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Comparing Higher-Order Encodings in LF and TL

Part II: TL and TL encodings, comparison with LF, conclusion

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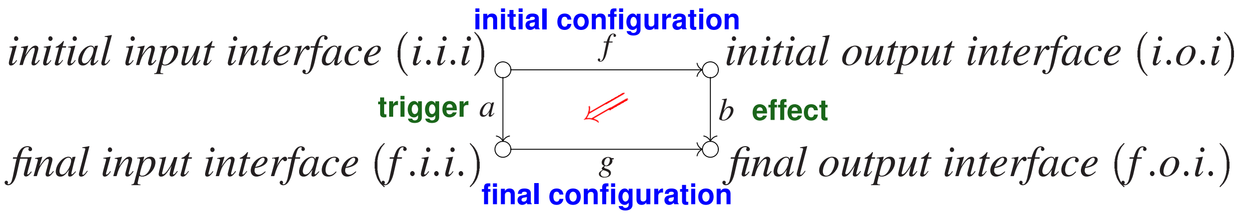
What Marina left to say

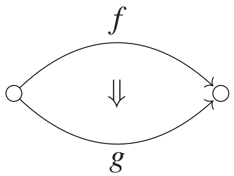
- tile logic (**TL**)
- higher-order tiles (**HOTL**) and cartian closed double categories (**CCDC**'s)
- a snapshot of late **π-calculus** encoding
- a snapshot of lazy simply typed **λ-calculus** encoding
- **comparison** between LF and TL (based on the two encodings)
- a few **conclusions** and **future work**

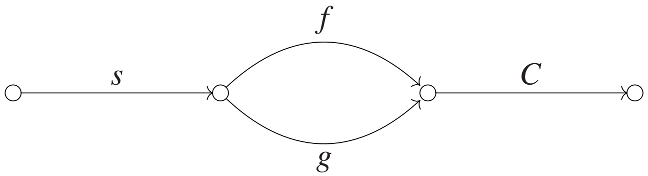
aim: modular description of the dynamics of concurrent systems (in **space** and in **time**)



RL

$f(x) \Rightarrow g(x)$

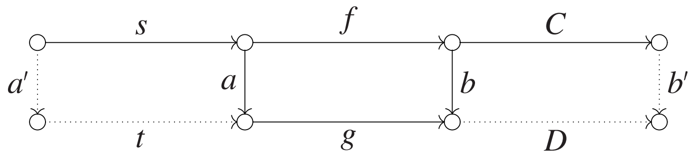




$\forall s, C[\cdot]$, then $C[f(s)] \Rightarrow C[g(s)]$

TL

$$\frac{x \xrightarrow{a} y}{f(x) \xrightarrow{b} g(y)} \qquad f \xrightarrow[b]{a} g$$



$$\forall s, C[\cdot], \text{ then } \frac{s \xrightarrow[a]{a'} t \ \wedge\ C \xrightarrow[b']{b} D}{C[f(s)] \xrightarrow[b']{a'} D[g(t)]}$$

sequents can be composed horizontally, vertically and in parallel

features: open configurations, visual presentation, categorical models

Theory of Configurations Fix the signature of **configurations**identifying the syntactic constructors and freely adjoin the **auxiliary structure** needed in the category of configurations (e.g., symmetries, cartesian operations);

Theory of Observations Fix the signature of **observations** (e.g. ordinary LTS's labels are usually seen as unary operators) and freely adjoin the **auxiliary structure** needed in the category of observations;

Theory of Tiles Define the basic operational rules for the dynamics of system components, and fix the **auxiliary tiles** to be freely adjoined.

issues: tile bisimilarity, tile traces, congruence formats, graphical proofs, applications

Moving to Higher-Order Tile Logic

monoidal structure: minimal tile model

symmetric monoidal structure: (consistent) permutations in the two dimensions

cartesianity: (consistent) pairing and projections

cartesian closed structure: higher-order configurations and observations

Example a *chemical solution* (à la CHAM) where functions and arguments float together

$$\begin{tikzcd} \Gamma \arrow[r, "\mathbf{P}:\tau^{\sigma},\mathbf{M}:\sigma,\mathbf{N}:\sigma"] \arrow[d, "\mathbf{id}_{\Gamma}"'] \arrow[dr, phantom, "\Leftarrow"] & f:\tau^{\sigma},x:\sigma,y:\sigma \arrow[d, "\mathbf{f\ x,y}"] \\ \Gamma \arrow[r, "\mathbf{PM}:\tau,\mathbf{N}:\sigma"'] & z:\tau,w:\sigma \end{tikzcd}$$

