# PhD Dissertation, Mathematics for Economic Decisions Leonardo Fibonacci School 

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## Introduction

(1) Introduction

- The Thesis
- Overview (Thesis) \& main themes (presentation)
(2) THE PRELIMINARIES
- The motivations
- The actors
(3) ThE MAIN BODY
- The auction models
- The barter models
- Coalitions for problem solving
- Deciding within a competition
(4) CONCLUSIONS


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4 CONCLUSIONS

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The main body

## Conclusions

## The Thesis

The main body
Conclusions

## The Thesis

$\Rightarrow$ Title
Methods and Models for Environmental Conflicts Analysis and Resolution
$\Rightarrow$ Tutor
Professor Giorgio Gallo

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## An overview of the Thesis

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The present Thesis:
$\Rightarrow$ defines and characterizes interactions among actors;
$\Rightarrow$ within different settings;
$\Rightarrow$ at different levels of abstraction and complexity;
$\Rightarrow$ by using different (abstract) models;
$\Rightarrow$ by using different (high level) methods;
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$\Rightarrow$ The actors.
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$\Rightarrow$ Barter models.
$\Rightarrow$ Coalitions for prob em solving.
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## The motivations

## THE MOTIVATIONS



## The motivations



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## Actors: DECIDERS, STAKEHOLDERS, EXPERTS

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Actors: deciders, stakeholders and experts
$\Rightarrow$ The deciders "drive" the procedures and bear the main responsibilities.
$\Rightarrow$ The stakeholders are part of the affected reactive environment of the deciders.
$\Rightarrow$ The experts are part of the affected reactive environment of the deciders.
$\Rightarrow$ Stakeholders and experts form dynamically interacting subsets.
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Monolithic deciders versus structured deciders.
> $\Rightarrow$ Monolithic deciders: as single players, no inner dynamics. $\Rightarrow$ Monolithic deciders: mainly decisions, competitive approaches.
> $\Rightarrow$ Structured deciders: inner structure, dynamics, local data and value systems.
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4 CONCLUSIONS

The main body

## The main body

## The roles of auctions

## The roles of auctions

$\sqrt{616}$
$\Rightarrow$ Autonomous tools for allocation or distribution.
$\Rightarrow$ Ancillary procedures for the definition of:
initial allocations,
initial distributions,
to be followed by negotiation or redistribution as a post auction phase.

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REASONS FOR AUCTIONING

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We can use auctions :
$\Rightarrow$ to describe one-to-many relations auctioneer versus bidders;
$\Rightarrow$ to allocate single bads or chores;
$\Rightarrow$ to share benefits and costs, initial distribution proportional
to the bids and redistribution (dissatisfaction, post auction);
$\Rightarrow$ to define initial endowments made of fractions of benefits and costs (link with barter mechanisms);
$\Rightarrow$ to compensate a lack of knowledge from the auctioneer of
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## Conclusions

## The auction models

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## The auction models



## Models of auctions.

$\Rightarrow$ Positive.
The bidders bid for getting the auctioned item. $\Rightarrow$ Negative. The bidders bid for not getting the auctioned item.

## The auction models



## Models of auctions.

$\Rightarrow$ Positive. $\Rightarrow$ Negative.

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NEGATIVE AUCTIONS

## The main body

Conclusions
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## NEGATIVE AUCTIONS

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Negative auctions
$\Rightarrow$ bidders bid for not getting an item (bad or chore),
$\Rightarrow$ the less offering bidder gets it together with a compensation,
$\Rightarrow$ the other bidders proportionally compensate him,
$\Rightarrow$ weakly dominant strategy: bid $=$ evaluation $+\delta$ with $\delta \rightarrow 0$
as $n \rightarrow \infty$.

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## Conclusions

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NEGATIVE AUCTIONS

## The main body

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$\left(p h_{1}\right)$ A auctions $\zeta ;$
$\left(p h_{2}\right)$ the $b_{i}$ make their bids $x_{i}$ in a sealed bid one shot auction;
$\left(p h_{3}\right)$ the bids are revealed;
$\left(p h_{4}\right)$ the lowest bidding bidder $b_{1}$ gets $\zeta$ and $x_{1}$ as a compensation for this allocation;
$\left(p h_{5}\right)$ each of the other bidders $b_{i}$ pays to $b_{1}$ a fraction $c_{i}$ of $x_{1}$ such that:

$$
\sum_{i \neq 1} c_{i}=x_{1} \quad c_{i}=x_{1} \frac{x_{i}}{X} \quad X=\sum_{j \neq 1} x_{j}
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The main body

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## Uses of negative auctions

