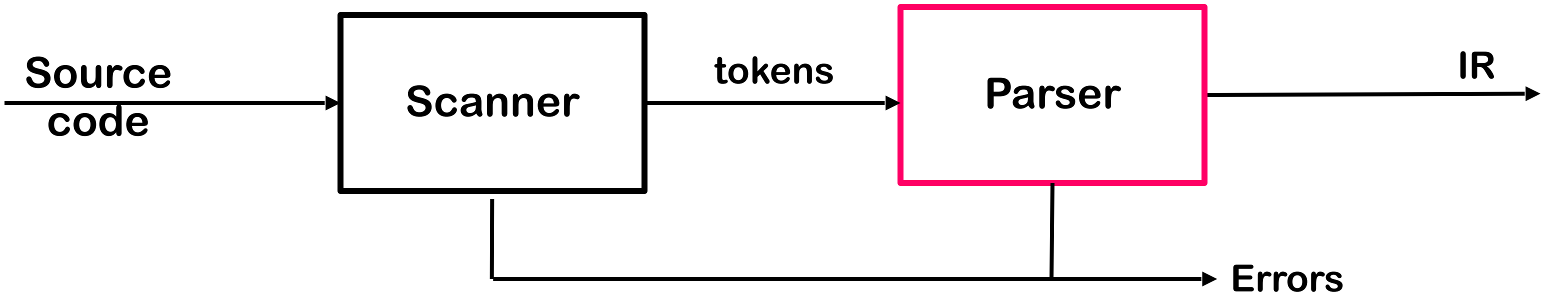
Introduction to Parsing

The Front End



Parser

- Checks the stream of words and their parts of speech (produced by the scanner) for grammatical correctness
- Determines if the input is syntactically well formed
- Guides checking at deeper levels than syntax
- Builds an IR representation of the code

The Study of Parsing

The process of discovering a derivation for some sentence

- Need a mathematical model of syntax — a grammar *G*
- Need an algorithm for testing membership in L(*G*)

Roadmap for our study of parsing

1. Context-free grammars and derivations
2. Top-down parsing
   - Generated LL(1) parsers & hand-coded recursive descent parsers
3. Bottom-up parsing
   - Generated LR(1) parsers

Why Not Use Regular Languages & DFAs?

Not all languages are regular (RL's ⊂ CFL's ⊂ CSL's)

You cannot construct DFA's to recognize these languages

- L = { $p^k q^k$ } (correspondence between declarations and variables)
- L = { $wcw^r \mid w \in \Sigma^*$ } (parenthesis languages)

Neither of these is a regular language

To recognize these features requires an arbitrary amount of context (left or right …)

But, this issue is somewhat subtle. You can construct DFA's for

- Strings with alternating 0's and 1's
  $(\varepsilon \mid 1) (01)^* (\varepsilon \mid 0)$
- Strings with an even number of 0's and 1's

RE's can count bounded sets and bounded differences

⇒ Cannot add parenthesis, brackets, begin-end pairs, …

A More Useful Grammar Than Sheep Noise

To explore the uses of CFGs, we need a more complex grammar

0	Expr	→	Expr Op Expr
1		|	<u>num</u>
2		|	<u>id</u>
3	Op	→	+
4		|	-
5		|	*
6		|	/

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,<u>x</u>> Op Expr
4	<id,<u>x</u>> - Expr
0	<id,<u>x</u>> - Expr Op Expr
1	<id,<u>x</u>> - <num,<u>2</u>> Op Expr
5	<id,<u>x</u>> - <num,<u>2</u>> * Expr
2	<id,<u>x</u>> - <num,<u>2</u>> * <id,<u>y</u>>

- Such a sequence of rewrites is called a derivation
- Process of discovering a derivation is called parsing

for the sequence of tokens <id,x> <op, -> < num,2> <op,*> < id,y>

Derivations

The goal of parsing is to construct a derivation

- At each step, we choose a nonterminal to replace
- Different choices can lead to different derivations

Two kind of derivations are of interest

- Leftmost derivation — replace leftmost NT at each step
- Rightmost derivation — replace rightmost NT at each step

These are the two systematic derivations

(We don't care about randomly-ordered derivations!)

The example on the preceding slide was a leftmost derivation

- Of course, there is also a rightmost derivation
- Interestingly, it turns out to be different

The rightmost derivation of id - num * id

0	Expr	→	Expr Op Expr
1		|	num
2		|	id
3	Op	→	+
4		|	-
5		|	*
6		|	/

Rightmost

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,x> Op Expr
4	<id,x> - Expr
0	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>

Leftmost

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	Expr Op <id,y>
5	Expr * <id,y>
0	Expr Op Expr * <id,y>
1	Expr Op <num,2> * <id,y>
4	Expr - <num,2> * <id,y>
2	<id,x> - <num,2> * <id,y>

They are different !

Derivations

The goal of parsing is to construct a derivation

A derivation consists of a series of rewrite steps

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \ldots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \text{sentence}$$

- Each γ_i is a sentential form
  - If γ contains only terminal symbols, γ is a sentence in L(G)
  - If γ contains 1 or more non-terminals, γ is a sentential form
- To get γ_i from γ_{i-1}, expand some NT $A \in \gamma_{i-1}$ by using $A \rightarrow \beta$
  - Replace the occurrence of $A \in \gamma_{i-1}$ with β to get γ_i
  - In a leftmost derivation, it would be the first NT $A \in \gamma_{i-1}$

A left-sentential form occurs in a leftmost derivation

A right-sentential form occurs in a rightmost derivation

The Two Derivations for x - 2 * y

Leftmost derivation

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,x> Op Expr
4	<id,x> - Expr
0	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>

Rightmost derivation

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	Expr Op <id,y>
5	Expr * <id,y>
0	Expr Op Expr * <id,y>
1	Expr Op <num,2> * <id,y>
4	Expr - <num,2> * <id,y>
2	<id,x> - <num,2> * <id,y>

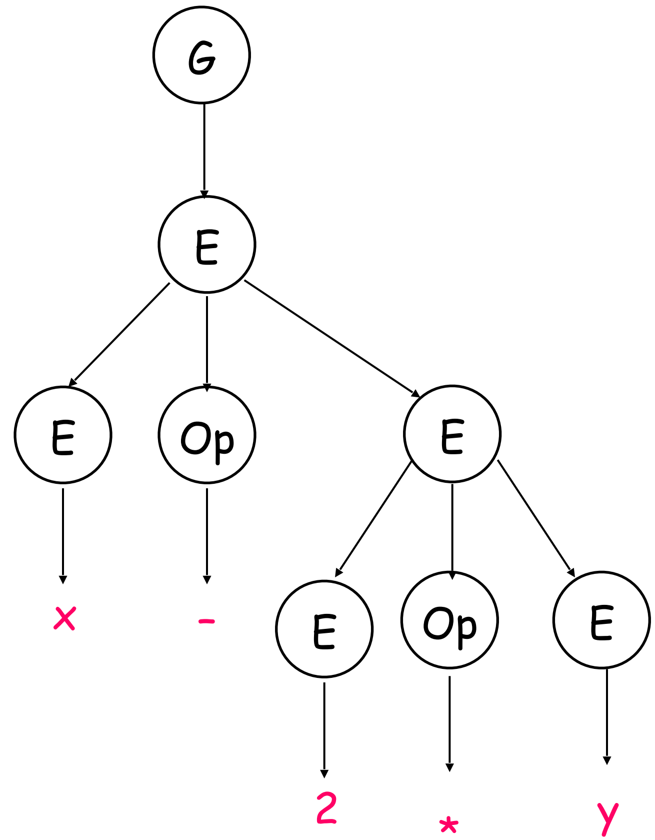
In both cases, Expr ⇒ id - num * id

- The two derivations produce different parse trees
- The parse trees imply different evaluation orders!

