

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{array}{l} \leftarrow \text{somma } -\frac{a_{21}}{a_{11}} \cdot R_1 \\ \leftarrow \text{somma } -\frac{a_{31}}{a_{11}} \cdot R_1 \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{a_{21}}{a_{11}} & 1 & 0 & 0 \\ -\frac{a_{31}}{a_{11}} & 0 & 1 & 0 \\ -\frac{a_{41}}{a_{11}} & 0 & 0 & 1 \end{bmatrix}}_{L_1} A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & b_{32} & b_{33} & b_{34} \\ 0 & b_{42} & b_{43} & b_{44} \end{bmatrix}}_{L_1 A} \begin{array}{l} \leftarrow \frac{b_{32}}{b_{22}} R_2 \\ \leftarrow \frac{b_{42}}{b_{22}} R_2 \end{array}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{b_{32}}{b_{22}} & 1 & 0 \\ 0 & -\frac{b_{42}}{b_{22}} & 0 & 1 \end{bmatrix}}_{L_2} L_1 A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & c_{43} & c_{44} \end{bmatrix}}_{L_2 L_1 A}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{c_{43}}{c_{33}} & 1 \end{bmatrix}}_{L_3} L_2 L_1 A = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & b_{22} & b_{23} & b_{24} \\ 0 & 0 & c_{33} & c_{34} \\ 0 & 0 & 0 & d_{44} \end{bmatrix}}_{L_3 L_2 L_1 A = U}$$

$$A = \underbrace{L_1^{-1} L_2^{-1} L_3^{-1}}_L U$$

↑ upper
↑ lower

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{a_{21}}{a_{11}} & 1 & 0 & 0 \\ \frac{a_{31}}{a_{11}} & 0 & 1 & 0 \\ \frac{a_{41}}{a_{11}} & 0 & 0 & 1 \end{bmatrix}}_{L_1^{-1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{b_{32}}{b_{22}} & 1 & 0 \\ 0 & \frac{b_{42}}{b_{22}} & 0 & 1 \end{bmatrix}}_{L_2^{-1}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{c_{43}}{c_{33}} & 1 \end{bmatrix}}_{L_3^{-1}}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{a_{21}}{a_{11}} & 1 & 0 & 0 \\ \frac{a_{31}}{a_{11}} & \frac{b_{32}}{b_{22}} & 1 & 0 \\ \frac{a_{41}}{a_{11}} & \frac{b_{42}}{b_{22}} & \frac{c_{43}}{c_{33}} & 1 \end{bmatrix}}_{L^{-1}}$$