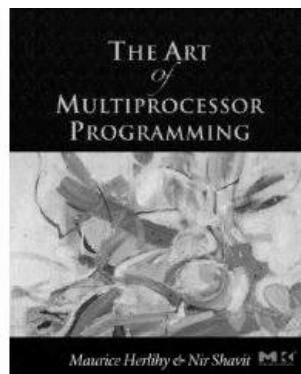




---

# PROGRAMMAZIONE CONCORRENTE

1



The Art of Multiprocessor Programming  
Maurice Herlihy & Nir Shavit

2

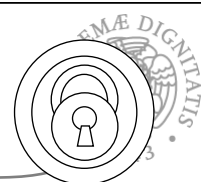
## Mutual Exclusion



- ✎ Problema essenziale della programmazione concorrente
  - Coordinamento di azioni su risorse condivise
- ✎ Come si dimostrano proprietà di astrazioni in presenza di thread
  - Problema essenziale nello sviluppo di applicazioni su sistemi multicore

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## Mutual Exclusion



E. W. Dijkstra [1965]:

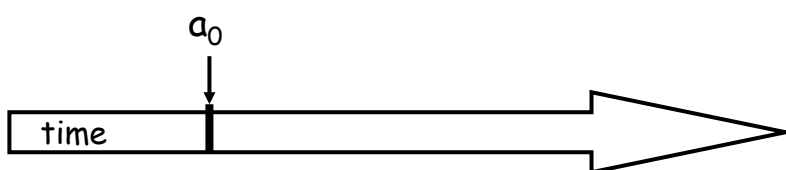
"Given in this paper is a solution to a problem which, to the knowledge of the author, has been an open question since at least 1962, irrespective of the solvability. [...] Although the setting of the problem might seem somewhat academic at first, the author trusts that anyone familiar with the logical problems that arise in computer coupling will appreciate the significance of the fact that this problem indeed can be solved."

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## Scala temporale



- ✎ Un *evento*  $a_0$  di un thread A e'
  - istantaneo
  - Non si assume la simultaneità di eventi

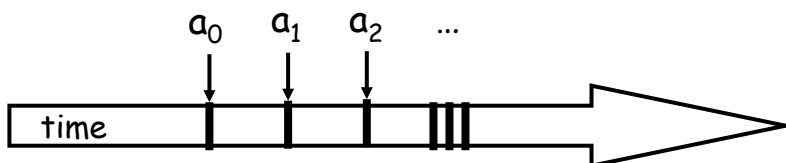


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## Thread (visione astratta)



- ✎ Un *thread* A una sequenza  $a_0, a_1, \dots$  di eventi
  - Modellato a "tracce" di esecuzione
  - $a_0 \rightarrow a_1$  indica l'ordine di occorrenza degli eventi



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## Cosa sono gli eventi?



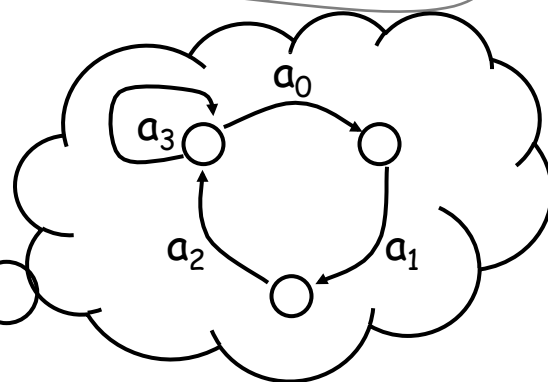
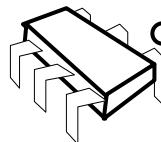
- ✎ Assegnamento aa variabili condivise (tra thread)
- ✎ Assegnamento a una variabile locale
- ✎ Invocazione di un metodo
- ✎ :
- ✎ ....