Derivations and Parse Trees

Leftmost derivation

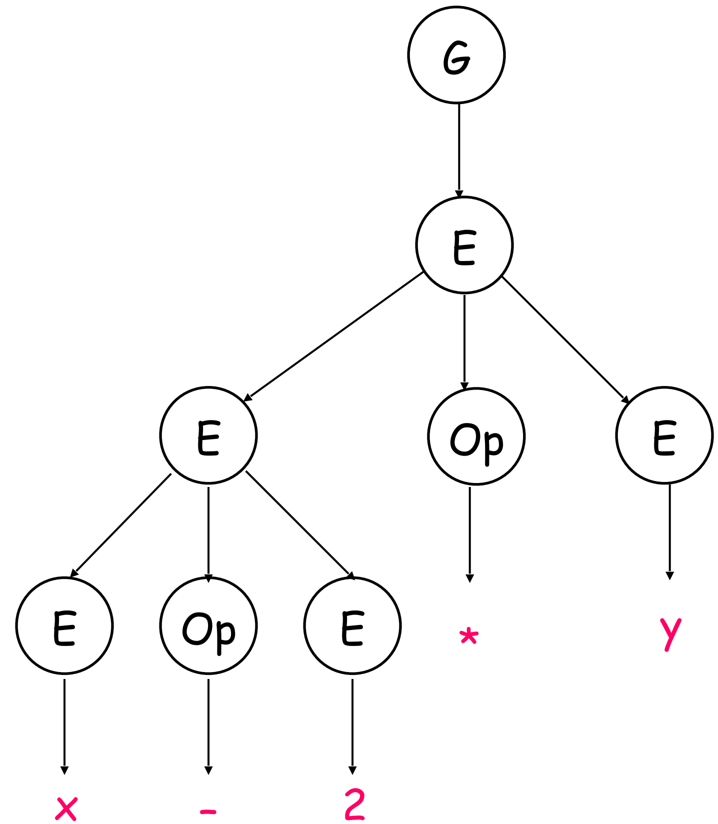
Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,x> Op Expr
4	<id,x> - Expr
0	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>



This evaluates as x - (2 * y)

Derivations and Parse Trees

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	Expr Op <id,y>
5	Expr * <id,y>
0	Expr Op Expr * <id,y>
1	Expr Op <num,2> * <id,y>
4	Expr - <num,2> * <id,y>
2	<id,x> - <num,2> * <id,y>



This evaluates as (x - 2) * y

This ambiguity is NOT good

Derivations and Precedence

These two derivations point out a problem with the grammar:

It has no notion of <u>precedence</u>, or implied order of evaluation

To add precedence

- Create a nonterminal for each level of precedence
- Isolate the corresponding part of the grammar
- Force the parser to recognize high precedence subexpressions first

For algebraic expressions

- Parentheses first (level 1)
- Multiplication and division, next (level 2)
- Subtraction and addition, last (level 3)

Derivations and Precedence

Adding the standard algebraic precedence produces:

level 3	0	Goal	→	Expr
	1	Expr	→	Expr + Term
	2		|	Expr - Term
	3		|	Term
level 2	4	Term	→	Term * Factor
	5		|	Term / Factor
	6		|	Factor
level 1	7	Factor	→	(Expr)
	8		|	<u>number</u>
	9		|	<u>id</u>

This grammar is slightly larger

- Takes more rewriting to reach some of the terminal symbols
- Encodes expected precedence
- Produces same parse tree under leftmost & rightmost derivations
- Correctness trumps the speed of the parser

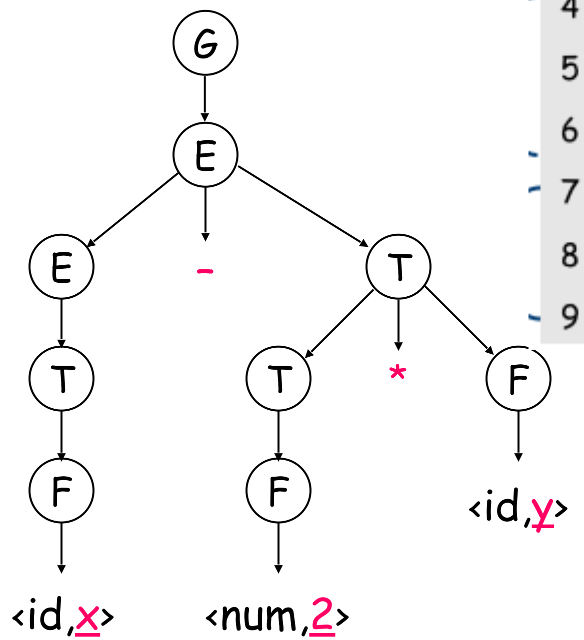
Let's see how it parses x - 2 * y

Introduced parentheses, too (beyond power of an RE)

Derivations and Precedence for x - (2 * y)

0	Goal	→	Expr	
1	Expr	→	Expr + Term	
2		|	Expr - Term	
3		|	Term	
4	Term	→	Term * Facto	
5		|	Term / Facto	
6		|	Factor	
7	Factor	→	(Expr)	
8		|	number	
9		|	id	

Rule	Sentential Form
—	Goal
0	Expr
2	Expr - Term
4	Expr - Term * Factor
9	Expr - Term * <id,y>
6	Expr - Factor * <id,y>
8	Expr - <num,2> * <id,y>
3	Term - <num,2> * <id,y>
6	Factor - <num,2> * <id,y>
9	<id,x> - <num,2> * <id,y>



Its parse tree

Both the leftmost and rightmost derivations give the same parse tree, because the grammar explicitly encodes the desired precedence.

Ambiguous Grammars

Let's leap back to our original expression grammar.

It had other problems.

0	Expr	→	Expr Op Expr
1		|	number
2		|	id
3	Op	→	+
4		|	-
5		|	*
6		|	/

Ambiguous!

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,x> Op Expr
4	<id,x> - Expr
0	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>

we have alternatives here

- This grammar allows multiple leftmost derivations for x - 2 * y
- Hard to automate derivation if > 1 choice

Two Leftmost Derivations for x – 2 * y

The Difference:

- Different productions chosen on the second step

Original choice

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,x> Op Expr
4	<id,x> - Expr
0	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
1	<id,x> - <num,2> * <id,y>

New choice

Rule	Sentential Form
—	Expr
0	Expr Op Expr
0	Expr Op Expr Op Expr
2	<id,x> Op Expr Op Expr
4	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>

- Both derivations succeed in producing x - 2 * y

Two Leftmost Derivations for x – 2 * y

The Difference:

- Different productions chosen on the second step

Rule	Sentential Form
—	Expr
0	Expr Op Expr
2	<id,x> Op Expr
4	<id,x> - Expr
0	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>

Rule	Sentential Form
—	Expr
0	Expr Op Expr
0	Expr Op Expr Op Expr
2	<id,x> Op Expr Op Expr
4	<id,x> - Expr Op Expr
1	<id,x> - <num,2> Op Expr
5	<id,x> - <num,2> * Expr
2	<id,x> - <num,2> * <id,y>

New choice

We are in the same situation! A different choice is possible !

Ambiguous Grammars

Definitions

- If a grammar has more than one leftmost derivation for a single sentential form, the grammar is ambiguous
- If a grammar has more than one rightmost derivation for a single sentential form, the grammar is ambiguous
- The leftmost and rightmost derivations for a sentential form may differ, even in an unambiguous grammar
  - However, they must have the same parse tree!

