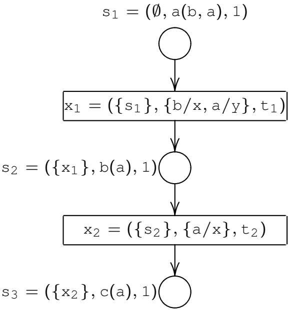

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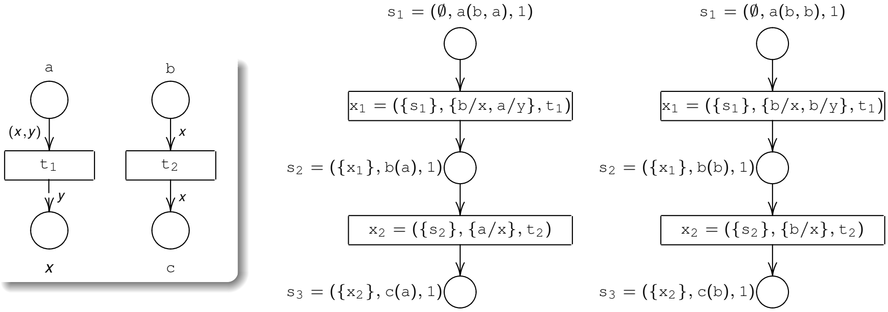

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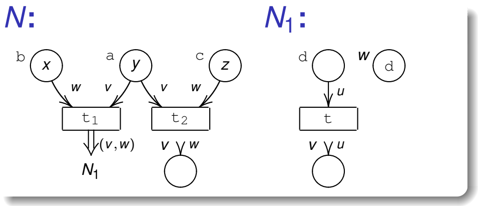

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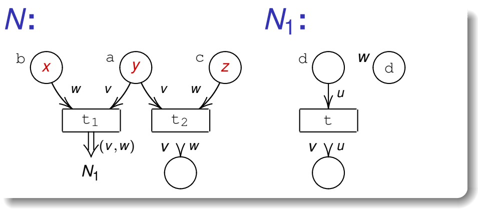
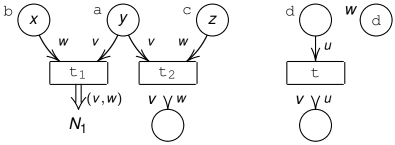


Unfolding pattern

N:

N_1 :

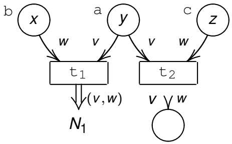


Dynamic Part:

Markings:

Unfolding pattern

N:



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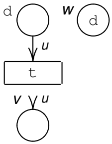
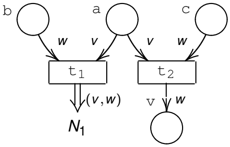


Dynamic Part:



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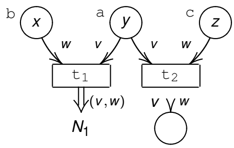

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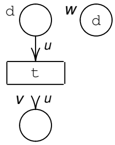

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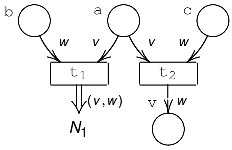

Markings:

$s_1 = (\emptyset, b(x), \emptyset, 1)$ ◯ $s_2 = (\emptyset, a(y), \emptyset, 1)$ ◯ ◯ $s_1 = (\emptyset, c(z), \emptyset, 1)$

Unfolding pattern

N:

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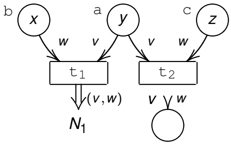

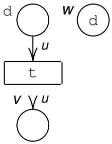

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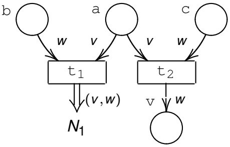

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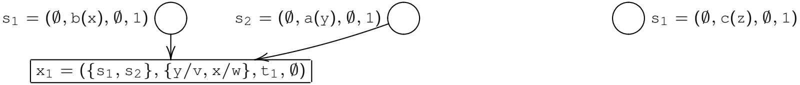