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## Conclusions

## USES OF NEGATIVE AUCTIONS

## Uses of negative auctions

Features and uses: benefit not only the auctioneer but also other bidders; $\Rightarrow$ the influence (as a damage) on other bidders is neqliaib e; $\Rightarrow$ the influence (as a damage) on actors distinct from the bidders is negligible; $\Rightarrow$ all the costs and damages can be summarized with m; $\Rightarrow$ solid waste disposal plants, hazardous waste disposal plants, incinerators;
$\Rightarrow$ energy production plants, chemical plants (point-wide allocations)

## Uses of negative auctions

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Features and uses:
$\Rightarrow$ the auctioned item involves a single bidder though it may benefit not only the auctioneer but also other bidders;
$\Rightarrow$ the influence (as a damage) on actors distinct from the bidders is negligible;
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## Introduction

(1) Introduction

- The Thesis
- Overview (Thesis) \& main themes (presentation)
(2) ThE PRELIMINARIES
- The motivations
- The actors
(3) The MAIN BODY
- The auction models
- The barter models
- Coalitions for problem solving
- Deciding within a competition
(4) Conclusions


## REASONS FOR BARTERING

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We can use barters [models] for:
$\Rightarrow$ describing one-to-one or many-to-many relations between actors;
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## Models for bilateral barters

## MODELS FOR BILATERAL BARTERS



## Models for bilateral barters



Bilateral barters:
$\Rightarrow$ involve two actors each with a basket of items (endowments of benefits and costs),
$\Rightarrow$ explicit barter if each actor reveals his basket,
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The preliminaries
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The main body
Conclusions

## Bilateral explicit barter, MERGE AND SPLIT

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The main body

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## Bilateral explicit barter, MERGE AND SPLIT

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(1) $B_{i, j}^{h}=b k_{i}^{h} \oplus b k_{j}^{h}$
(2) if negotiation $\left(B_{i, j}^{h}\right)$ is successful then

- $i$ takes $b k_{i}^{h+1} \succ_{i} b k_{i}^{h}$
- $j$ takes $b k_{j}^{h+1} \succ_{j} b k_{j}^{h}$
else if negotiation $\left(B_{i, j}^{h}\right)$ fails
- $i$ takes back $b k_{i}^{h}$
- $j$ takes back $b k_{j}^{h}$
(3) end;


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The preliminaries
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## BILATERAL EXPLICIT BARTER, NEGOTIATION

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The main body

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## Bilateral explicit barter, negotiation

## 3

(1) random selection to choose player 1 ;
(2) 1 proposes a split of the set $B_{i, j}^{h}$ as $b k_{1}^{h+1}, b k_{2}^{h+1}$;
(3) if 2 accepts then

- negotiation successful, go to (5);
(4) if 2 refuses then
(4a) 2 proposes a split of the set $B_{i, j}^{h}$ as $b k_{2}^{h+1}, b k_{1}^{h+1}$;
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- negotiation successful, go to (5);
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- negotiation fails, go to (5);
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## MULTILATERAL BARTERS

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## Conclusions

## MULTILATERAL BARTERS

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$\Rightarrow$ involve more than two actors each with a basket of items,
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(1) a merge operation is executed so to define $B_{S}^{h}=\oplus_{i \in S} b k_{i}^{h}$;
(2) one of the players $i \in S$ is randomly selected;
(3) the selected player $i$ proposes a basket $b k \subset B_{S}^{h}$ and passes it along to the others;
(4) if nobody modifies it in any way (so that $i$ is conventionally the last modifier) then the basket is assigned to $i$ and becomes $b k_{i}^{h+1}$ so that $i$ exits from $S$ (and so from the game);
(5) if other players modify it and if $j$ is the last modifier we have the following cases:
(5a) if $i$ accepts the modified basket he gets it so that it becomes $b k_{i}^{h+1}$ and then $i$ exits from $S$ (and so from the game);
(5b) if $i$ refuses the modified basket $j$ gets it so that it becomes $b k_{j}^{h+1}$ and then $j$ exits from $S$ (and so from the game);
(6) the items allocated to either $i$ or $j$ must be removed from $B_{S}^{h}$;
(7) if there are still at least two players go to (2) else end;

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## PARALLEL AND CASCADED BARTERS



Bilateral and multilateral barters may be:
$\Rightarrow$ executed in parallel,
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## The evaluation criteria, Basic definitions

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We say a barter is fair if the following conditions are satisfied, otherwise it is unfair.
> nobody would prefer the portion of somebody else to his own.
> each of the n players thinks to have received at least $1 / n$ of the total value.
> $\Rightarrow$ Equitability: each player thinks he has received a portion that is worth the same in one's evaluation as the other's portion in the other's evaluation.
> $\Rightarrow$ Pareto efficiency: there is no other allocation where one of the players is better off and none of the others is worse off.