Classic example — the if-then-else problem

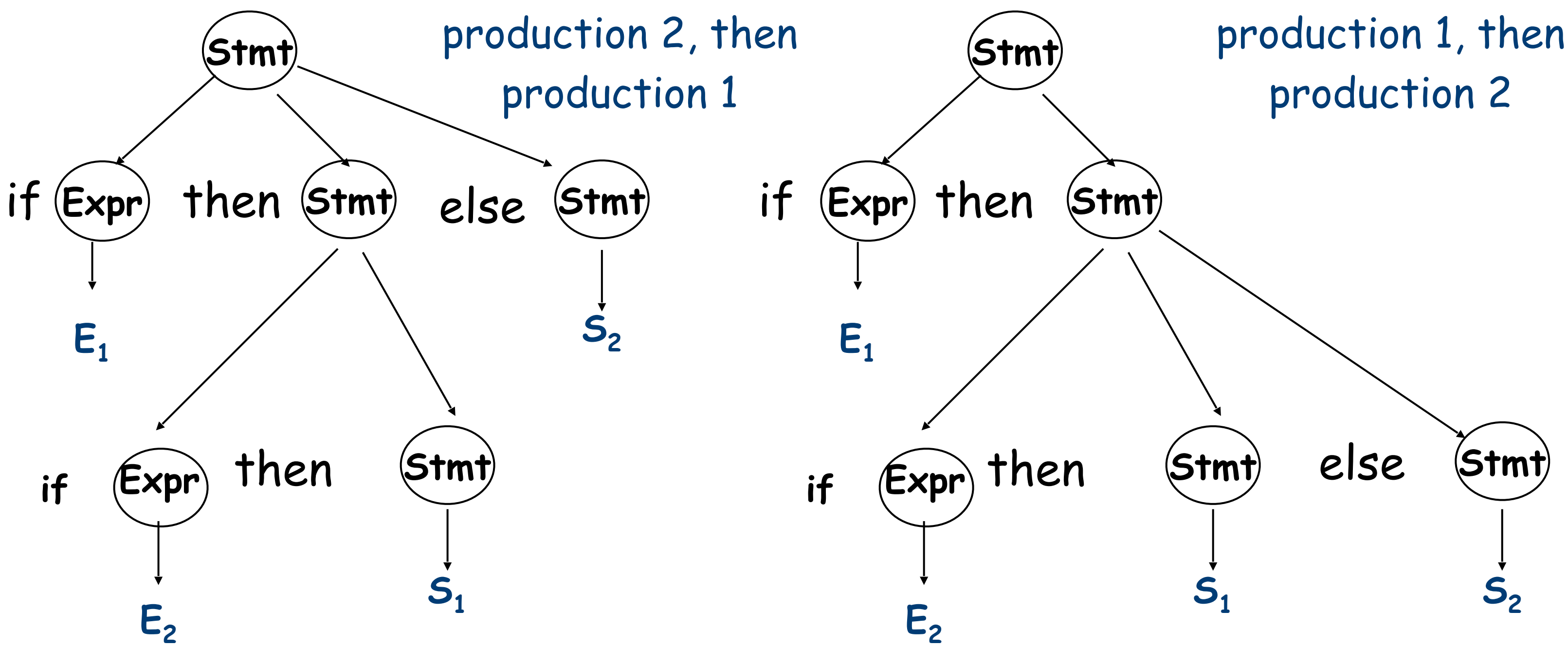
Stmt → if Expr then Stmt
| if Expr then Stmt else Stmt
| … other stmts …

This ambiguity is inherent in the grammar

Ambigous grammar

Stmt → <u>if</u> Expr <u>then</u> Stmt
      | <u>if</u> Expr <u>then</u> Stmt <u>else</u> Stmt

if E_1 then if E_2 then S_1 else S_2 has two different parse trees



The problem is that the structure built by the parser will determine the interpretation of the code, and these two forms have different meanings!

Ambiguity

The grammar forces the structure to match the desired meaning.

Removing the ambiguity

- Must rewrite the grammar to avoid generating the problem
- Match each else to innermost unmatched if (common sense rule)

0	Stmt	→	if Expr then Stmt
1		|	if Expr then WithElse else Stmt
2		|	Other Statements
3	WithElse	→	if Expr then WithElse else WithElse
4		|	Other Statements

With this grammar, the example has only one rightmost derivation

Intuition: once into WithElse, we cannot generate an unmatched else

… an if without an else can only come through rule 0…

Ambiguity

<u>if</u> E_1 <u>then</u> <u>if</u> E_2 <u>then</u> S_1 <u>else</u> S_2

Rule	Sentential Form
—	Stmt
0	<u>if</u> Expr <u>then</u> Stmt
1	<u>if</u> Expr <u>then</u> <u>if</u> Expr <u>then</u> WithElse <u>else</u> Stmt
2	<u>if</u> Expr <u>then</u> <u>if</u> Expr <u>then</u> WithElse <u>else</u> S_2
4	<u>if</u> Expr <u>then</u> <u>if</u> Expr <u>then</u> S_1 <u>else</u> S_2
?	<u>if</u> Expr <u>then</u> <u>if</u> E_2 <u>then</u> S_1 <u>else</u> S_2
?	<u>if</u> E_1 <u>then</u> <u>if</u> E_2 <u>then</u> S_1 <u>else</u> S_2

Other productions to derive Expr s

This grammar has only one rightmost derivation for the example

Deeper Ambiguity

Ambiguity usually refers to confusion in the CFG

Overloading can create deeper ambiguity

a = f(17)

In many Algol-like languages, f could be either a function or a subscripted variable

Disambiguating this one requires context

- Need values of declarations
- Really an issue of type, not context-free syntax
- Requires an extra-grammatical solution (not in CFG)
- Must handle these with a different mechanism
  - Step outside grammar rather than use a more complex grammar

Ambiguity - the Final Word

Ambiguity arises from two distinct sources

- Confusion in the context-free syntax (if-then-else)
- Confusion that requires context to resolve (overloading)

Resolving ambiguity

- To remove context-free ambiguity, rewrite the grammar
- To handle context-sensitive ambiguity takes cooperation
  - Knowledge of declarations, types, …
  - Accept a superset of L(G) & check it by other means (Context Sensitive analysis)
  - This is a language design problem

Sometimes, the compiler writer accepts an ambiguous grammar

- Parsing techniques that "do the right thing"
- i.e., always select the same derivation

Exercises

Say if the following grammars are ambiguous:

A-> AbA|c

```
S ::= a | SS

S ::= () | (S) | S S

S ::=  ε| a | aR
R ::= b | bS
```

and propose equivalent non ambiguous grammars

Parsing Techniques

Top-down parsers (LL(1), recursive descent)

- Start at the root of the parse tree and grow toward leaves
- Pick a production & try to match the input
- Bad "pick" $\Rightarrow$ may need to backtrack
- Some grammars are backtrack-free (predictive parsing)

Bottom-up parsers (LR(1), operator precedence)

- Start at the leaves and grow toward root
- As input is consumed, encode possibilities in an internal state
- Start in a state valid for legal first tokens
- Bottom-up parsers handle a large class of grammars

Top-down Parsing

A top-down parser starts with the root of the parse tree

The root node is labeled with the goal symbol of the grammar

Top-down parsing algorithm:

Construct the root node of the parse tree

Repeat until lower fringe of the parse tree matches the input string

1. At a node labeled *A*, select a production with *A* on its lhs and, for each symbol on its rhs, construct the appropriate child
2. When a terminal symbol is added to the border and it doesn't match the border, backtrack
3. Find the next node to be expanded (label $\in$ NT)

The key is picking the right production in step 1

- That choice should be guided by the input string

Remember the expression grammar?

We will call this version "the classic expression grammar"

0	Goal	→	Expr
1	Expr	→	Expr + Term
2		|	Expr - Term
3		|	Term
4	Term	→	Term * Factor
5		|	Term / Factor
6		|	Factor
7	Factor	→	(Expr)
8		|	number
9		|	id

And the input x – 2 * y

Example

Let's try x – 2 * y :

Rule	Sentential Form	Input
—	Goal	↑x - 2 * y

↑ is the position in the input buffer

0	Goal	→	Expr
1	Expr	→	Expr + Term
2		|	Expr - Term
3		|	Term
4	Term	→	Term * Factor
5		|	Term / Factor
6		|	Factor
7	Factor	→	(Expr)
8		|	number
9		|	id

