domani 14:00 ricevimento
e tutti i mercoledì

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow \begin{bmatrix} b_1 \\ b_2 + 2b_1 \\ b_3 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

mar 3-13:54

$$\underbrace{\begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}}_{A} x = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 \\ 0 & 1 & 1 \\ -2 & -3 & 7 \end{bmatrix}$$

$E \cdot A$

Prodotto di matrici

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 9 \\ -3 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$$

OPERAZIONI SU MATRICI

$$3\left\{\begin{bmatrix} 1 & 2 & 5 & -3 & \pi \\ 5/2 & 0 & 1 & 5 & 9 \\ 1 & -2 & -5 & 0 & 9 \end{bmatrix}\right. \in Mat_{3\times5}(\mathbb{R})$$

$\underbrace{\qquad\qquad\qquad}_{5}$

Matrice 3×5

Notazioni:

$$Mat_{m\times n}(\mathbb{R}) \qquad M_{m\times n}(\mathbb{R}) \qquad \mathbb{R}^{m\times n}$$

$$E \in Mat_{3\times3}(\mathbb{R})$$

Somme

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix}$$

Se hanno le stesse dim

$A+B$

$A, B \in Mat_{m,n}(\mathbb{R})$

Prodotto per scalare (numero)

$$\frac{3}{2} \cdot \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 3/2 \\ 3 & 9/2 \end{bmatrix}$$

$c \cdot A$ sempre

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 26 & 31 \end{bmatrix}$$

$0 \cdot 5 + 1 \cdot 7$

$2 \cdot 4 + 3 \cdot 6$

$2 \cdot 5 + 3 \cdot 7$

$A \cdot B$

posso farlo se

$A \in Mat_{m \times n}(\mathbb{R})$

$B \in Mat_{n \times p}(\mathbb{R})$

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 2 & 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 7 & x & x & x \\ x & x & x & x \end{bmatrix}$$

$$2\times 3 \quad 3\times 4 = 2\times 4$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \in Mat_{m\times n}(\mathbb{R}) \qquad B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ \vdots & & \ddots & \vdots \\ b_{n1} & \cdots & & b_{np} \end{bmatrix} \in Mat_{n\times p}(\mathbb{R})$$

$C = AB$ Gli elementi di C sono

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots + a_{in}b_{nj} = \sum_{k=1}^{n} a_{ik}b_{kj}$$

mar 3-14:28

$C \in Mat_{m \times p}(\mathbb{R})$

Proprietà

$A+B=B+A$ $(A+B)+C=A+(B+C)$

$c(A+B)=cA+cB$

$C(A+B)=CA+CB$ per ogni A,B,C di dim. compatibili

$(AB)C=A(BC)=ABC$

$$(A+B)C = AC + BC$$

$$\underset{2\times 3}{A}\ \underset{3\times 4}{B} = \underset{2\times 4}{C} \qquad \underset{3\times 4}{B}\ \underset{2\times 3}{A} = ?$$

$$\begin{bmatrix} \times & \times & \times \\ \times & \times & \times \end{bmatrix} \begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix} \qquad \begin{bmatrix} \times & \times & \times & \times \\ \times & \times & \times & \times \\ \times & \times & \times & \times \end{bmatrix} \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}} \cdot \underset{B}{\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}} = \underset{C}{\begin{bmatrix} 3 & 6 \\ 0 & 15 \end{bmatrix}} \qquad \underset{B}{\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}} \underset{A}{\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}} = \begin{bmatrix} 3 & 10 \\ 0 & 15 \end{bmatrix}$$

$AB \neq BA$ bisogna sempre stare attenti all'ordine

$$\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \begin{bmatrix} x \\ x \\ x \end{bmatrix} = \begin{bmatrix} x \\ x \\ x \end{bmatrix}$$

$n \times n \quad n \times 1 \quad n \times 1$

$$\begin{bmatrix} x & x & x \end{bmatrix} \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} = \begin{bmatrix} x & x & x \end{bmatrix}$$ vettore riga

$1 \times n \quad n \times n \quad 1 \times n$

mar 3-14:44

$$\begin{bmatrix} x & x & x \end{bmatrix} \begin{bmatrix} x \\ x \\ x \end{bmatrix} = \begin{bmatrix} x \end{bmatrix}$$