7

## Threads sono sistemi di transizione



Eventi  
sono le  
transizioni



**Sistemi di Transizione  
= State Machine**

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## Stati



### Thread State

- Informazioni di controllo (continuation stack)
- Informazioni di ambiente (la struttura degli stack a run time)

### System state

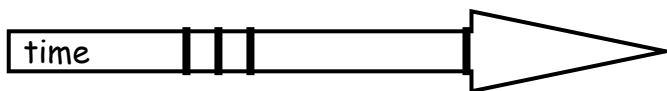
- Heap con gli oggetti condivisi
- Stato di tutti i thread in esecuzione (ready, wait, run)

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## Concorrenza




### Thread

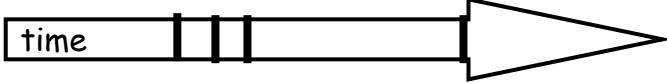


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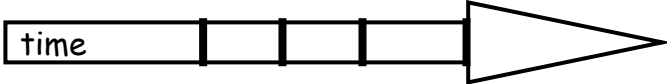
## Concorrenza



☞ Thread A




☞ Thread B



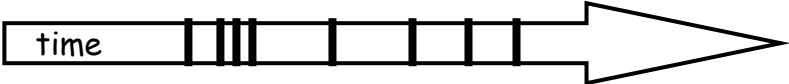
11

## Interleaving



☞ Eventi dei thread

- Eventi dei thread in esecuzione sono intercalati
- Non sono sempre indipendenti

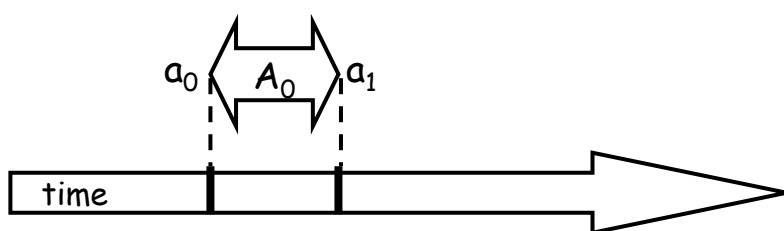


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## Intervallo

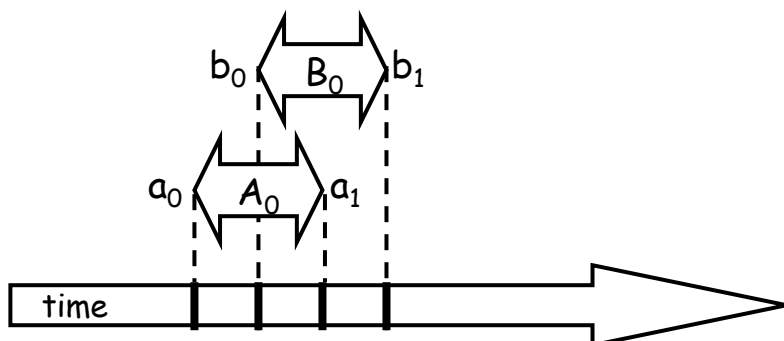
- Un *intervallo*  $A_0 = (a_0, a_1)$ 
  - Periodo di tempo trascorso tra l'occorrenza dell'evento  $a_0$  e l'evento  $a_1$



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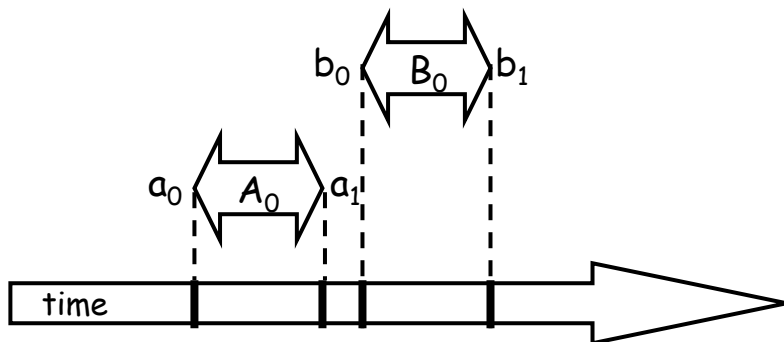


## Intervalli si possono sovrapporre



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## Intervalli possono essere disgiunti

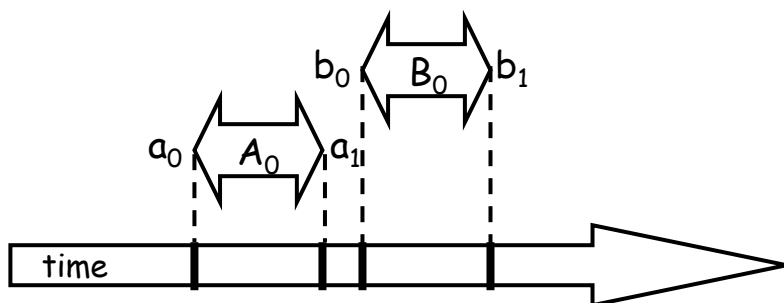


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## Precedenza




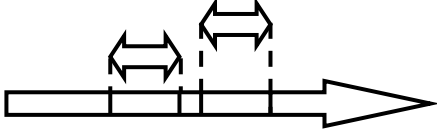
Intervallo  $A_0$  precede intervallo  $B_0$