## The evaluation criteria, Basic definitions

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## THE EVALUATION CRITERIA, BASIC DEFINITIONS

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## THE EVALUATION CRITERIA, THE PARAMETERS

## THE EVALUATION CRITERIA, THE PARAMETERS



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## THE EVALUATION CRITERIA, THE PARAMETERS



We define the following parameters for player $i$ :
$\Rightarrow a_{i}$ the value of what $i$ gets from the barter,
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$\Rightarrow\left(a_{i}\right)_{i}$ the value of what $i$ gets from the barter in $i$ 's opinion,
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## THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (1)

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## THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (1)

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The main body
Conclusions

## The Evaluation criteria, modified definitions (1)

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$$
v_{i}^{h+1}=v_{i}^{h}-l_{i}+a_{i}
$$

so that player $i$ accepts a proposed barter (since $v_{i}^{h+1} \geq v_{i}^{h}$ ) if and only if:

$$
a_{i} \geq l_{i}
$$

In the case of two players a barter is envy-free if we have for player $i$ :

$$
\frac{a_{i}}{l_{i}} \geq 1
$$

In the case of more than two players if we consider player $i$ we have that the following relation must hold for all $j \neq i$ :

$$
a_{i} \geq\left(a_{j}\right)_{i}
$$

In the case of two players we want to maintain the equivalence between proportionality and envy-freeness

$$
\frac{a_{i}}{a_{i}+l_{i}} \geq \frac{1}{2}
$$

In the general case of more than two players

$$
\begin{aligned}
& \text { envy }- \text { freeness } \Rightarrow \text { proportionality } \\
& \text { proportionality } \nRightarrow \text { envy }- \text { freeness } \\
& \qquad \frac{a_{i}}{a_{i}+A} \geq \frac{1}{n}
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## The Evaluation criteria, modified definitions (2)

The preliminaries
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Conclusions
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## The Evaluation criteria, modified definitions (2)

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## Conclusions

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## The Evaluation criteria, modified definitions (2)

$$
\frac{a_{i}}{v_{i}^{h+1}} \geq \frac{l_{i}}{v_{i}^{h}} \quad \frac{a_{j}}{v_{j}^{h+1}} \geq \frac{l_{j}}{v_{j}^{h}}
$$

If both relations hold we say that the barter is equitable.

$$
\begin{aligned}
v_{i}^{h+1}=v_{i}^{h}+a_{i}-l_{i} & \bar{v}=v_{i}^{h+1}-a_{i}=v_{i}^{h}-l_{i} \\
v_{i}^{h+1}=\bar{v}+a_{i} & v_{i}^{h}=\bar{v}+l_{i} \\
\frac{a_{i}}{\bar{v}+a_{i}} \geq \frac{l_{i}}{\bar{v}+l_{i}} & \text { we can easily derive } a_{i} \geq l_{i}
\end{aligned}
$$

from equitability we derive envy-freeness envy-freeness can be expressed as $a_{i} \geq l_{i}$ (and $v_{i}^{h+1} \geq v_{i}^{h}$ )

$$
1 \leq \frac{v_{i}^{h+1}}{v_{i}^{h}}=\frac{\bar{v}+a_{i}}{\bar{v}+l_{i}} \leq \frac{a_{i}}{l_{i}}
$$

In this way we get that, in the case of two players, envy-freeness necessarily implies equitability and vice versa.

## The Evaluation criteria, modified definitions (2)

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## THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (3)

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## THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (3)

$\square$

The main body
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## The Evaluation criteria, modified definitions (3)

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$a_{i j}$ the value of what $i$ gets from $j$
$l_{i j}$ the value of what $i$ gives to $j$
bilaterally equitable if for a pair $i, j$ :

$$
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If such relations (that scale easily to the two players case) are satisfied for every $i$ and for every $j \neq i$ we say that the barter satisfies bilateral equitability.
If, for a given $i$, we sum all the relations over all the $j \neq i$ we get:

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& \frac{a_{i}}{v_{i}^{h+1}} \geq \frac{l_{i}}{v_{i}^{h}} \quad a_{i}=\sum_{j \neq i} a_{i j} \quad l_{i}=\sum_{j \neq i} l_{i j} \text { an hypothesis of additivity } \\
& \frac{a_{i}}{\bar{v}+a_{i}} \geq \frac{l_{i}}{\bar{v}+l_{i}} \quad \text { or: } \quad a_{i} \geq l_{i} \text { and: } v_{i}^{h+1} \geq v_{i}^{h}
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## The Evaluation criteria, modified definitions (3)