$1 \times n \quad n \times 1 \quad 1 \times 1$

$$\begin{bmatrix} 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 2 \cdot 1 + 4 \cdot 0 + 6 \cdot (-1) = -4$$

$$\begin{bmatrix} x \\ x \\ x \end{bmatrix} \begin{bmatrix} x & x & x \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

$n \times 1 \quad 1 \times n$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 0 & 0 & 0 \\ -2 & -4 & -6 \end{bmatrix}$$

mar 3-14:46

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

matrice identità $n \times n$ $\quad I \in Mat_{n \times n}(\mathbb{R})$

$I_n \in Mat_{n \times n}(\mathbb{R})$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

mar 3-14:49

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \in Mat_{2\times 2}(\mathbb{R})$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = A \cdot A \cdot A = \begin{bmatrix} 1 & 26 \\ 0 & 27 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 10 & 5 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

Nell'eliminazione di Gauss serve anche scambiare righe

$$A \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_1 \\ b_3 \end{bmatrix}$$

Matrici di permutazione

Scambiano elementi di un vettore

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_2 \\ b_3 \\ b_1 \end{bmatrix}$$

mar 3-15:01

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad P^3 = I \quad \text{per cosa}$$

$Ax = b$

$$\begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} x = \begin{bmatrix} 2 \\ 8 \\ 10 \end{bmatrix}$$

Augmented matrix: $[A \ b]$

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -2 & : & 2 \\ 4 & 9 & -3 & : & 8 \\ -2 & -3 & 7 & : & 10 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 & : & 2 \\ 0 & 1 & 1 & : & 4 \\ -2 & -3 & 7 & : & 10 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\-2&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}2&4&-2&\vdots&2\\4&9&-3&\vdots&8\\-2&-3&7&\vdots&10\end{bmatrix}=\begin{bmatrix}2&4&-2&\vdots&2\\0&1&1&\vdots&4\\-2&-3&7&\vdots&10\end{bmatrix}$$

$$E_2=\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}\quad\begin{bmatrix}b_1\\b_2\\b_1+b_3\end{bmatrix}$$

$$\underbrace{\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix}}_{E_2}\cdot\underbrace{\begin{bmatrix}2&4&-2&2\\0&1&1&4\\-2&-3&7&10\end{bmatrix}}_{E_1[A\,\vdots\,b]}=\begin{bmatrix}2&4&-2&2\\0&1&1&4\\0&1&5&12\end{bmatrix}=E_2E_1[A\,\vdots\,b]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ -2 & -3 & 7 & 10 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 12 \end{bmatrix}$$

$$\begin{cases} 2x_1 + 4x_2 - 2x_3 = 2 \\ x_2 + x_3 = 4 \\ 4x_3 = 8 \end{cases}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}}_{E_3} \underbrace{\begin{bmatrix} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 5 & 12 \end{bmatrix}}_{E_2 E_1 [A \vdots b]} = \underbrace{\left[\begin{array}{ccc:c} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 4 & 8 \end{array}\right]}_{E_3 E_2 E_1 [A \vdots b]}$$

mar 3-15:15

$$\begin{matrix}(a)\\(b)\\(c)\end{matrix}\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 4 & 8 \end{array}\right] = E_3 E_2 E_1 [A \,\vdots\, b]$$

$$(b) \leftarrow (b) - \frac{1}{4}(c)$$

$$E_4 \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/4 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 - \frac{1}{4} b_3 \\ b_3 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1/4 \\ 0 & 0 & 1 \end{bmatrix}}_{E_4} \underbrace{\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 4 & 8 \end{array}\right]}_{E_3 E_2 E_1 [A \vdots b]} = \underbrace{\left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right]}_{E_4 E_3 E_2 E_1 [A \vdots b]}$$

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$$\begin{matrix} (a) \\ (b) \\ (c) \end{matrix} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right] \qquad (a) \to (a) + \tfrac{1}{2}(c)$$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 + \frac{1}{2} b_3 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} 2 & 4 & -2 & 2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & 4 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right] \begin{array}{c} (a) \\ (b) \\ (c) \end{array}$$

$$(a) \to (a) - 4(b)$$

$$\underbrace{\begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E6} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} b_1 - 4b_2 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} 2 & 4 & 0 & 6 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

vorrei che qui ci fosse $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

moltiplico la prima riga per $\frac{1}{2}$
e la terza per $\frac{1}{4}$

$$\underbrace{\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}}_{E_7} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 1/2\, b_1 \\ b_2 \\ 1/4\, b_3 \end{bmatrix}$$

$$E_7 \left[\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 4 & 8 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