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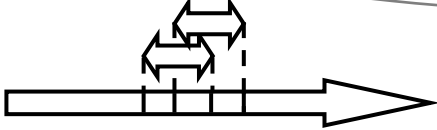
## Ordinamento



- ✎ Notazione:  $A_0 \rightarrow B_0$ ,
  - Evento terminale di  $A_0$  occorre prima dell'evento iniziale di  $B_0$
  - "happens before" nozione introdotta da Leslie Lamport (Time, clocks and ordering of events in distributed systems)

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## Ordinamento



- ✎  $A \rightarrow A$  (FALSO)
- ✎ Se  $A \rightarrow B$  allora  $B \rightarrow A$  (FALSO)
- ✎ Se  $A \rightarrow B$  &  $B \rightarrow C$  allora  $A \rightarrow C$
- ✎ True Concurrency:  $A \rightarrow B$  &  $B \rightarrow A$  potrebbero essere entrambe false

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## Ordinamento parziale



- ✎ Irriflessivo:
  - Non vale  $A \rightarrow A$
- ✎ Antisimmetrico:
  - $A \rightarrow B$  non implica che  $B \rightarrow A$
- ✎ Transitivo:
  - Se  $A \rightarrow B$  &  $B \rightarrow C$  allora  $A \rightarrow C$

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## Ordine totale



- ✎ Ordine totale = ordine parziale
- ✎ Vicolo: per ogni coppia di elementi distinti  $A, B$ ,
  - Deve valere  $A \rightarrow B$  o  $B \rightarrow A$

20

## Ripetizione di eventi



```
while (mumble) {  
    a0; a1;  
}
```

$K$ -sima occorrenza  
di  $a_0$

$a_0^k$

$A_0^k$

$k$ -sima occorrenza  
dell'intervallo  
 $A_0 = (a_0, a_1)$

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## Il solito contatore



```
public class Counter {  
    private long value;  
  
    public long getAndIncrement() {  
        temp = value;  
        value = temp + 1;  
        return temp;  
    }  
}
```

Azione  
Atomica

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## Locks (Mutual Exclusion)



```
public interface Lock {  
    public void lock();  
    public void unlock();  
}
```

**Definizione dei lock usata  
in Java**

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## Locks (Mutual Exclusion)



```
public interface Lock {  
    public void lock();  
    public void unlock();  
}
```

**acquire lock**

24

## Locks (Mutual Exclusion)



```
public interface Lock {  
    public void lock();  
    public void unlock();  
}
```

**lock()** acquire lock

**unlock()** release lock

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## Esempio



```
public class Counter {  
    private long value;  
    private Lock lock;  
    public long getAndIncrement() {  
        lock.lock();  
        try {  
            int temp = value;  
            value = value + 1;  
        } finally {  
            lock.unlock();  
        }  
        return temp;  
    }  
}
```

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## Using Locks



```
public class Counter {
    private long value;
    private Lock lock;
    public long getAndIncrement() {
        lock.lock();
        try {
            int temp = value;
            value = value + 1;
        } finally {
            lock.unlock();
        }
        return temp;
    }
}
```

**acquire Lock**

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## Using Locks



```
public class Counter {
    private long value;
    private Lock lock;
    public long getAndIncrement() {
        lock.lock();
        try {
            int temp = value;
            value = value + 1;
        } finally {
            lock.unlock();
        }
        return temp;
    }
}
```

**Release lock**

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## Using Locks



```
public class Counter {
    private long value;
    private Lock lock;
    public long getAndIncrement() {
        lock.lock();
        try {
            int temp = value;
            value = value + 1;
        } finally {
            lock.unlock();
        }
        return temp;
    }
}
```

Critical section

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## Il problema della mutua esclusione



- Con la notazione  $CS_i^k$
- indichiamo che il thread  $i$  sta eseguendo per la  $k$ -sima volta sezione critica

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## Mutua esclusione



- ☞  $CS_i^k \Leftrightarrow$
- ☞  $CS_j^m \Leftrightarrow$
- ☞ Allora deve accadere
  - ○
  - $\Leftrightarrow \Leftrightarrow \quad \Leftrightarrow \Leftrightarrow$