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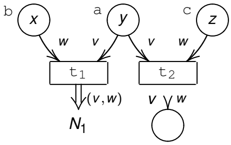

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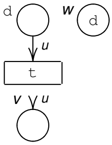

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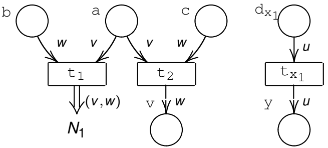

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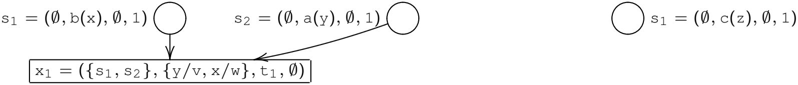

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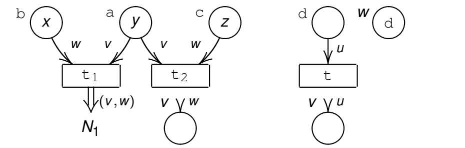

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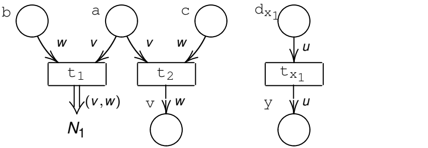

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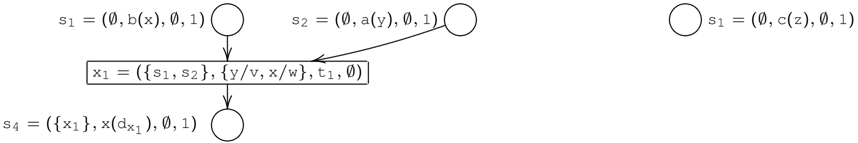

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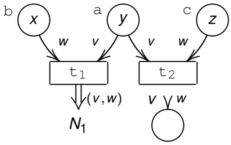

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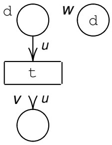

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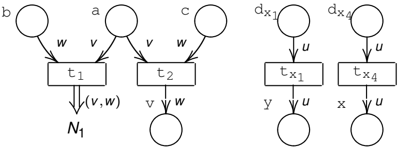

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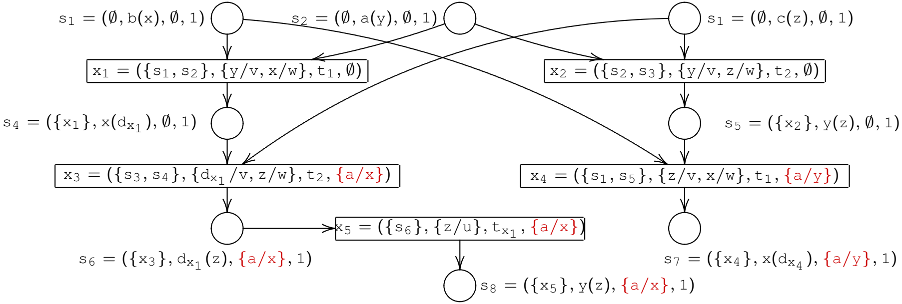


Dynamic Net Unfolding Pattern

The unfolding pattern of N

$\mathcal{U}[N] = (S, T, \delta_0, \delta_1, \mathcal{S}, \mathcal{T}, \xi_0, \xi_1)$:

- $(S, T, \delta_0, \delta_1)$ is an (reconfigurable occurrence net)
- $(\mathcal{S}, \mathcal{T}, \xi_0, \xi_1, \emptyset) \in$ DN,

(INI-PL-PATT)

$$\frac{\mathtt{a} \in S_N}{\mathtt{a} \in \mathcal{S}}$$

(INI-TR-PATT)

$$\frac{\mathtt{t} \in T_N}{\mathtt{t} \in \mathcal{T}, \quad \xi_0(\mathtt{t}) = \delta_{0N}(\mathtt{t}), \quad \xi_1(\mathtt{t}) = \delta_{1N}(\mathtt{t})}$$