we expect consistency of horizontal/vertical applications

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exploit the correspondence between ccc and simply typed λ-calculus to define a **double λ-notation** on the basis of **cartesian closed double categories**
note: **only object (interface) abstraction is possible**

double λ-signature: $\Sigma = \langle \overset{\textit{type const.}}{\mathbb{B}}, \overset{\textit{horiz. term const.}}{\mathbb{H}}, \overset{\textit{vert. term const.}}{\mathbb{V}}, \overset{\textit{tile term const.}}{\mathbb{C}} \rangle$

$h \in \mathbb{H}$ and $v \in \mathbb{V}$ (with $\varnothing \vdash h : \sigma$ and $\varnothing \vdash v : \tau$) can have h.o. types σ and τ;
$\alpha \in \mathbb{C}$ has a *closed contour type* (defining the tile border)

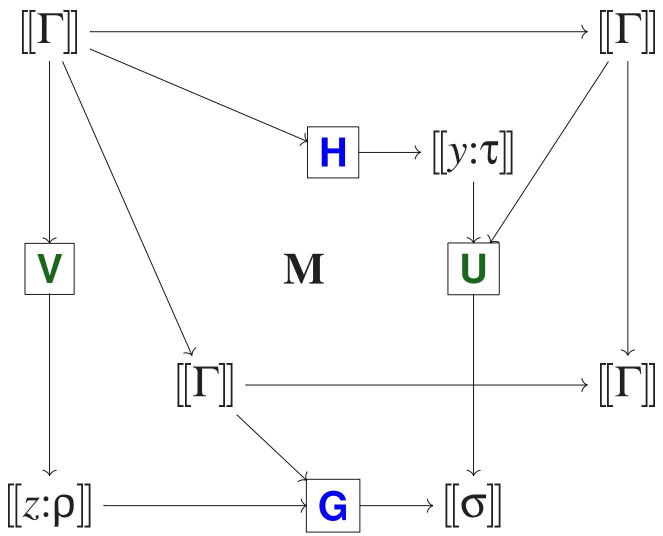
a generic contour type $\mathbf{t}$ has the form $\mathbf{H} : \tau \xrightarrow[\mathbf{U}:\sigma^\tau]{\mathbf{V}:\rho} \mathbf{G} : \sigma^\rho$; $\mathbf{t}$ is *well-typed* under Γ if H, G, V and U are well-typed under Γ, and it is *closed* if H, G, V and U are closed.

$M ::= \alpha \mid h \mid v \mid x \mid \star \mid \langle M, M \rangle^\xi \mid Proj_i^\xi M \mid \lambda^\xi x : \sigma . M \mid M @^\xi M \mid M \xi M$ (for $\xi \in \{*, \cdot, \triangleleft, \triangleright\}$)

type sentences have the form: $\Gamma \vdash M \square\ \mathbf{H} : \tau \xrightarrow[\mathbf{U}:\sigma^\tau]{\mathbf{V}:\rho} \mathbf{G} : \sigma^\rho$

Γ is a global environment for M (names in Γ can be used in H, V, U and G)

$$\Gamma \vdash M \,\square\, \mathbf{H} : \tau \xrightarrow[\lambda y.U:\sigma^{\tau}]{\mathbf{V}:\rho} \lambda z.G : \sigma^{\rho}$$



this makes evident that Γ is a global environment

Encoding the Pi-Calculus in TL

reverse observations for handling names

final configuration

final input interface — *final output interface*

trigger — **effect**

$$\begin{tikzcd} \circ \arrow[r, "H"] \arrow[d, "V"'] \arrow[dr, phantom, "\Rightarrow"] & \circ \arrow[d, "U"] \\ \circ \arrow[r, "G"'] & \circ \end{tikzcd}$$

initial input interface — *initial output interface*

initial configuration

fix a compact notation: $\mathbf{H} : \tau \xrightarrow[\lambda y:\tau.U:\sigma^{\tau}]{\mathbf{V}:\rho} \lambda z:\rho.G : \sigma^{\rho}$ is written $\mathbf{G}[\mathbf{V}/z] \Rightarrow \mathbf{U}[\mathbf{H}/y] : \sigma$