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$a_{i j}$ the value of what $i$ gets from $j$
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## THE EVALUATION CRITERIA, SATISFACTION (1)

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## The Evaluation criteria, satisfaction (1)

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The multilateral barter models in general satisfy:
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## Introduction

(1) Introduction

- The Thesis
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(2) ThE PRELIMINARIES
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(4) CONCLUSIONS


## Introductory remarks

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The main body

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We define a two stage procedure and two conditions:
$\Rightarrow$ dynamic setting: the sets of deciders $N$, issues I and criteria $C$ are defined from seminal sets;
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The main body

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## THE BASIC FRAMEWORK

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## The static setting

Conclusions

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In the static setting :
$\Rightarrow$ the deciders of N can:
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## LATE MERGE

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## Late merge of Rankings with ties (2)

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## *

One decider, four issues [1,2,3,4], four criteria, the graphs.

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## Late merge of rankings with ties (3)

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## LATE MERGE OF RANKINGS WITH TIES (3)

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The main body

## The main body

## The dynamic setting

## The dynamic setting

Conclusions

## The main body <br> 000000000000000000000000000000000000

$\square$

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## In the dynamic setting:

$\Rightarrow$ from the seminal sets $N_{0}, l_{0}, C_{0}$,
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## DECIDING WITHIN A COMPETITION

The main body
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## ONE PROJECT, THE SETS

## ONE PROJECT, THE SETS

Conclusions

## ONE PROJECT, THE SETS

## *

$$
\begin{gathered}
C_{a}=\left\{C_{a_{i}} \mid i \in I\right\} \\
B_{a}=\left\{B_{a_{j}} \mid j \in J\right\} \\
C_{1}=\left\{\alpha_{i} C_{a_{i}} \mid i \in I\right\} \quad B_{1}=\left\{\beta_{j} B_{a_{j}} \mid j \in J\right\} \\
C_{2}=\left\{\left(1-\alpha_{i}\right) C_{a_{i}} \mid i \in I\right\} B_{2}=\left\{\left(1-\beta_{j}\right) B_{a_{j}} \mid j \in J\right\}
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The negotiations involve the following sets:
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$\Rightarrow$ the sets $C_{2}$ and $B_{2}$ for the other decider,
$\Rightarrow$ a key role is played by the sets $\left\{\alpha_{i}\right\}$ and $\left\{\beta_{i}\right\}$ (how).

## ONE PROJECT, THE SETS

## ,

$$
\begin{gathered}
C_{a}=\left\{C_{a_{i}} \mid i \in I\right\} \\
B_{a}=\left\{B_{a_{j}} \mid j \in J\right\} \\
C_{1}=\left\{\alpha_{i} C_{a_{i}} \mid i \in I\right\} \quad B_{1}=\left\{\beta_{j} B_{a_{j}} \mid j \in J\right\} \\
C_{2}=\left\{\left(1-\alpha_{i}\right) C_{a_{i}} \mid i \in I\right\} B_{2}=\left\{\left(1-\beta_{j}\right) B_{a_{j}} \mid j \in J\right\}
\end{gathered}
$$

The negotiations involve the following sets:
$\Rightarrow$ the set $C_{a}$ of the [shareable] costs (what),
$\Rightarrow$ the set $B_{a}$ of the [shareable] benefits (what),
$\Rightarrow$ the sets $C_{1}$ and $B_{1}$ for one decider,
$\Rightarrow$ the sets $C_{2}$ and $B_{2}$ for the other decider,
$\Rightarrow$ a key role is played by the sets $\left\{\alpha_{i}\right\}$ and $\left\{\beta_{i}\right\}$ (how).

## ONE PROJECT, THE STRUCTURE

## One project, THE STRUCTURE

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## One project, THE STRUCTURE



## OnE PROJECT, THE STRUCTURE

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## One project, THE COARSE GRAIN NEGOTIATION

