On the efficiency of promoters and of cooperative rules in Membrane Systems

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Introduction (1)

In Membrane Systems with promoters each evolution rule is enriched with a multiset p of promoter objects

$$u \longrightarrow v|_p$$

Promoter objects

- are required to be present in the membrane to enable the rule
- are not consumed when the rule is applied
- are such that the presence of a single occurrence of a promoter object may enable more than one rule

Introduction (2)

Universality:

- Membrane Systems with promoters and non-cooperative rules are known to be universal by using only one membrane
- The same holds for Membrane Systems with cooperative rules

Our aim is to compare promoters and cooperation under the point of view of efficiency:

- there exist computational problems whose complexity with promoters is strictly smaller (or greater) than with cooperation?
- we are not interested here in polynomial solutions of NP complete problems

On the efficiency of promoters (1)

One might think that promoter objects are very efficient control tools:

- A number of rules may become applicable in one step by just producing one copy of their promoter objects

For instance, the following cooperative Membrane System generates a multiset a^{2^n} in about 2^{n+2} steps:

Membrane 1:

$$EDa \rightarrow (ED, here)(aa, in_2)$$
$$aD \rightarrow (\#, here) \qquad a$$
$$E \rightarrow (G, here)(ED, in_2)$$
$$FDa \rightarrow (FD, here)(a, out)$$
$$\# \rightarrow (\#, here) \qquad GD \rightarrow \lambda$$

Membrane 2:

$$EDa \rightarrow (ED, here)(a, out)$$
$$aD \rightarrow (\#, out) \qquad ED$$
$$E \rightarrow (G, here)(ED, out)$$
$$E \rightarrow (G, here)(FD, out)$$
$$GD \rightarrow \lambda$$

On the efficiency of promoters (2)

One might think that promoter objects are very efficient control tools:

- A number of rules may become applicable in one step by just producing one copy of their promoter objects

whereas the following Membrane System with promoters generates the same multiset in about *n* steps:

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$$a' \rightarrow (a, here)|_b \qquad b \rightarrow (b, here)$$

$$a \rightarrow (aa, here)|_b \qquad b \rightarrow (c, here)$$

$$a \rightarrow (a, out)|_c$$

$$a'b$$

On the efficiency of cooperation (1)

However, there are problems that can be solved more efficiently with cooperation:

- For example, accepting the multiset language a^{2n}

Definition: An acceptor Membrane System for a multiset language L over an alphabet Σ is a Membrane System

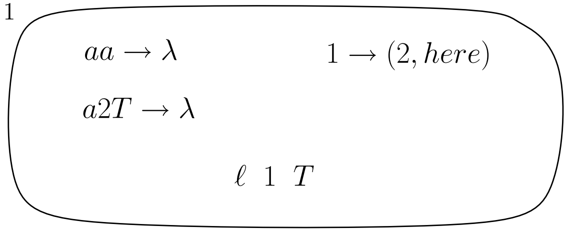
$$\Pi_L = (\Sigma \cup \mathcal{C} \cup \{T\}, \mu, w_1 \cup \ell, w_2, \dots, w_n, R_1, \dots, R_n)$$

where:

- $\mathcal{C}$ is a set of *control objects* such that $\Sigma \cap \mathcal{C} = \varnothing$;
- T is a special object not contained in $\Sigma \cup \mathcal{C}$;
- w_i, for $1 \leq i \leq n$, are multisets of objects in $\mathcal{C}$;
- when the placeholder ℓ is replaced by a multiset of objects the output of the computation says whether it belongs to L as follows: the multiset is accepted (belongs to L) if and only if a final configuration can be reached with T appearing in the skin membrane.

On the efficiency of cooperation (2)

With cooperation it is very easy to give an acceptor for the language a^{2n}.



Actually, the complexity of this (deterministic) solution is $O(1)$.

On the efficiency of cooperation (3)

Giving an acceptor for the language a^{2n} with promoters but without cooperation is not as easy as before...

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$$a \rightarrow (a, here)|_1 \quad T \rightarrow (F, here)|_{bb2} \quad T \rightarrow (F, here)|_{b3}$$

$$a \rightarrow (b, here)|_1 \quad T \rightarrow (F, here)|_{cc2} \quad T \rightarrow (F, here)|_{c3}$$

$$a \rightarrow (c, here)|_1 \quad T \rightarrow (T, here)|_{bc3}$$

$$b \rightarrow \lambda|_3 \quad c \rightarrow \lambda|_3$$

$$1 \rightarrow (2, here) \quad 2 \rightarrow (3, here) \quad 3 \rightarrow (4, here) \quad 4 \rightarrow (1, here)|_a$$

$$\ell \ \ 1 \ \ T$$

Actually, the complexity of this (non-deterministic) solution is $O(n)$.

Current result

Theorem. The multiset language $\{a^{2n} \mid n \geq 0\}$ cannot be accepted in constant time without cooperation.

Idea of the proof: By contradiction, let us assume that the language can be accepted without cooperation in k steps:

- let us assume that the acceptor system consists of one membrane
- since the set of rules is finite there must exist an infinity of words that are accepted by applying the same rules at the same execution steps, but a different number of times
- let a^{2i} be one of such words, with i very great
- since rule are non-cooperative, it can be shown that a^{2i+1} can be accepted as well as a^{2i} by applying the same sets of rules at the same steps, but a different number of times

Further developments

Are Membrane Systems with cooperation strictly more efficient than with promoters?

- In general it seems that there are problems that can be solved more efficiently with promoters (e.g. generation of a^{2^n})
- What happens if we consider only acceptor systems? Currently, we are looking for a language that can be accepted more efficiently with promoters than with cooperation

What is the role of non-determinism?

- Non-determinism (and traps) are used differently in Membrane Systems with promoters and with cooperation (e.g. the systems we have shown)
- Generative Membrane Systems for non-trivial multiset languages require non-determinism
- Currently, we are investigating the power and efficiency of deterministic acceptor systems with promoters