Interfaces	**Configurations**	**Observations**	**ProcessMapping**
n	$\mathbf{0} : p$	$\tau : p \to p$	$[\![0]\!] = \mathbf{0}$
p	$\tau_p : p \to p$	$out : \langle n,n \rangle \to p \to p$	$[\![\tau.P]\!] = \tau_p\ [\![P]\!]$
	$in_p : n \to (n \to p) \to p$	$in : n \to (n \to p) \to p$	$[\![x(y).P]\!] = in_p\ x\ \lambda y : n.[\![P]\!]$
	$out_p : \langle n,n \rangle \to p \to p$	$bout : n \to (n \to p) \to p$	$[\![\bar{x}y.P]\!] = out_p\ \langle x,y \rangle\ [\![P]\!]$
	$\mid\ : \langle p,p \rangle \to p$		$[\![P_1 \mid P_2]\!] = [\![P_1]\!] \mid [\![P_2]\!]$
	$\nu : (n \to p) \to p$		$[\![(\nu x)P]\!] = \nu\ \lambda x : n.[\![P]\!]$
	$if : \langle n,n \rangle \to p \to p$		$[\![[x=y]P]\!] = if\ \langle x,y \rangle [\![P]\!]$

Tile term constants

$$q : p \vdash \mathtt{Tau}@^{\triangleleft} q \,\Box\, \tau_p\ [q] \ \Rightarrow\ \tau\ [q]\ : p$$

$$x : n, r : n \to (p \times n) \vdash \mathtt{Open}@^{\triangleleft} x @^{\triangleleft} r \,\Box\, \nu[\lambda y : n.out\ \langle x, Proj_2(ry) \rangle Proj_1(ry)] \ \Rightarrow\ bout\ x[\lambda y : n.Proj_1(ry)]\ : p$$

Encoding the Pi-Calculus in TL: an Example

$$\frac{P \xrightarrow{\bar{x}y} P'}{(\nu y)P \xrightarrow{\bar{x}(w)} P'\{w/y\}} \; y \neq x, w \notin fn((\nu y)P') \qquad \text{(OPEN)}$$

$$\begin{tikzcd}
\Gamma, y : n \arrow[r, "\langle [\![P']\!], y\rangle"] \arrow[d] \arrow[dr, phantom, "\mathbf{M}"] & q : p, y' : n \arrow[d, "out\langle x, y'\rangle q"] \\
\varepsilon \arrow[r, "[\![P]\!]"'] & : p
\end{tikzcd}
\qquad
\Gamma, y : n \vdash \mathtt{M} \,\square\, [\![P]\!][\,] \;\Rightarrow\; out\ \langle x, [y]_1\rangle[[\![P]\!]]_2 \ : p$$

$$\begin{tikzcd}
\Gamma \arrow[r, "\lambda y : n.\langle [\![P']\!], y\rangle"] \arrow[d] \arrow[dr, phantom, "\lambda^{*}\mathbf{y}.\mathbf{M}"] & s : (p \times n)^n \arrow[d, "\lambda y' : n.out\langle x, Proj_2(sy')\rangle Proj_1(sy)"] \\
\varepsilon \arrow[r, "\lambda y : n.[\![P]\!]"'] & : p^n
\end{tikzcd}
\qquad
\begin{tikzcd}
s : (p \times n)^n \arrow[r, "\lambda w : n.Proj_1(sw)"] \arrow[d] \arrow[dr, phantom, "\mathtt{Open@}^{\triangleleft}x"] & q_2 : p^n \arrow[d, "bout\ x\ q_2"] \\
q_1 : p^n \arrow[r, "\nu\ q_1"'] & : p
\end{tikzcd}$$

$$\begin{tikzcd}
\Gamma \arrow[r, "\lambda w : n.[\![P'\{w/y\}]\!]"] \arrow[d] \arrow[dr, phantom, "(\lambda^{*}\mathbf{y}.\mathbf{M}) * (\mathtt{Open@}^{\triangleleft}\mathbf{x})"] & q : p^n \arrow[d, "bout\ x\ q"] \\
\varepsilon \arrow[r, "[\![(\nu y)P]\!]"'] & : p
\end{tikzcd}$$

Proposition If $P \xrightarrow{\mu} Q$ then $fn(P) \vdash M \Box S \xrightarrow[\llbracket\mu\rrbracket]{id} \llbracket P \rrbracket$, in $\mathcal{R}_{late}$

where $S = \llbracket Q \rrbracket$ if μ is free,

while $S = \lambda y : n.\llbracket Q \rrbracket$ if μ is bound with object y.