## One project, THE COARSE GRAIN NEGOTIATION

## One project, THE COARSE GRAIN NEGOTIATION

(1) $d_{b}$ [refuses the sets $B_{a}$ and $C_{a}$ and] presents the sets $B_{a}^{\prime}$ and $C_{a}^{\prime}$;
(2) $d_{a}$ presents the sets $B_{a}^{\prime \prime}$ and $C_{a}^{\prime \prime}$;
(3) we have the following cases:
(3.a) if $B_{a}^{\prime \prime}=B_{a}^{\prime}$ and $C_{a}^{\prime \prime}=C_{a}^{\prime}$ then go to (4);
(3.b) if $B_{a}^{\prime \prime} \neq B_{a}^{\prime}$ or $C_{a}^{\prime \prime} \neq C_{a}^{\prime}$ then:
(a) with a random device we define an ordering between the two players;
(b) the player who comes first in the ordering gets a token only if he declares he wishes to rethink about his proposal otherwise the token assignment procedure is repeated with the other player;
(c) if $d_{a}$ has the token then:
(i) $d_{a}$ presents the modified sets $B_{a}^{\prime \prime}$ and $C_{a}^{\prime \prime}$;
(ii) go to (3);
(d) if $d_{b}$ has the token then:
(i) $d_{b}$ presents the modified sets $B_{a}^{\prime}$ and $C_{a}^{\prime}$;
(ii) go to (3);
(e) if none of them has the token then go to (5);
(4) go to the fine grain negotiation algorithm;
(5) $d_{a}$ and $d_{b}$ may either agree to go on or to give up;
(6) if they agree to go on then go to (1);
(7) if they agree to give up then call mediator procedure;

## One project, THE COARSE GRAIN NEGOTIATION

(1) $d_{b}$ [refuses the sets $B_{a}$ and $C_{a}$ and] presents the sets $B_{a}^{\prime}$ and $C_{a}^{\prime}$;
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(ii) go to (3);
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(6) if they agree to go on then go to (1);
(7) if they agree to give up then call mediator procedure;

## One project, THE FINE GRAIN NEGOTIATION

## One project, The fine grain negotiation

$\square$

## One project, The fine grain negotiation

(1) $d_{a}$ presents the sets $\alpha$ and $\beta$;
(2) $d_{b}$ presents the sets $\alpha^{\prime}$ and $\beta^{\prime}$;
(3) we have the following cases:
(3.a) if $\alpha=\alpha^{\prime}$ and $\beta=\beta^{\prime}$ then go to (4);
(3.b) if $\alpha \neq \alpha^{\prime}$ or $\beta \neq \beta^{\prime}$ then:
(a) with a random device we define on ordering between the two players;
(b) the player who comes first in the ordering gets a token only if he declares he wishes to rethink about his proposal otherwise the token assignment procedure is repeated with the other player;
(c) if $d_{a}$ has the token then:
(i) $d_{a}$ presents the modified sets $\alpha$ and $\beta$;
(ii) go to (3);
(d) if $d_{b}$ has the token then:
(i) $d_{b}$ presents the modified sets $\alpha^{\prime}$ and $\beta^{\prime}$;
(ii) go to (3);
(e) if none of them has the token then go to (5);
(4) end;
(5) go back to the coarse grain negotiation procedure;

## One project, The fine grain negotiation

(1) $d_{a}$ presents the sets $\alpha$ and $\beta$;
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(3.a) if $\alpha=\alpha^{\prime}$ and $\beta=\beta^{\prime}$ then go to (4);
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## Two projects, THE STRUCTURE

Conclusions

## The main body

## Two projects, THE STRUCTURE

The main body
$\square$

## Two projects, THE STRUCTURE



## Two projects, THE STRUCTURE




## MANY PROJECTS \& MANY DECIDERS

## MANY PROJECTS \& MANY DECIDERS



The main body

## The main body Conclusions

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## MANY PROJECTS \& MANY DECIDERS



In the thesis we have also examined the following cases:
$\Rightarrow$ one project and more than two deciders, $\Rightarrow$ two projects and more than two deciders,
$\Rightarrow$ more than two projects and more than two deciders, $\Rightarrow$ more than two projects and two deciders.

## MANY PROJECTS \& MANY DECIDERS



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The main body

## The possible extensions

## The possible Extensions

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The main body
Conclusions

## THE POSSIBLE EXTENSIONS

As to the extensions we have examined:
$\Rightarrow$ the structured deciders (inner dynamics, two levels of negotiation and decision),
$\Rightarrow$ the presence of the stakeholders (pressures, reactive environment),
$\Rightarrow$ the presence of the experts (lack of autonomy, reactive environment),
$\Rightarrow$ the dynamic setting (variable sets).

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## The possible extensions

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| :---: | :---: |
| 40 |}

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## Conclusions

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## Conclusions

$\Rightarrow$ What this thesis is（description of a particular approach）．
$\Rightarrow$ What this thesis is not（not a general and exhaustive description）．
$\Rightarrow$ Open issues：＂practical＂applications to real cases．
$\Rightarrow$ Open issues：deeper and more complete formalization．

## CONCLUSIONS


$\Rightarrow$ What this thesis is (description of a particular approach).
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