Pre-nets, read arcs and unfolding: a functorial presentation*

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Extended abstract

P/T Petri nets are one of the most widely known models of concurrency. Since their introduction [Pet62], the conceptual simplicity of the model and its intuitive graphical presentation have attracted the interest of both theoreticians and practitioners. Nevertheless, the concurrent semantics of Petri nets present several subtleties that still cannot be considered fully understood.

In this presentation, we concentrate on the semantic interpretations arising from the so-called individual token philosophy (ITPh) as opposed to the collective token philosophy (CTPh), the latter being less amenable to the full variety of aspects that can be studied in the ITPh. (The two terminologies have been introduced in [vGP95].) Our twofold aim is: (1) to emphasise the missing fragments of the overall picture arising from the different semantic frameworks for P/T Petri nets (roughly classified in process-oriented, unfolding, algebraic, logical, categorical); (2) to outline some preliminary ideas and results for bridging the gaps in a formal and satisfactory way.

The *process* approach focuses on non-sequential computations. Several notions of (deterministic) processes have been proposed that rely on different abstractions in modelling resources, executed events and concurrent computations [BD87,GR83].

The *unfolding approach* is built, to some extent, on top of deterministic processes to account for a broader view of computations, which includes e.g. branching information. Starting from the seminal work of Winskel [Win87], which focuses on the simpler class of *safe* nets, several authors have contributed to the generalisation of the approach to the full class of P/T Petri nets. More precisely, the paper [Win87] shows that a chain of coreflections leads from the category **Safe** of safe nets to the category **PES** of prime event structures, equivalent the category **Dom** of coherent finitary prime algebraic domains, through the category **Occ** of nondeterministic occurrence nets. In [MMS92,MMS97a] it has been shown that essentially the same construction applies to the wider category of *semi-weighted* nets. The generalisation to the whole category of P/T nets requires some original technicality and and leads to a chain of proper adjunctions (rather than coreflections) from **PTNets** to **DecOcc** and then to **Occ**, with **PTNets** be the category of P/T Petri nets and **DecOcc** be the category of a special kind of occurrence nets, called decorated occurrence nets (see [MMS96] for details). We remind that when functors are left/right adjoints they preserve colimits/limits yielding good compositionality properties, as (co)limits can be used to express complex models in terms of simpler constituents.

The *algebraic approach*, originally proposed in [MM90] for the CTPh, recasts the process approach in universal algebras. The concurrent model of computation $\mathcal{M}[N]$ of a net N is obtained by lifting to computations the monoidal structure of states (multisets of places). The CTPh construction is expressed as an adjunction that associates with each net a monoidal category that is naturally isomorphic to the category of (commutative) processes. More importantly, it can be expressed at the logical level by a simple theory morphism in partial membership equational logic between the theories of nets and of commutative monoidal categories [BMMS01]. The extension of the algebraic approach to the ITPh presents several problems related to the fact that the monoidal operation on computations is only commutative up to a natural symmetric isomorphism. As a consequence, the first extension proposed in [DMM96] fails to preserves some ordinary simulation morphisms between nets. The situation has been improved through [MMS97b] up to a pseudo-functorial construction [Sas98].

* Research supported by the IST Project AGILE and by the Italian MIUR Project COMETA. The second author is also supported by an Italian CNR fellowship for research on Information Sciences and Technologies, and by the CS Dept. of the Univ. of Illinois at Urbana-Champaign.

A relevant advance of the theory has been the introduction of *pre-nets* [BMMS01], a variant of ordinary nets where a total ordering is imposed on the places occurring in the pre- and post-set of transitions. Pre-nets allow to obtain a fully satisfactory categorical treatment, where the construction of the model of computation yields an adjunction between the category of pre-nets and the category of models (symmetric monoidal categories). Any pre-net can be seen as a concrete "implementation" of the Petri net obtained by forgetting about the ordering of places in pre- and post-sets. Notably, the construction of the model of computation of any two pre-nets implementing the same Petri nets yields the same result, in a way that we can define the semantic of a net as the algebraic semantic of any of its implementations.

Still several results are needed to complete the missing links in the theory.

A domain semantics for pre-nets can be obtained by generalising a construction proposed for ordinary nets in [MMS96]. More specifically, the comma category $(m \downarrow \mathcal{M}[N])$, where m is the initial marking of the net is a preorder, inducing a partial order whose ideal completion is a prime algebraic domain. Roughly this domain consists of the set of deterministic processes of the net (starting from the initial marking), endowed with a kind of prefix ordering.

Pre-nets lack an unfolding semantics, in the style of Winskel, associating to any pre-net first an acyclic pre-net representing any possible computation of the original net in a single branching structure, then an event structure and finally a prime algebraic domain.

Since the unfolding is essentially a nondeterministic process that completely describes the behaviour of a net, a clear link between the unfolding and the algebraic approaches is called for. Indeed, as for ordinary Petri nets [MMS96], we expect the domain originating from the algebraic model of computation through the comma contruction and the one extracted from the unfolding to be isomorphic.

The above result, as a side effect, would show that the outlined constructions associate the same domain to any pre-net implementing a given Petri net.

Finally, the pre-net and Petri net framework should be reconciled by explaining how the domain semantics of a net and of its pre-net implementations are related.

Similar considerations holds in the case of Petri nets with *read arcs*. Recall that the introduction of read arcs is motivated by a common phenomenon of distributed and parallel systems, namely the concurrent read-only access to shared resources. Exploiting previous works on the different semantic approaches for nets with contextual arcs (processes [MR95]; unfolding [BCM01,Bal00]; algebraic [GM96,BS01,MOS01]; logical [BMMS02]) we expect to be able to add the missing pieces of the ITPh theory, extending the analogous results obtained in the absence of read arcs.

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