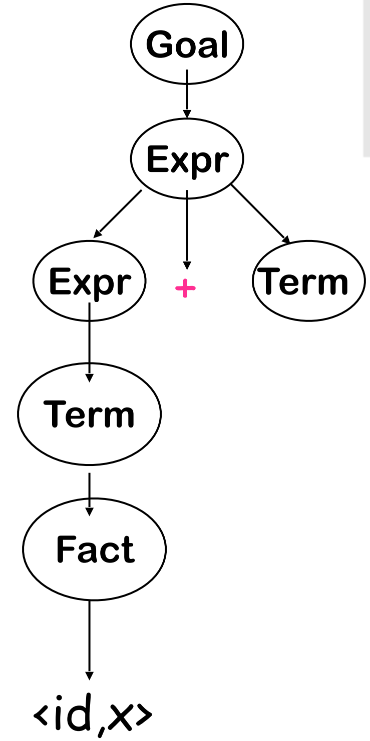
↑ is the position in the input buffer

Example

0	Goal	→	Expr
1	Expr	→	Expr + Term
2		|	Expr - Term
3		|	Term
4	Term	→	Term * Factor
5		|	Term / Factor
6		|	Factor
7	Factor	→	(Expr)
8		|	number
9		|	id

Let's try x – 2 * y :

Rule	Sentential Form	Input
—	Goal	↑x - 2 * y
0	Expr	↑x - 2 * y
1	Expr +Term	↑x - 2 * y
3	Term +Term	↑x - 2 * y
6	Factor +Term	↑x - 2 * y
9	<id,x> +Term	↑x - 2 * y
→	<id,x> +Term	x ↑- 2 * y



This worked well, except that "–" doesn't match "+"

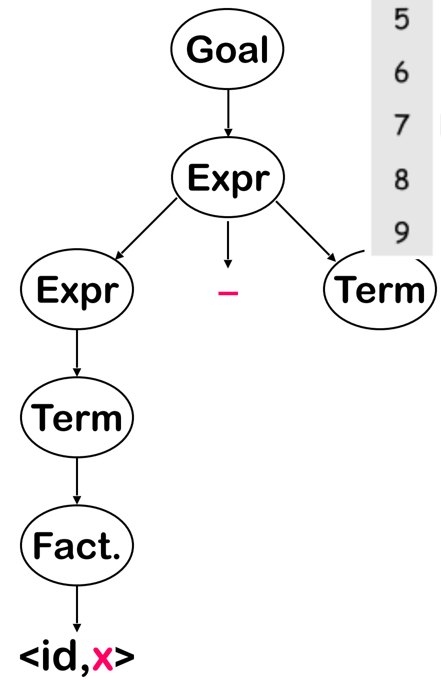
The parser must backtrack to here

0	Goal	→	Expr
1	Expr	→	Expr + Term
2		|	Expr - Term
3		|	Term
4	Term	→	Term * Factor
5		|	Term / Factor
6		|	Factor
7	Factor	→	(Expr)
8		|	number
9		|	id

Example

Continuing with x - 2 * y :

Rule	Sentential Form	Input
—	Goal	↑x - 2 * y
0	Expr	↑x - 2 * y
2	Expr -Term	↑x - 2 * y
3	Term -Term	↑x - 2 * y
6	Factor -Term	↑x - 2 * y
9	<id,x> - Term	↑x - 2 * y
→	<id,x> -Term	x ↑- 2 * y
→	<id,x> -Term	x - ↑2 * y



Now, "-" and "-" match

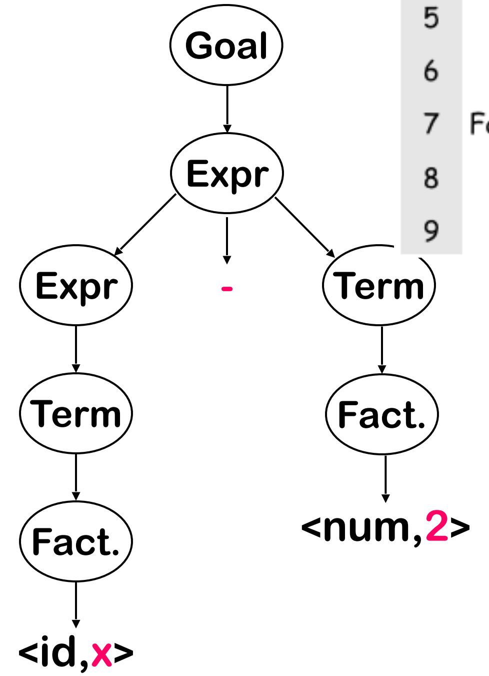
Now we can expand Term to match "2"

Example

0	Goal	→	Expr
1	Expr	→	Expr + Term
2		|	Expr - Term
3		|	Term
4	Term	→	Term * Facto
5		|	Term / Facto
6		|	Factor
7	Factor	→	(Expr)
8		|	number
9		|	id

Trying to match the "2" in x - 2 * y :

Rule	Sentential Form	Input
→	<id,x> - Term	x - ↑2 * y
6	<id,x> - Factor	x - ↑2 * y
8	<id,x> - <num,2>	x - ↑2 * y
→	<id,x> - <num,2>	x - 2 ↑* y



Where are we?

- "2" matches "2"
- We have more input, but no NTs left to expand
- The expansion terminated too soon

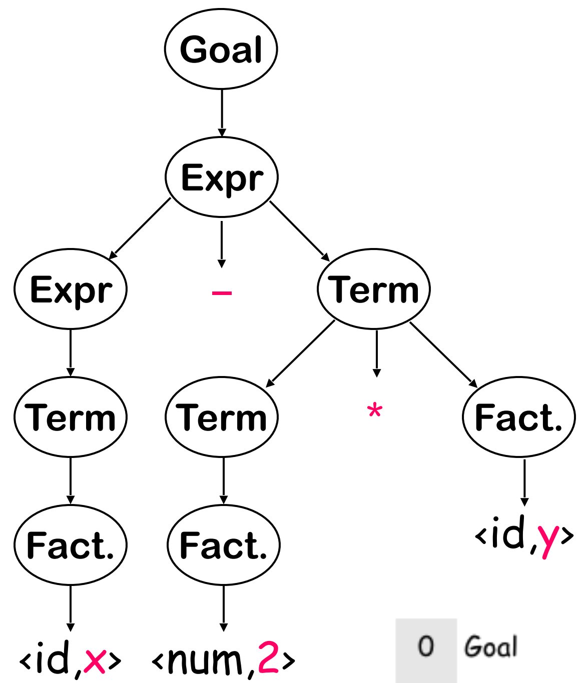
⇒ Need to backtrack

Example

The Point: The parser must make the right choice when it expands a NT. Wrong choices lead to wasted effort.

Trying again with "2" in x - 2 * y :

Rule	Sentential Form	Input
→	<id,x> - Term	x - ↑2 * y
4	<id,x> - Term * Factor	x - ↑2 * y
6	<id,x> - Factor * Factor	x - ↑2 * y
8	<id,x> - <num,2> * Factor	x - ↑2 * y
→	<id,x> - <num,2> * Factor	x - 2 ↑* y
→	<id,x> - <num,2> * Factor	x - 2 * ↑y
9	<id,x> - <num,2> * <id,y>	x - 2 * ↑y
→	<id,x> - <num,2> * <id,y>	x - 2 * y↑



This time, we matched & consumed all the input

⇒ Success!

0	Goal	→	Expr
1	Expr	→	Expr + Term
2		|	Expr - Term
3		|	Term
4	Term	→	Term * Fact
5		|	Term / Fact
6		|	Factor
7	Factor	→	(Expr)
8		|	number
9		|	id

Another possible parse

Other choices for expansion are possible

Rule	Sentential Form	Input
—	Goal	↑x - 2 * y
0	Expr	↑x - 2 * y
1	Expr +Term	↑x - 2 * y
1	Expr + Term +Term	↑x - 2 * y
1	Expr + Term +Term + Term	↑x - 2 * y
1	And so on	↑x - 2 * y

Consumes no input!