(INI-MK-PATT)

$$\frac{m_N(\mathtt{a})(c) = n}{\{(\emptyset, \mathtt{a}(c), \emptyset)\} \times [n] \subseteq S}$$

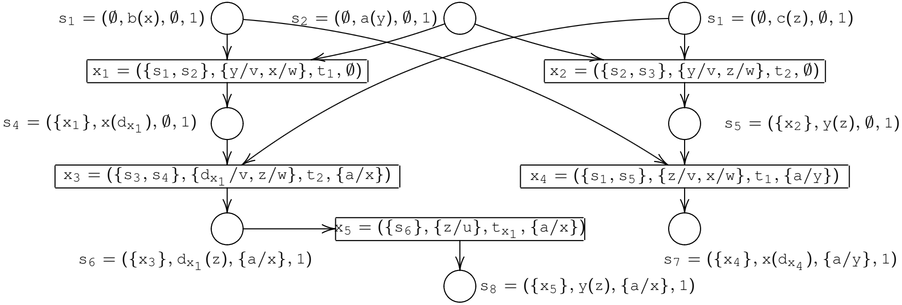
(PRE-PATT)

$$\frac{B = \{(\epsilon_j, b_j, \mu_j, i_j) | j \in J\} \subseteq S, \quad Co(B), \quad \mathtt{t} \in \mathcal{T}, \quad \xi_0(\mathtt{t})\sigma = \oplus_{j \in J} b_j \mu_{\mathtt{t}}, \quad range(\mu_{\mathtt{t}}) \subseteq S_N, \quad \mu = \mu_{\mathtt{t}} \cup \bigcup_j \mu_j \ well-defined\ substitution}{(B, \sigma, \mathtt{t}, \mu) \in T, \quad \delta_0(B, \sigma, \mathtt{t}, \mu) = B}$$

(POST-PATT)

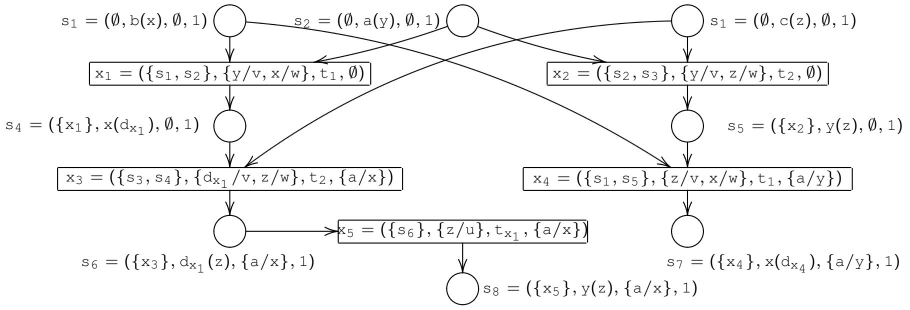
$$\frac{x = (B, \sigma, \mathtt{t}, \mu) \in T, \quad \xi_1(\mathtt{t}) = N_1}{Q = \{(\{x\}, \mathtt{b}(c), \mu, i) \mid 0 < i \leq m_{N_1}\rho_x\sigma(\mathtt{b})(c)\} \subseteq S, \quad \delta_1(x) = Q, \quad S_{N_1}\rho_x \subseteq \mathcal{S}, \quad T_{N_1}\rho_x \subseteq \mathcal{T}, \quad for\ \mathtt{t} \in T_{N_1} : \xi_0(\mathtt{t}\rho_x) = \delta_{0N_1}(\mathtt{t})\rho_x, \xi_1(\mathtt{t}\rho_x) = \delta_{1N_1}(\mathtt{t})\rho_x\sigma}$$

- Given the unfolding pattern for $p = \mathrm{b}(x) \oplus \mathrm{a}(y) \oplus \mathrm{c}(z)$



Instance of a pattern

- Given the unfolding pattern for $p = \mathrm{b}(x) \oplus \mathrm{a}(y) \oplus \mathrm{c}(z)$
- How do we obtain the unfolding for $m_0 = \mathrm{b}(\mathrm{a}) \oplus \mathrm{a}(\mathrm{b}) \oplus \mathrm{c}(\mathrm{a}) = p\theta$, with $\theta = \{\mathrm{a}/x, \mathrm{b}/y, \mathrm{a}/z\}$.