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## Mutua esclusione




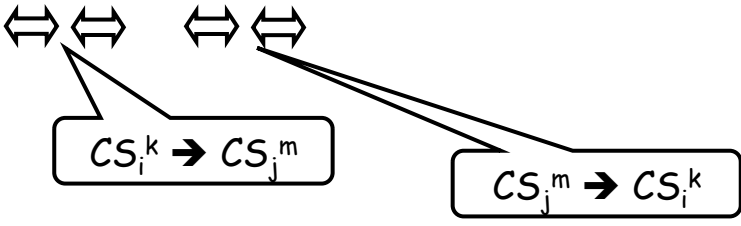
- ☞  $CS_i^k \Leftrightarrow$
- ☞  $CS_j^m \Leftrightarrow$
- ☞ Allora deve accadere
  - ○
  - $\Leftrightarrow \Leftrightarrow \quad \Leftrightarrow \Leftrightarrow$
  - $CS_i^k \rightarrow CS_j^m$

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## Mutua esclusione



- ✎  $CS_i^k \Leftrightarrow$
- ✎  $CS_j^m \Leftrightarrow$
- ✎ Allora deve accadere
  - ○
  - 

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## Deadlock-Free



- ✎ Assumiamo che un thread esegua una operazione di **lock()**
  - E non restituisce mai il lock
  - Allora altri thread possono eseguire chiamate di **lock()** e **unlock()** “infinitely often”
- ✎ Sistema nella sua globalità continua a evolvere anche se una sua sottocomponente “starve”

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## Starvation-Free



- ✎ Se un thread esegue una operazione di lock lock() allora “eventually return”
- ✎ Ogni singolo thread si muove !!

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```
class ... implements Lock {  
    ...  
    // thread-local index, 0 or 1  
    public void lock() {  
        int i = ThreadID.get();  
        int j = 1 - i;  
        ...  
    }  
}
```

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```
class ... implements Lock {  
    ...  
    // thread-local index, 0 or 1  
    public void lock() {  
        int i = ThreadID.get();  
        int j = 1 - i;  
        ...  
    }  
}
```

*i* thread in esecuzione,  
*j* thread sospeso

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## LockOne



```
class LockOne implements Lock {  
    private boolean[] flag = new boolean[2];  
    public void lock() {  
        flag[i] = true;  
        while (flag[j]) {}  
    }  
}
```

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## LockOne



```
class LockOne implements Lock {  
    private boolean[] flag = new boolean[2];  
    public void lock() {  
        flag[i] = true;  
        while (flag[j]) {}  
    }  
}
```

Each thread has flag

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## LockOne



```
class LockOne implements Lock {  
    private boolean[] flag = new boolean[2];  
    public void lock() {  
        flag[i] = true;  
        while (flag[j]) {}  
    }  
}
```

Set my flag

40

## LockOne



```
class LockOne implements Lock {  
    private boolean[] flag = new boolean[2];  
    public void lock() {  
        flag[i] = true;  
        while (flag[j]) {}  
    }  
}
```

Wait for other flag to  
become false

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LockOne verifica la proprieta' di mutua  
esclusione?



- ✎ Ipotesi (per assurdo):  $CS_A^j$  si sovrappone con  $CS_B^k$
- ✎ Consideriamo l'occorrenza (j-sima e k-sima)
- ✎ Si deriva una contraddizione

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## Analizziamo il codice



✎  $\text{write}_A(\text{flag}[A]=\text{true}) \rightarrow \text{read}_A(\text{flag}[B]==\text{false})$   
 $\rightarrow \text{CS}_A$

✎  $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow \text{read}_B(\text{flag}[A]==\text{false})$   
 $\rightarrow \text{CS}_B$

```
class LockOne implements Lock {  
    ...  
    public void lock() {  
        flag[i] = true;  
        while (flag[j]) {}  
    }  
}
```

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## Ipotesi



✎  $\text{read}_A(\text{flag}[B]==\text{false}) \rightarrow \text{write}_B(\text{flag}[B]=\text{true})$

✎  $\text{read}_B(\text{flag}[A]==\text{false}) \rightarrow \text{write}_A(\text{flag}[A]=\text{true})$

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## Mettiamo tutto assieme



### Ipotesi:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

### Il codice

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

45

### Ipotesi:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

### codice

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

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Ipotesi:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