This expansion doesn't terminate

- Wrong choice of expansion leads to non-termination
- Non-termination is a bad property for a parser to have
- Parser must make the right choice

The property that we just saw: Left Recursion

Top-down parsers cannot handle left-recursive grammars

Formally,

A grammar is left recursive if $\exists\, A \in NT$ such that $\exists$ a derivation $A \Rightarrow^{+} A\alpha$, for some string $\alpha \in (NT \cup T)^{+}$

Our classic expression grammar is left recursive

- This can lead to non-termination in a top-down parser
- In a top-down parser, any recursion must be right recursion
- We would like to convert the left recursion to right recursion

Non-termination is <u>always</u> a bad property in a compiler

0	Goal	→	Expr
1	Expr	→	Expr + Term
2			| Expr - Term
3			| Term
4	Term	→	Term * Factor
5			| Term / Factor
6			| Factor
7	Factor	→	(Expr)
8			| <u>number</u>
9			| <u>id</u>

Eliminating Left Recursion

To remove left recursion, we can transform the grammar

Consider a grammar fragment of the form

$$Fee \rightarrow Fee\ \alpha$$
$$\quad |\ \beta$$

where β does not start with Fee

We can rewrite this fragment as

$$Fee \rightarrow \beta\ Fie$$
$$Fie \rightarrow \alpha\ Fie$$
$$\quad |\ \varepsilon$$

where Fie is a new non-terminal

The new grammar defines the same language as the old grammar, using only right recursion.

Added a reference to the empty string

Eliminating Left Recursion

The expression grammar contains two cases of left recursion

Expr	$\rightarrow$	Expr + Term	Term	$\rightarrow$	Term * Factor
	|	Expr - Term		|	Term * Factor
	|	Term		|	Factor

Applying the transformation yields

Expr	$\rightarrow$	Term Expr′	Term	$\rightarrow$	Factor Term′
Expr′	$\rightarrow$	+ Term Expr′	Term′	$\rightarrow$	* Factor Term′
	|	- Term Expr′		|	/ Factor Term′
	|	ε		|	ε

These fragments use only right recursion

Right recursion often means right associativity. In this case, the grammar does not display any particular associative bias.

Eliminating Left Recursion

Substituting them back into the grammar yields

0	Goal	→	Expr
1	Expr	→	Term Expr′
2	Expr′	→	+ Term Expr′
3		|	- Term Expr′
4		|	ε
5	Term	→	Factor Term′
6	Term′	→	* Factor Term′
7		|	/ Factor Term′
8		|	ε
9	Factor	→	(Expr)
10		|	number
11		|	id

- This grammar is correct, but somewhat non-intuitive.
- It is left associative, as was the original
  - ⇒ The naïve transformation yields a right recursive grammar, which changes the implicit associativity
- A top-down parser will terminate using it.
- even if it may still need to backtrack with it.

Eliminating Left Recursion

The transformation eliminates immediate left recursion

What about more general, indirect left recursion ?

The general algorithm:

arrange the NTs into some order $A_1, A_2, \ldots, A_n$

for i ← 1 to n

for s ← 1 to i - 1

replace each production $A_i \rightarrow A_s\gamma$ with $A_i \rightarrow \delta_1\gamma \mid \delta_2\gamma \mid \ldots \mid \delta_k\gamma$,

where $A_s \rightarrow \delta_1 \mid \delta_2 \mid \ldots \mid \delta_k$ are all the current productions for A_s

eliminate any immediate left recursion on A_i using the direct transformation

This assumes that the initial grammar has no cycles ($A_i \Rightarrow^+ A_i$), and no epsilon productions

Eliminating Left Recursion

How does this algorithm work?

1. Impose arbitrary order on the non-terminals
2. Outer loop cycles through NT in order
3. Inner loop ensures that a production expanding A_i has no non-terminal A_s in its rhs, for $s < i$
4. Last step in outer loop converts any direct recursion on A_i to right recursion using the transformation showed earlier
5. New non-terminals are added at the end of the order & have no left recursion

At the start of the i^{th} outer loop iteration

For all $k < i$, no production that expands A_k contains a non-terminal A_s in its rhs, for $s < k$

Example

$$Fee \rightarrow Fee\ \alpha \mid \beta \quad \xrightarrow{\text{rec elim}} \quad Fee \rightarrow \beta\ Fie, \quad Fie \rightarrow \alpha\ Fie \mid \varepsilon$$

- Order of symbols: G, E, T

1. A_i = G

G → E
E → E + T
E → T
T → E * T
T → id

no sub
no rec elim

2. A_i = E

G → E
E → T E'
E' → + T E'
E' → ε
T → E * T
T → id

no sub
rec elim

3. A_i = T, A_s = E

G → E
E → T E'
E' → + T E'
E' → ε
T → T E' * T
T → id

sub

4. A_i = T

G → E
E → T E'
E' → + T E'
E' → ε
T → id T'
T' → E ' * T T'
T' → ε

rec elim

Exercises

Eliminate left recursion from the following grammars:

grammatica I

S--> Aa | b
A--> Ac | Sd | e;

grammatica II

A -> Aa | AAb | c

Picking the "Right" Production

If it picks the wrong production, a top-down parser may backtrack

Alternative is to look ahead in input & use context to pick correctly

How much lookahead is needed?

- In general, an arbitrarily large amount

Fortunately,

- Large subclasses of CFGs can be parsed with limited lookahead
- Most programming language constructs fall in those subclasses

Among the interesting subclasses are LL(1) and LR(1) grammars

We start with LL(1) grammars & predictive parsing

LL(k) grammars

Predictive Parsing

Basic idea

Given $A \rightarrow \alpha \mid \beta$, the parser should be able to choose (between α & β) the right production to expand A in the parser tree at each step

How can it do it?

Guided by the input string!

LL(k) grammars

- An LL(k) grammar is a context-free grammar that can be parsed by predictive parser (no backtracking) which reads the input Left to right and construct a Leftmost derivation looking to k symbols in the input string

- A language that has a LL(k) grammar is said an LL(k) language

- LL(k) is a grammar that can predict the right production to apply with lookhead of most k symbols

$$LL(0) \subset LL(1) \subset LL(2) \subset ... \subset LL(*)$$

Predictive Parsing

Basic idea

Given $A \rightarrow \alpha \mid \beta$, the parser should be able to choose between α & β

The parser will decide what to choose on the base of the input and of the following sets:

- The FIRST set: FIRST(α) with $\alpha \in (T \cup NT)^*$
- The FOLLOW set: FOLLOW(A) with $A \in NT$

The FIRST set

FIRST sets

For some rhs $\alpha \in G$, define FIRST(α) as the set of tokens that appear as the first symbol in some string that derives from α

That is, x ∈ FIRST(α) iff $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

We will learn how to compute it!

The LL(1) Property

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ both appear in the grammar, we would like

$$\text{FIRST}(\alpha) \cap \text{FIRST}(\beta) = \varnothing$$

This is almost correct See the next slide

This would allow the parser to make a correct choice with a lookahead of exactly one symbol !

Example

0	Goal	→	Expr
1	Expr	→	Term Expr'
2	Expr'	→	+ Term Expr'
3		|	- Term Expr'
4		|	ε
5	Term	→	Factor Term'
6	Term'	→	* Factor Term'
7		|	/ Factor Term'
8		|	ε
9	Factor	→	(Expr)
10		|	number
11		|	id

first(Expr')={ +,-, ε}

But what else I need to consider?

{ eof,) }

Predictive Parsing

What about ε-productions?

⇒ They complicate the definition of LL(1)

If $A \rightarrow \alpha$ and $A \rightarrow \beta$ and $\varepsilon \in$ FIRST(α), then we need to ensure that FIRST(β) is disjoint from FOLLOW(A), too, where

FOLLOW(A) = the set of terminal symbols that can immediately follow A in a sentential form

Later we will learn how to compute them!

Define FIRST⁺($A \rightarrow \alpha$) as

- FIRST(α) $\cup$ FOLLOW(A), if $\varepsilon \in$ FIRST(α)
- FIRST(α), otherwise

Then, a grammar is LL(1) iff $A \rightarrow \alpha$ and $A \rightarrow \beta$ implies

FIRST⁺($A \rightarrow \alpha$) $\cap$ FIRST⁺($A \rightarrow \beta$) = $\varnothing$

Predictive Parsing

Given a grammar that has the LL(1) property

- Can write a simple routine to recognize each lhs
- Code is both simple & fast

One kind of predictive parser is the recursive descent parser.