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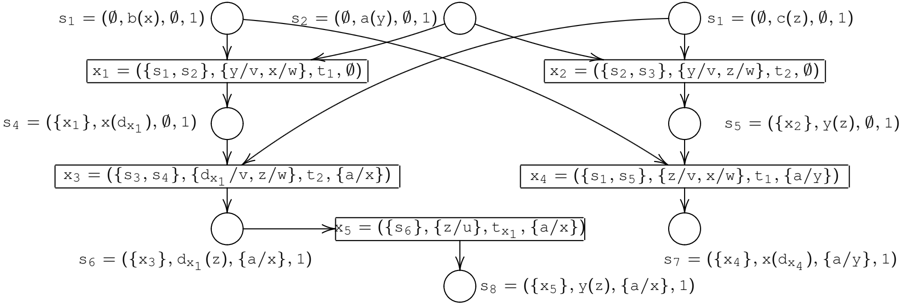
- By removing the elements that are not consistent with θ



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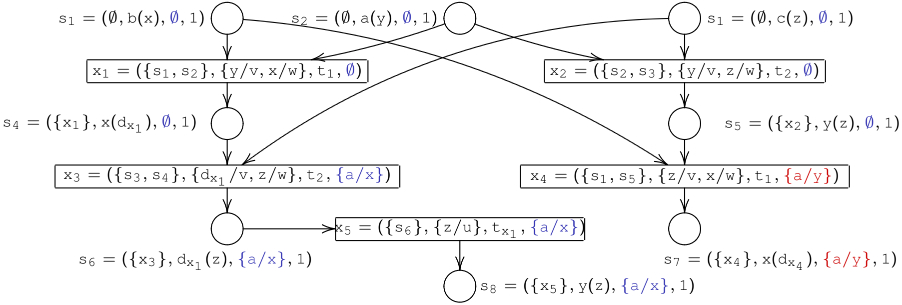

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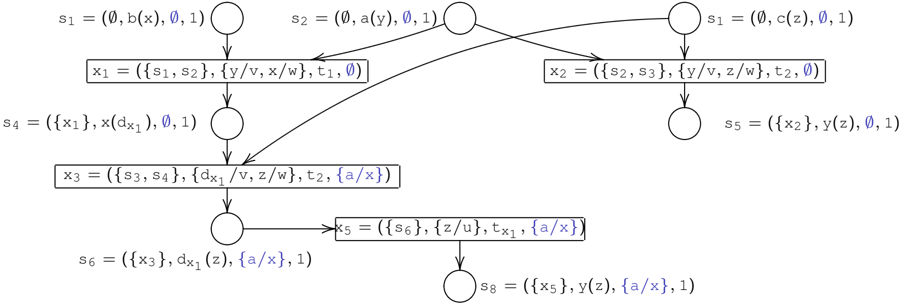

Instance of a pattern

Instances of $\mathcal{UP}[N]$ (initial marking p)

- Given a substitution θ, s.t. $dom(\theta) \subseteq col_{\mathcal{X}}(p)$ and $range(\theta) \subseteq S_N$,
- $\mathcal{UP}[N]\theta = (S\theta, T\theta, \delta\theta_0, \delta\theta_1, \mathcal{S}\theta, \mathcal{T}\theta, \xi\theta_0, \xi\theta_1)$ is , defined as:

$$\frac{(H, \mathtt{a}(c), \mu, n) \in S,\ (\mu \cup \theta)\ well-defined}{(H, \mathtt{a}(c), \mu, n) \in S\theta} \qquad \frac{\mathtt{a}_x \in \mathcal{S},\ x \in T\theta}{\mathtt{a}_x \in \mathcal{S}\theta} \qquad \frac{\mathtt{x} \in T\theta}{\delta\theta_i(\mathtt{x}) = \delta_i(\mathtt{x})}$$

$$\frac{(B, \sigma, \mathtt{t}, \mu) \in T,\ (\mu \cup \theta)\ well-defined}{(B, \sigma, \mathtt{t}, \mu) \in T\theta} \qquad \frac{\mathtt{t}_x \in \mathcal{T},\ x \in T\theta}{\mathtt{t}_x \in \mathcal{T}\theta} \qquad \frac{\mathtt{t} \in \mathcal{T}\theta}{\xi\theta_i(\mathtt{t}) = \xi_i(\mathtt{t})}$$