Codice

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

47



Ipotesi:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

Codice

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

48





Ipotesi:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$

Codice

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$



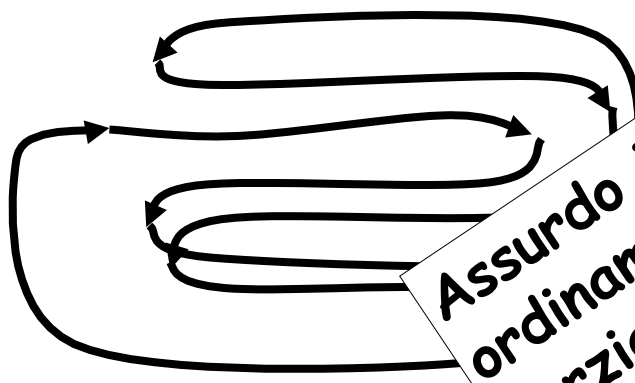
Ipotesi:

- $\text{read}_A(\text{flag}[B] == \text{false}) \rightarrow \text{write}_B(\text{flag}[B] = \text{true})$
- $\text{read}_B(\text{flag}[A] == \text{false}) \rightarrow \text{write}_A(\text{flag}[A] = \text{true})$


Codice

- $\text{write}_A(\text{flag}[A] = \text{true}) \rightarrow \text{read}_A(\text{flag}[B] == \text{false})$
- $\text{write}_B(\text{flag}[B] = \text{true}) \rightarrow \text{read}_B(\text{flag}[A] == \text{false})$

Otteniamo un ciclo!



Assurdo in un ordinamento parziale




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## Deadlock Freedom

LockOne non soddisfa la proprieta' di deadlock-freedom

- Concurrent execution can deadlock

```
flag[i] = true;   flag[j] = true;
while (flag[j]){ while (flag[i]){
```



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## LockTwo



```
public class LockTwo implements Lock {
    private int victim;
    public void lock() {
        victim = i;
        while (victim == i) {};
    }
    public void unlock() {}
}
```

53

## LockTwo



```
public class LockTwo implements Lock {
    private int victim;
    public void lock() {
        victim = i;
        while (victim == i) {};
    }
    public void unlock() {}
}
```

**Let other go first**

54

## LockTwo



```
public class LockTwo implements Lock {  
    private int victim;  
    public void lock() {  
        victim = i;  
        while (victim == i) {};  
    }  
    public void unlock() {}  
}
```

**Wait for permission**

55

## LockTwo



```
public class Lock2 implements Lock {  
    private int victim;  
    public void lock() {  
        victim = i;  
        while (victim == i) {};  
    }  
    public void unlock() {}  
}
```

**Nothing to do**

56

## LockTwo



- ✎ Verifica la mutual exclusion
  - Se il thread **I** e in CS
  - Allora **victim == j**
  - Non puo' avere il valore 0 e 1 allo stesso tempo!!
- ✎ Non vala le deadlock free
  - Sequential deadlock
  - Concurrent no

```
public void LockTwo() {  
    victim = i;  
    while (victim == i) {};  
}
```

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## Algoritmo di Peterson



```
public void lock() {  
    flag[i] = true;  
    victim = i;  
    while (flag[j] && victim == i) {};  
}  
public void unlock() {  
    flag[i] = false;  
}
```

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Announce I'm

```
public void lock() {
  flag[i] = true;
  victim = i;
  while (flag[j] && victim == i) {};
}
public void unlock() {
  flag[i] = false;
}
```

interested

59



Announce I'm

```
public void lock() {
  flag[i] = true;
  victim = i;
  while (flag[j] && victim == i) {};
}
public void unlock() {
  flag[i] = false;
}
```

interested

Defer to other

60



```
public void lock() {
    flag[i] = true;
    victim = i;
    while (flag[j] && victim == i) {};
}
public void unlock() {
    flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim

61



```
public void lock() {
    flag[i] = true;
    victim = i;
    while (flag[j] && victim == i) {};
}
public void unlock() {
    flag[i] = false;
}
```

Announce I'm interested

Defer to other

Wait while other interested & I'm the victim

No longer interested

62



## Mutual Exclusion

(1)  $\text{write}_B(\text{Flag}[B]=\text{true}) \rightarrow \text{write}_B(\text{victim}=B)$