Consider $A \rightarrow \beta_1 \mid \beta_2 \mid \beta_3$, with

$$FIRST^{+}(A \rightarrow \beta_i) \cap FIRST^{+}(A \rightarrow \beta_j) = \varnothing \text{ if } i \neq j$$

```
/* find an A */
if (current_word ∈ FIRST+(A→β1))
 recognise a β1 and return true
else if (current_word ∈ FIRST+(A→β2))
   recognise a β2 and return true
else if (current_word ∈ FIRST+(A→β3))
   recognise a β3 and return true
else
   report an error and return false
```

Of course, there is more detail to " recognize a β_i" a procedure for each nonterminal

Recursive Descent Parsing

Recall the expression grammar, after transformation

0	Goal	→	Expr
1	Expr	→	Term Expr'
2	Expr'	→	+ Term Expr'
3		|	- Term Expr'
4		|	ε
5	Term	→	Factor Term'
6	Term'	→	* Factor Term'
7		|	/ Factor Term'
8		|	ε
9	Factor	→	(Expr)
10		|	<u>number</u>
11		|	<u>id</u>

This produces a parser with six <u>mutually recursive</u> routines:

- Goal
- Expr
- EPrime
- Term
- TPrime
- Factor

Each recognizes one NT or T

The term <u>descent</u> refers to the direction in which the parse tree is built.

Recursive Descent Parsing (Procedural)

A couple of routines from the expression parser

they return a boolean

```
Goal( )
    token ← next_token( );
    if (Expr( ) = true & token = EOF)
        then next compilation step;
        else
            report syntax error;
            return false;

Expr( )
  if (Term( ) = false)
     then return false;
     else return Eprime( );
```

0	Goal	→	Expr
1	Expr	→	Term Expr′

Recursive Descent Parsing II

```
Eprime( )
    if (token = '+' OR  token = '-' )
    then begin
        token ← next_token( );
        if Term( ) then return Eprime ();
        else report syntax error;
        end;
    else if  (token = ')' OR  token = EOF )
        then return true;
        else  return false;
```

2	Expr'	→	+ Term Expr'
3			| - Term Expr'
4			| ε

FIRST⁺(Expr'-> + Term Expr')={+}

FIRST⁺(Expr'-> - Term Expr')={-}

FIRST⁺(Expr'-> ε)={EOF ,) }

Term, & Tprime follow the same basic lines

Recursive Descent Parsing III

```
Factor( )
  if (token = Number) then
     token ← next_token( );
     return true;
  else if (token = Identifier) then
     token ← next_token( );
     return true;
  else if (token = Lparen)
     token ← next_token( );
     if (Expr( ) = true & token = Rparen) then
        token ← next_token( );
        return true;
  // fall out of if statement
  report syntax error;
     return false;
```

9	Factor	→	(Expr)
10		|	number
11		|	id

FIRST+(Factor-> (Expr))={(}

FIRST+(Factor-> number)= number}

FIRST+(Factor-> id)={id}

looking for Number, Identifier, or "(", found token instead, or failed to find Expr or ")" after "("

Roadmap (Where are we?)

We set out to study parsing

- Specifying syntax
  - Context-free grammars ✓
- Top-down parsers
  - Algorithm & its problem with left recursion ✓
  - Ambiguity ✓
  - Left-recursion removal ✓
- Predictive top-down parsing
  - The LL(1) condition ✓
  - Simple recursive descent parsers ✓
  - Transforming a grammar to be LL(1)
  - First and Follow sets
  - Table-driven LL(1) parsers

What If My Grammar Is Not LL(1) ?

Can we transform a non-LL(1) grammar into an LL(1) grammar?

- In general, the answer is no, however, sometime it is yes

Assume a grammar *G* with productions $A \rightarrow \alpha\,\beta_1$ and $A \rightarrow \alpha\,\beta_2$

- If α derives anything other than ε, then

$$\text{FIRST}^{+}(A \rightarrow \alpha\,\beta_1) \cap \text{FIRST}^{+}(A \rightarrow \alpha\,\beta_2) \neq \emptyset$$

- And the grammar is not LL(1)
- If we pull the common prefix, α, into a separate production, we may make the grammar LL(1).

$$A \rightarrow \alpha\, A',\; A' \rightarrow \beta_1 \text{ and } A' \rightarrow \beta_2$$

Now, if $\text{FIRST}^{+}(A' \rightarrow \beta_1) \cap \text{FIRST}^{+}(A' \rightarrow \beta_2) = \emptyset$, *G* may be LL(1)

What If My Grammar Is Not LL(1) ?

Left Factoring

For each nonterminal A

find the longest prefix α common to 2 or more alternatives for A

if $\alpha \neq \varepsilon$ then

replace all of the productions

$A \rightarrow \alpha\, \beta_1 \mid \alpha\, \beta_2 \mid \alpha\, \beta_3 \mid \ldots \mid \alpha\, \beta_n \mid \gamma$

with

$A \rightarrow \alpha\, A' \mid \gamma$

$A' \rightarrow \beta_1 \mid \beta_2 \mid \beta_3 \mid \ldots \mid \beta_n$

Repeat until no nonterminal has alternative rhs' with a common prefix

This transformation makes some grammars into LL(1) grammars

There are languages for which no LL(1) grammar exists

Left Factoring Example

Consider a simple right-recursive expression grammar

0	Goal	→	Expr
1	Expr	→	Term + Expr
2		|	Term - Expr
3		|	Term
4	Term	→	Factor * Term
5		|	Factor / Term
6		|	Factor
7	Factor	→	<u>number</u>
8		|	<u>id</u>

To choose between 1, 2, & 3, an LL(1) parser must look past the <u>number</u> or <u>id</u> to *see* the operator.

FIRST+(1) = FIRST+(2) = FIRST+(3)

and

FIRST+(4) = FIRST+(5) = FIRST+(6)

Let's left factor this grammar.

Left Factoring Example

After Left Factoring, we have

0	Goal	→	Expr
1	Expr	→	Term Expr'
2	Expr'	→	+ Expr
3		|	- Expr
4		|	ε
5	Term	→	Factor Term'
6	Term'	→	* Term
7		|	/ Term
8		|	ε
9	Factor	→	<u>number</u>
10		|	<u>id</u>

Clearly,

$\text{FIRST}^+(2)$, $\text{FIRST}^+(3)$, & $\text{FIRST}^+(4)$

are disjoint, as are

$\text{FIRST}^+(6)$, $\text{FIRST}^+(7)$, & $\text{FIRST}^+(8)$

The grammar now has the LL(1) property

FIRST and FOLLOW Sets

FIRST(α)

For some $\alpha \in (T \cup NT)^*$, define FIRST(α) as the set of symbols that appear as the first one in some string that derives from α

That is, $\underline{x} \in$ FIRST(α) iff $\alpha \Rightarrow^* \underline{x}\,\gamma$, for some γ

FOLLOW(A)

For some $A \in NT$, define FOLLOW(A) as the set of symbols that can occur immediately after A in a valid sentential form

FOLLOW(S) = {EOF}, where S is the starting symbol

To build FOLLOW sets, we need FIRST sets …

Computing FIRST Sets

For a grammar symbol X, FIRST(X) is defined as follows.