- the proof is by induction on the proof of $P \xrightarrow{\mu} Q$;
- define a denotational mapping from proof terms of late LTS to well-typed tile terms;
- $\mathcal{R}_{late}$ defines a *richer* LTS.

Conjecture Tile bisimilarity is finer than late bisimilarity.

processes with different sets of free names are not tile bisimilar

Interfaces	Configurations	Observations	Tile term constants
ι	$c\colon \star \to \iota$	$\hat{c}\colon \star \to \iota$	$\vdash \mathtt{Dyn}_c \,\square\, \hat{c}[\star] \;\Rightarrow\; c[\star] \;:\iota$
κ	$0\colon \star \to \kappa$	$\downarrow_c\colon \iota \to \kappa$	$\vdash \mathtt{Const}_c \,\square\, \downarrow_c [c] \;\Rightarrow\; 0[\star] \;:\kappa$

- observing configurations;
- since $\star$ is terminal we cannot take $\downarrow_c\colon \iota \to \star$ (otherwise $\downarrow_c = \downarrow_{c'}$ for all c, c');
- thus type constant κ and horizontal term constant 0 are used;
- this time tile *are not* reversed:

$$
\begin{tikzcd}
\circ \arrow[r, "\star"] \arrow[d, "\star"'] \arrow[dr, phantom, "\mathtt{Dyn}_c"] & \circ \arrow[d, "\hat{c}"] \\
\circ \arrow[r, "c"'] & \circ
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
\circ \arrow[r, "c"] \arrow[d, "\star"'] \arrow[dr, phantom, "\mathtt{Const}_c"] & \circ \arrow[d, "\downarrow_c"] \\
\circ \arrow[r, "0"'] & \circ
\end{tikzcd}
$$

Proposition For any closed configuration M, we have that $\mathcal{R}_\lambda \vdash \star \xrightarrow[\hat{M}]{\star} M$,
where $\hat{M}$ is the term obtained by the syntactic replacement of all c by $\hat{c}$.

Corollary For any closed configurations $M \in \Lambda_\emptyset^{\sigma\to\tau}$ and $N \in \Lambda_\emptyset^{\sigma}$, we have $\mathcal{R}_\lambda \vdash M \xrightarrow[\star]{@\hat{N}} MN$,
where $@ \triangleq \lambda x : \tau.\lambda f : \sigma \to \tau. fx$.

Proposition **Tile bisimilarity coincides with applicative bisimilarity.**

- τ moves of the applicative LTS are mapped to vertical identities;
- the two kinds of labeled steps have their counterparts in TL;
- the type system allows for correct applications only.

Comparison Between Encodings

the choice of π and λ is instrumental in comparing TL and LF (not for the novelty of the encoding technique)

	LF	TL
π	**encoding driven by syntax**	**encoding driven by strong typing**
	labels are separate entities $\overset{\tau}{\longmapsto}$, $\overset{out(x,y)}{\longmapsto}$, $\overset{in(x)}{\longmapsto\!\!\!\!\rightarrow}$, $\overset{bout(x)}{\longmapsto\!\!\!\!\rightarrow}$	**labels are h.o. vertical operators** τ, $out\ xy$, $in\ x$, $bout\ x$
	only curryed operators	**cartesian pairing**
	transition proved by inhabitance	**semantic domain for proofs**
	late bis. needs stronger type theory	**tile bisimilarity or ad hoc def.**
	handle name abstraction, creation, passing, application	
λ	**transition proved by inhabitance**	**dynamic application is built-in**
	can handle other evaluation strategies **can handle extrinsic type systems**	**up-to α, β, η** **untyped λ needs some extension**
	handle name abstraction, application	

Overall Comparison

LF	TL
more expressive power	**only object abstraction**
tools (automatization)	**visual formalism**
flat view	**space/time distinction**
functor categories	**categorical view**
delegate to the metalanguage many aspects of object systems (e.g. name binding, name passing, name creation, name freshness, capture-avoiding substitutions, α-conversion	
several case studies presented in the literature	
HOA**S** (**S**=Syntax)	HOA**S** (**S**=Semantics)

Future Work

- extend the comparison via a thorough exploration of other semantics for the two object languages considered here (e.g. call-by-value and untyped λ-calculus, early π-calculus, barbed equivalence);

- consider other object systems (e.g. ambient, spi-calculus, polyadic π-calculus);

- syntactical encoding of HOTL in LF (feasible);

- semantical encoding of LF in HOTL (difficult, because of dependent types)