```
public void lock() {  
    flag[i] = true;  
    victim = i;  
    while (flag[j] && victim == i) {};  
}
```

63  
63



(2)  $\text{write}_A(\text{victim}=A) \rightarrow \text{read}_A(\text{flag}[B])$   
 $\rightarrow \text{read}_A(\text{victim})$

```
public void lock() {  
    flag[i] = true;  
    victim = i;  
    while (flag[j] && victim == i) {};  
}
```

64  
64





## Ipotesi

(3)  $\text{write}_B(\text{victim}=B) \rightarrow \text{write}_A(\text{victim}=A)$

65  
65



## Mescoliamo ben bene

(1)  $\text{write}_B(\text{flag}[B]=\text{true}) \rightarrow$

(3)  $\text{write}_B(\text{victim}=B) \rightarrow \text{write}_A(\text{victim}=A)$

(2)   $\rightarrow \text{read}_A(\text{flag}[B])$

$\rightarrow \text{read}_A(\text{victim})$

**Pertanto,  $A$  read  $\text{flag}[B] == \text{true}$**

**e**

**$\text{victim} == A$ , non intra in CS**

**QED**

66  
66



## Deadlock Free

```
public void lock() {  
    ...  
    while (flag[j] && victim == i) {};  
}
```

- ✎ Il thread viene bloccato
  - **while** loop
  - Se il flag dell'altro thread e' true
  - Solo se e' la "vittima" **victim**
- ✎ Una sola flag ha valore false

67  
67




## Starvation Free

- ✎ Thread **i** e' bloccato solo se **j** esegue sempre
- ✎ **flag[j] == true and victim == i**
- ✎ Ma se **j** entra nuovamente
  - **victim** diventa **j**.
  - Pertanto entra **i**

```
public void lock() {  
    flag[i] = true;  
    victim = i;  
    while (flag[j] && victim == i) {};  
}  
  
public void unlock() {  
    flag[i] = false;  
}
```

68  
68

## Filter



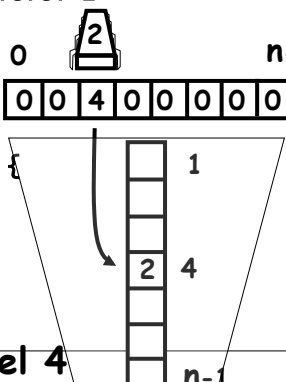
```

class Filter implements Lock {
    int[] level; // level[i] for thread i
    int[] victim; // victim[L] for level L

    public Filter(int n) {
        level = new int[n];
        victim = new int[n];
        for (int i = 1; i < n; i++) {
            level[i] = 0;
        }
        ...
    }
}
    
```

level


0	0	4	0	0	0	0	0	0	n-1
---	---	---	---	---	---	---	---	---	-----



**Thread 2 at level 4**

victim 69

## Filter



```

class Filter implements Lock {
    ...

    public void lock(){
        for (int L = 1; L < n; L++) {
            level[i] = L;
            victim[L] = i;
            while ((∃ k != i level[k] >= L) &&
                victim[L] == i ) {};
        }
    }
    public void unlock() {
        level[i] = 0;
    }
}
    
```

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## Filter



```
class Filter implements Lock {
  ...
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while (( $\exists$  k != i) level[k] >= L) &&
        victim[L] == i) {}
    }
  }
  public void release(int i) {
    level[i] = 0;
  }
}
```

One level at a time

71

## Filter



```
class Filter implements Lock {
  ...
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while (( $\exists$  k != i) level[k] >= L) &&
        victim[L] == i) {}
    }
  }
  public void release(int i) {
    level[i] = 0;
  }
}
```

Announce intention to enter level L

72

## Filter



```
class Filter implements Lock {
    int level[n];
    int victim[n];
    public void lock() {
        for (int L = 1; L < n; L++) {
            level[i] = L;
            victim[L] = i;
            while ((∃ k != i) level[k] >= L) &&
                victim[L] == i) {};
        }
    }
    public void release(int i) {
        level[i] = 0;
    }
}
```

**Give priority to anyone but me**

73

## Filter



**Wait as long as someone else is at same or higher level, and I'm designated victim**

```
public void lock() {
    for (int L = 1; L < n; L++) {
        level[i] = L;
        victim[L] = i;
        while ((∃ k != i) level[k] >= L) &&
            victim[L] == i) {};
    }
}
public void release(int i) {
    level[i] = 0;
}
```

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## Filter