- For every terminal X, FIRST(X) = {X}.
- For every nonterminal X, if $X \rightarrow Y_1 Y_2 \ldots Y_n$ is a production, then
  - $\text{FIRST}(Y_1) \subseteq \text{FIRST}(X)$.
  - Furthermore, if $Y_1, Y_2, \ldots, Y_k$ are nullable ($Y_i \stackrel{*}{\rightarrow} \varepsilon$) then

    $$\text{FIRST}(Y_{k+1}) \subseteq \text{FIRST}(X).$$

FIRST

- We are concerned with FIRST(X) only for the nonterminals of the grammar
- FIRST(X) for terminals is trivial
- According to the definition, to determine FIRST(*A*), we must inspect all productions that have *A* on the left

FIRST Example

Let the grammar be

$E \rightarrow T\ E'$

$E' \rightarrow +\ T\ E' \mid \varepsilon$

$T \rightarrow F\ T'$

$T' \rightarrow *\ F\ T' \mid \varepsilon$

$F \rightarrow (E) \mid$ **id** $\mid$ **num**

Find FIRST(E)

- E occurs on the left in only one production

  $E \rightarrow T\ E'$

- Therefore, FIRST(T) $\subseteq$ FIRST(E)

- Furthermore, T is not nullable

  Therefore, FIRST(E) = FIRST(T)

- We have yet to determine FIRST(T)

FIRST Example

Let the grammar be

$E \rightarrow T\ E'$

$E' \rightarrow +\ T\ E' \mid \varepsilon$

$T \rightarrow F\ T'$

$T' \rightarrow *\ F\ T' \mid \varepsilon$

$F \rightarrow (E) \mid$ **id** $\mid$ **num**

Find FIRST(T)

- T occurs on the left in only one production

  $T \rightarrow F\ T'$

- Therefore, FIRST(F) $\subseteq$ FIRST(T)
- Furthermore, F is not nullable
- Therefore, FIRST(T) = FIRST(F)
- We have yet to determine FIRST(F)

FIRST Example

Let the grammar be

$E \rightarrow T\ E'$

$E' \rightarrow +\ T\ E'\ |\ \varepsilon$

$T \rightarrow F\ T'$

$T' \rightarrow *\ F\ T'\ |\ \varepsilon$

$F \rightarrow$ (E) | **id** | **num**

- Find FIRST(F).

  FIRST(F) = {(, **id**, **num**}

- Therefore,
  - FIRST(E) = {(, **id**, **num**}
  - FIRST(T) = {(, **id**, **num**}

- Find FIRST(E')
  - FIRST(E') = {+}
- Find FIRST(T')
  - FIRST(T') = {*}

Computing FOLLOW Sets

- For a grammar symbol X, FOLLOW(X) is defined as follows
  - If S is the start symbol, then EOF $\in$ FOLLOW(S)
  - If $A \rightarrow \alpha B \beta$ is a production, then FIRST(β) $\subseteq$ FOLLOW(B)
  - If $A \rightarrow \alpha B$ is a production, or $A \rightarrow \alpha B \beta$ is a production and β is nullable, then FOLLOW(A) $\subseteq$ FOLLOW(B)

FOLLOW

- We are concerned about FOLLOW(X) only for the nonterminals of the grammar.
- According to the definition, to determine FOLLOW(A), we must inspect all productions that have A on the right.

FOLLOW Example

Let the grammar be

$E \rightarrow T\,E'$

$E' \rightarrow +\,T\,E' \mid \varepsilon$

$T \rightarrow F\,T'$

$T' \rightarrow *\,F\,T' \mid \varepsilon$

$F \rightarrow (E) \mid$ **id** $\mid$ **num**

Find FOLLOW(E).

- E is the start symbol, therefore EOF $\in$ FOLLOW(E).
- E occurs on the right in only one production.

  $F \rightarrow (E)$.
- Therefore FOLLOW(E) = {EOF,) }

FOLLOW Example

Let the grammar be

$E \rightarrow T\ E'$

$E' \rightarrow +\ T\ E'\ |\ \varepsilon$

$T \rightarrow F\ T'$

$T' \rightarrow *\ F\ T'\ |\ \varepsilon$

$F \rightarrow$ (E) | **id** | **num**

Find FOLLOW(E').

- E' occurs on the right in two productions.

  $E \rightarrow T\ E'$

  $E' \rightarrow +\ T\ E'$

- Therefore,
  FOLLOW(E') = FOLLOW(E) = {EOF,) }

FOLLOW Example

Let the grammar be

$$E \rightarrow T\,E'$$

$$E' \rightarrow +\,T\,E' \mid \varepsilon.$$

$$T \rightarrow F\,T'$$

$$T' \rightarrow *\,F\,T' \mid \varepsilon.$$

$$F \rightarrow (E) \mid \mathbf{id} \mid \mathbf{num}$$

Find FOLLOW(T)

- T occurs on the right in two productions

  $E \rightarrow T\,E'$

  $E' \rightarrow +\,T\,E'$

- Therefore,
  FOLLOW(T) contains FIRST(E') = {+}

- However, E' is nullable, therefore it also contains
  FOLLOW(E) = {EOF,) } and
  FOLLOW(E') = {EOF,) }

- Therefore, FOLLOW(T) = {+, EOF,) }

FOLLOW Example

Let the grammar be

$E \rightarrow T\ E'$

$E' \rightarrow +\ T\ E'\ |\ \varepsilon.$

$T \rightarrow F\ T'$

$T' \rightarrow *\ F\ T'\ |\ \varepsilon.$

$F \rightarrow (E)$ | **id** | **num**

Find FOLLOW(T')

- T' occurs on the right in two productions.

  $T \rightarrow F\ T'$

  $T' \rightarrow *\ F\ T'$

- Therefore, FOLLOW(T') = FOLLOW(T) = {EOF,), +}.

FOLLOW Example

Let the grammar be

$E \rightarrow T\ E'$

$E' \rightarrow +\ T\ E'\ |\ \varepsilon$

$T \rightarrow F\ T'$

$T' \rightarrow *\ F\ T'\ |\ \varepsilon$

$F \rightarrow (E)\ |$ **id** $|$ **num**

Find FOLLOW(F)

- F occurs on the right in two productions.

  $T \rightarrow F\ T'$

  $T' \rightarrow *\ F\ T'$
- Therefore, FOLLOW(F) contains FIRST(T') = {*}
- However, T' is nullable, therefore it also contains

  FOLLOW(T) = {+, EOF,)} and

  FOLLOW(T') = {EOF,), +}
- Therefore, FOLLOW(F) = {*, EOF,), +}.

Classic Expression Grammar

0	Goal	→	Expr
1	Expr	→	Term Expr'
2	Expr'	→	+ Term Expr'
3		|	- Term Expr'
4		|	ε
5	Term	→	Factor Term'
6	Term'	→	* Factor Term'
7		|	/ Factor Term'
8		|	ε
9	Factor	→	<u>number</u>
10		|	<u>id</u>
11		|	<u>(</u> Expr <u>)</u>

Symbol	FIRST	FOLLOW
<u>num</u>	<u>num</u>	Ø
<u>id</u>	<u>id</u>	Ø
+	+	Ø
-	-	Ø
*	*	Ø
/	/	Ø
<u>(</u>	<u>(</u>	Ø
<u>)</u>	<u>)</u>	Ø
<u>eof</u>	<u>eof</u>	Ø
ε	ε	Ø
Goal	<u>(</u>,<u>id</u>,<u>num</u>	EOF
Expr	<u>(</u>,<u>id</u>,<u>num</u>	<u>), EOF</u>
Expr'	+, -, ε	<u>)</u>, EOF
Term	<u>(</u>,<u>id</u>,<u>num</u>	+,-,),EOF
Term'	*, /, ε	+,-,),EOF
Factor	<u>(</u>,<u>id</u>,<u>num</u>	+,-,*,/,),EOF

Classic Expression Grammar

Symbol	FIRST	FOLLOW
Goal	(,id,num	EOF
Expr	(,id,num	), EOF
Expr'	+, -, ε	), EOF
Term	(,id,num	+,-,),EOF
Term'	*, /, ε	+,-,),EOF
Factor	(,id,num	+,-,*,/,),EOF

Define FIRST+($A \rightarrow \alpha$) as

- FIRST(α) ∪ FOLLOW(A), if $\varepsilon \in$ FIRST(α)
- FIRST(α), otherwise

Prod'n	FIRST+	
0	(,id,num	Goal ->Expr
1	(,id,num	Expr ->Term Expr'
2	+	Expr'-> +Term Expr'
3	-	Expr'-> -Term Expr'
4	),EOF	Expr'-> ε
5	(,id,num	Term-> Factor Term'
6	*	Term'->*Factor Term'
7	/	Term'->/ Factor Term'
8	+,-,), EOF	Term'-> ε
9	number	Factor-> number
10	id	Factor-> id
11	(	Factor-> (Expr)