Unfoldings are instances of patterns

Lemma (Correspondence)

Let $\mathcal{U}[N]$ *and* $\mathcal{UP}[N]\theta$ *s.t.* $m_{0N} = p\theta$*. Then, there exists a bijective* $f = (f_S : S \to S_P\theta, f_T : T \to T_P\theta, f_{\mathcal{S}} : \mathcal{S} \to \mathcal{S}_P\theta, f_{\mathcal{T}} : \mathcal{T} \to \mathcal{T}_P\theta)$ *s.t.:*

1. *Dynamic structures are isomorphic:* $\forall \mathtt{t} \in \mathcal{T} : f_{\mathcal{S}}({}^{\bullet}\mathtt{t})[\rangle f_{\mathcal{S}}(\mathtt{t}^{\bullet}) \equiv_{\alpha} {}^{\bullet}f_{\mathcal{T}}(\mathtt{t})[\rangle f_{\mathcal{T}}(\mathtt{t})^{\bullet}$;
2. *Causal nets are isomorphic:* $\forall \mathtt{x} \in T : f_S({}^{\bullet}\mathtt{x})[\rangle f_S(\mathtt{x}^{\bullet}) = {}^{\bullet}f_T(\mathtt{t})[\rangle f_T(\mathtt{t})^{\bullet}$;
3. *Mapped events correspond to the same transition:* $\forall \mathtt{x} = (B, \sigma, \mathtt{t}) \in T : f_T(\mathtt{x}) = (B', \sigma', f_{\mathcal{T}}(\mathtt{t}), \mu)$;
4. *Mapped places correspond to the same token:* $\forall \mathtt{s} = (H, \mathtt{a}(c), i) \in S : f_S(\mathtt{s}) = (H', \mathtt{b}(c'), \mu, j)$ *and* $\mathtt{a}(c) = f_{\mathcal{S}}(\mathtt{b}(c')\theta)$.

Process Pattern

Process Pattern

A *process pattern* PP is a net morphism

- $PP : K \rightarrow (S, T, \delta_0, \delta_1)$, where $\mathcal{UP}[N] = (S, T, \delta_0, \delta_1, \mathcal{S}, \mathcal{T}, \xi_0, \xi_1)$;
- $\forall \mu_1, \mu_2 \in inst(PP(D(PP))) : \mu_1 \cup \mu_2$ is a well-defined substitution.

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Process pattern instantiation

A substitution θ is a *compatible instantiation* of *PP* iff is compatible with the instantiations of final elements of *PP*: $\forall \mu \in inst(PP(D(PP))), \mu \cup \theta$ is a well-defined substitution

Process instances

Lemma

Let PP be a process pattern of N for the unfolding pattern $\mathcal{UP}[N]$*, and* θ *a compatible instantiation. Then, PP is a process of* $\mathcal{UP}[N]\theta$*.*

Process instances

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Let PP be a process pattern of N for the unfolding pattern $\mathcal{UP}[N]$*, and* θ *a compatible instantiation. Then, PP is a process of* $\mathcal{UP}[N]\theta$*.*

Theorem (Correspondence)

Let PP be a process pattern of N for the linear pattern p s.t. $m_{0N} = p\theta$*. Then,* $PP\theta$ *describes a process of* $\mathcal{U}[N]$*.*

Conclusions

- We have extended the ordinary notion of unfolding and process to the more expressive setting of dynamic nets.
- We give a more general notion of unfolding patterns and processes that account that capture the behaviour of several initial markings.
- Indirectly, we provide a process semantics for the Join calculus, which is a name passing calculus.