↳ questa è la soluzione

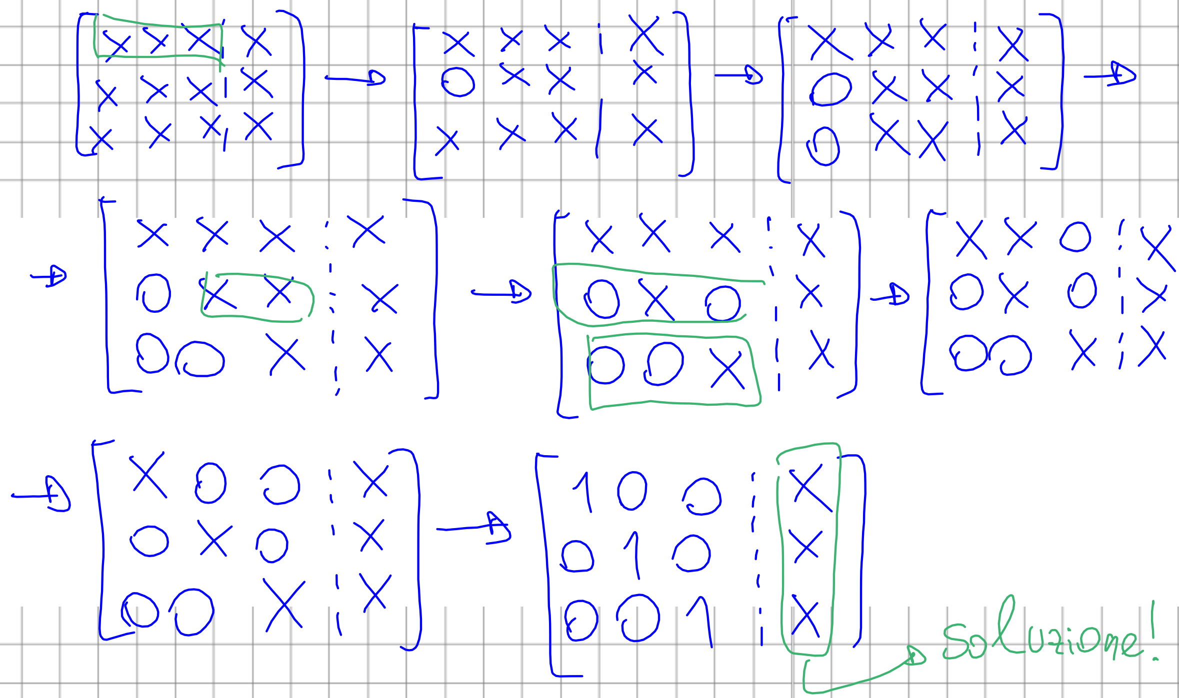
$$\Downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$$

$$\Updownarrow$$

$x_1 = -1$
$x_2 = 2$
$x_3 = 2$

Eliminazione di Gauss-Jordan



$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} x_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \to$$

$$(c) \leftarrow (c) + \frac{2}{3}(b)$$

$$\to \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & \frac{4}{3} & \frac{1}{3} & \frac{2}{3} & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & -1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 & \frac{3}{4} & \frac{3}{2} & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \end{array}\right] \begin{array}{l} (a)+(a)+(b) \\ (b) \\ (c) \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & \frac{3}{2} & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{2}{4} & \frac{3}{4} \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{2} & 1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \end{array}\right]$$

I

soluzione di $Ax = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

soluzione x_2 di $Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

soluzione x_1 di $Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$A \begin{bmatrix} \frac{3}{4} \\ \frac{1}{2} \\ \frac{1}{4} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad A \begin{bmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad A \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ \frac{3}{4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A \cdot \underbrace{\begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix}}_{D} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

... lungo lavoro ... $A \cdot D = I$

mar 3-15:55