Building Top-down Parsers for LL(1) Grammars

Given an LL(1) grammar, and its FIRST & FOLLOW sets …

- Emit a routine for each non-terminal
  - Nest of if-then-else statements to check alternate rhs's
  - Each returns true on success and throws an error on false
  - Simple, working (perhaps ugly) code
- This automatically constructs a recursive-descent parser

Improving matters

- Nest of if-then-else statements may be slow
  - Good case statement implementation would be better
- What about a table to encode the options?
  - Interpret the table with a skeleton, as we did in scanning

Cannot expand Factor into an operator ⇒ error

Building Top-down Parsers

Strategy

- Encode knowledge in a table
- Use a standard "skeleton" parser to interpret the table

Example

- The non-terminal Factor has 3 expansions
  - (Expr) or Identifier or Number
- Table might look like:

0	Goal	→	Expr
1	Expr	→	Term Expr'
2	Expr'	→	+ Term Expr'
3		|	- Term Expr'
4		|	ε
5	Term	→	Factor Term'
6	Term'	→	* Factor Term'
7		|	/ Factor Term'
8		|	ε
9	Factor	→	number
10		|	id
11		|	(Expr)

Terminal Symbols

Non-terminal Symbols

	+	-	*	/	Id.	Num.	(	)	EOF
Factor	—	—	—	—	10	9	11	—	—

Expand Factor by rule 9 with input "number"

Building Top-down Parsers

Building the complete table

- Need a row for every NT & a column for every T

	+	-	*	/	Id	Num	(	)	EOF
Goal	—	—	—	—	0	0	0	—	—
Expr	—	—	—	—	1	1	1	—	—
Expr'	2	3	—	—	—	—	—	4	4
Term	—	—	—	—	5	5	5	—	—
Term'	8	8	6	7	—	—	—	8	8
Factor	—	—	—	—	10	9	11	—	—

Building Top-down Parsers

Building the complete table

- Need a row for every NT & a column for every T
- Need an interpreter for the table (skeleton parser)

LL(1) Skeleton Parser

```
word ← NextWord()              // Initial conditions, including
push $ onto Stack              // a stack to track the border of the parse tree
push the start symbol, S, onto Stack
TOS ← top of Stack             //we read the first symbol on the top of the stack

loop forever
  if TOS = $ and word = EOF then
     break & report success    // exit on success

   else if TOS is a terminal then
     if TOS matches word then
        pop Stack              // recognized TOS
        word ← NextWord()
     else report error looking for TOS  // error exit

   else                        // TOS is a non-terminal
     if TABLE[TOS,word] is A→ B1B2…Bk then
        pop Stack              // get rid of A
        push Bk, Bk-1, …, B1   // in that order
     else break & report error expanding TOS

  TOS ← top of Stack
```

Building Top-down Parsers

Building the complete table

- Need a row for every NT & a column for every T
- Need a table-driven interpreter for the table
- Need an algorithm to build the table

Filling the table

	+	-	*	/	Id	Num	(	)	EOF
Goal	—	—	—	—	0	0	0	—	—
Expr	—	—	—	—	1	1	1	—	—
Expr'	2	3	—	—	—	—	—	4	4
Term	—	—	—	—	5	5	5	—	—
Term'	8	8	6	7	—	—	—	8	8
Factor	—	—	—	—	10	9	11	—	—

Prod'n	FIRST+	
0	(,id,num	Goal ->Expr
1	(,id,num	Expr ->Term Expr'
2	+	Expr'-> +Term Expr'
3	-	Expr'-> -Term Expr'
4	),EOF	Expr'-> ε
5	(,id,num	Term-> Factor Term'
6	*	Term'->*Factor Term'
7	/	Term'->/ Factor Term'
8	+,-,), EOF	Term'-> ε
9	number	Factor-> number
10	id	Factor-> id
11	(	Factor-> (Expr)

Filling in TABLE[X,y], $X \in NT$, $y \in T$

1. write the rule $X \rightarrow \beta$, if $y \in FIRST^{+}(X \rightarrow \beta)$
2. write error if rule 1 does not apply

If any entry has more than one rule, *G* is not LL(1)

We call this algorithm the LL(1) table construction algorithm

Actions of the LL(1) Parser for x + y x z

Rule	Stack	Input
—	$ *Goal*	↑ name + name x name
0	$ *Expr*	↑ name + name x name
1	$ *Expr′ Term*	↑ name + name x name
5	$ *Expr′ Term′ Factor*	↑ name + name x name
11	$ *Expr′ Term′* name	↑ name + name x name
→	$ *Expr′ Term′*	name ↑ + name x name
8	$ *Expr′*	name ↑ + name x name
2	$ *Expr′ Term* +	name ↑ + name x name
→	$ *Expr′ Term*	name + ↑ name x name
5	$ *Expr′ Term′ Factor*	name + ↑ name x name
11	$ *Expr′ Term′* name	name + ↑ name x name
→	$ *Expr′ Term′*	name + name ↑ x name
6	$ *Expr′ Term′ Factor* x	name + name ↑ x name
→	$ *Expr′ Term′ Factor*	name + name x ↑ name
11	$ *Expr′ Term′* name	name + name x ↑ name
→	$ *Expr′ Term′*	name + name x name ↑
8	$ *Expr′*	name + name x name ↑
4		name + name x name ↑

Prod'n	FIRST+	
0	(,id,num	Goal ->Expr
1	(,id,num	Expr ->Term Expr'
2	+	Expr'-> +Term Expr'
3	-	Expr'-> -Term Expr'
4	),EOF	Expr'-> ε
5	(,id,num	Term-> Factor Term'
6	*	Term'->*Factor Term'
7	/	Term'->/ Factor Term'
8	+,-,), EOF	Term'-> ε
9	number	Factor-> number
10	id	Factor-> id
11	(	Factor-> (Expr)

	+	-	*	/	Id	Num	(	)	EOF
Goal	—	—	—	—	0	0	0	—	—
Expr	—	—	—	—	1	1	1	—	—
Expr'	2	3	—	—	—	—	—	4	4
Term	—	—	—	—	5	5	5	—	—
Term'	8	8	6	7	—	—	—	8	8
Factor	—	—	—	—	10	9	11	—	—

Actions of the LL(1) Parser for x + / y

Rule	Stack	Input
—	$ *Goal*	↑ name + ÷ name
0	$ *Expr*	↑ name + ÷ name
1	$ *Expr′ Term*	↑ name + ÷ name
5	$ *Expr′ Term′ Factor*	↑ name + ÷ name
11	$ *Expr′ Term′* name	↑ name + ÷ name
→	$ *Expr′ Term′*	name ↑ + ÷ name
8	$ *Expr′*	name ↑ + ÷ name
2	$ *Expr′ Term* +	name ↑ + ÷ name
→	$ *Expr′ Term*	name + ↑ ÷ name

Prod'n	FIRST+	
0	(,id,num	Goal ->Expr
1	(,id,num	Expr ->Term Expr'
2	+	Expr'-> +Term Expr'
3	-	Expr'-> -Term Expr'
4	),EOF	Expr'-> ε
5	(,id,num	Term-> Factor Term'
6	*	Term'->*Factor Term'
7	/	Term'->/ Factor Term'
8	+,-,), EOF	Term'-> ε
9	number	Factor-> number
10	id	Factor-> id
11	(	Factor-> (Expr)

	+	-	*	/	Id	Num	(	)	EOF
Goal	—	—	—	—	0	0	0	—	—
Expr	—	—	—	—	1	1	1	—	—
Expr'	2	3	—	—	—	—	—	4	4
Term	—	—	—	—	5	5	5	—	—
Term'	8	8	6	7	—	—	—	8	8
Factor	—	—	—	—	10	9	11	—	—

Exercises

Let G be the following grammar:
S::= prog B end
B::= L B l L
L::= x A
A::= a A l x A l ;

- Is G in LL(1)? If yes, write its parsing table. If not, explain why.

Let G be the grammar below:
S::= S U l x
U::= x U U l x z

- Is G in LL(1)? If yes, write its parsing table. If not, explain why

S ::= Au l bv
A = a l bAv

• G e` in LL(1)? If not modify the grammar (if it is possible) to make it LL(1) and then write its parsing table.