```
class Filter implements Lock {
  int level[n];
  int victim[n];
  public void lock() {
    for (int L = 1; L < n; L++) {
      level[i] = L;
      victim[L] = i;
      while (( $\exists k \neq i$ ) level[k] >= L) &&
        victim[L] == i) {};
```

Thread enters level L when it completes the loop

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## Bakery Algorithm



- 👁️ First-Come-First-Served
- 👁️ Quale caratteristica ?
  - Si prende un "ticket"
  - Si sttendo il proprio turno

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## Bakery Algorithm



```

class Bakery implements Lock {
    boolean[] flag;
    Label[] label;
    public Bakery (int n) {
        flag = new boolean[n];
        label = new Label[n];
        for (int i = 0; i < n; i++) {
            flag[i] = false; label[i] = 0;
        }
    }
    ...
}
    
```

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## Bakery Algorithm



```

class Bakery implements Lock {
    boolean[] flag;
    Label[] label;
    public Bakery (int n) {
        flag = new boolean[n];
        label = new Label[n];
        for (int i = 0; i < n; i++) {
            flag[i] = false; label[i] = 0;
        }
    }
    ...
}
    
```

0      2      6      n-1  

f	f	t	f	f	t	f	f
0	0	4	0	0	5	0	0

↓      ↓  
**CS**

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## Bakery Algorithm



```
class Bakery implements Lock {
  ...
  public void lock() {
    flag[i] = true;
    label[i] = max(label[0], ..., label[n-1])+1;
    while ( $\exists k$  flag[k]
          && (label[i],i) > (label[k],k));
  }
}
```

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## Bakery Algorithm



```
class Bakery implements Lock {
  ...
  public void lock() {
    flag[i] = true;
    label[i] = max(label[0], ..., label[n-1])+1;
    while ( $\exists k$  flag[k]
          && (label[i],i) > (label[k],k));
  }
}
```

**Doorway**

80



## Bakery Algorithm



```
class Bakery implements Lock {  
    ...  
    public void lock() {  
        flag[i] = true;  
        label[i] = max(label[0], ..., label[n-1])+1;  
        while ( $\exists k$  flag[k]  
            && (label[i],i) > (label[k],k));  
    }  
}
```

**I'm interested**

81

## Bakery Algorithm



```
class Bakery implements Lock {  
    ...  
    public void lock() {  
        flag[i] = true;  
        label[i] = max(label[0], ..., label[n-1])+1;  
        while ( $\exists k$  flag[k]  
            && (label[i],i) > (label[k],k));  
    }  
}
```

**Take increasing label (read labels in some arbitrary order)**

82

## Bakery Algorithm



```
class Bakery implements Lock {  
    ...  
    public void lock() {  
        flag[i] = true;  
        label[i] = max(label[0], ..., label[n-1])+1;  
        while ( $\exists k$  flag[k]  
                && (label[i],i) > (label[k],k));  
    }  
}
```

**Someone is interested**

83

## Bakery Algorithm



```
class Bakery implements Lock {  
    boolean flag[n];  
    int label[n];  
  
    public void lock() {  
        flag[i] = true;  
        label[i] = max(label[0], ..., label[n-1])+1;  
        while ( $\exists k$  flag[k]  
                && (label[i],i) > (label[k],k));  
    }  
}
```

**Someone is interested**

**With lower (label,i) in  
lexicographic order**

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## Bakery Algorithm



```
class Bakery implements Lock {  
    ...  
    public void unlock() {  
        flag[i] = false;  
    }  
}
```

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## Bakery Algorithm



```
class Bakery implements Lock {  
    ...  
    public void unlock() {  
        flag[i] = false;  
    }  
}
```

**No longer  
interested**

**labels are always increasing**

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## Bakery Y2<sup>32</sup>K Bug



```
class Bakery implements Lock {
    ...
    public void lock() {
        flag[i] = true;
        label[i] = max(label[0], ..., label[n-1])+1;
        while ( $\exists k$  flag[k]
                && (label[i],i) > (label[k],k));
    }
}
```

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## Bakery Y2<sup>32</sup>K Bug



```
class Bakery implements Lock { Mutex breaks if
    ... label[i] overflows
    public void lock() {
        flag[i] = true;
        label[i] = max(label[0], ..., label[n-1])+1;
        while ( $\exists k$  flag[k]
                && (label[i],i) > (label[k],k));
    }
}
```

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