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PhD Dissertation, Mathematics for Economic Decisions Leonardo Fibonacci School

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#### Department of Computer Science, University of Pisa December, 3 2010

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  - The actors
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  - The auction models
  - The barter models
  - Coalitions for problem solving
  - Deciding within a competition

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Methods and Models for Environmental Conflicts Analysis and Resolution

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- $\Rightarrow$  within different settings;
- $\Rightarrow$  at different levels of abstraction and complexity;
- $\Rightarrow$  by using different (abstract) models;
- $\Rightarrow$  by using different (high level) methods;
- $\Rightarrow$  within a unifying perspective (the environment and the conflicts).

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#### THE MAIN THEMES OF THIS PRESENTATION

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- $\Rightarrow$  The motivations.
- $\Rightarrow$  The actors.
- $\Rightarrow$  Auction models.
- $\Rightarrow$  Barter models.
- $\Rightarrow$  Coalitions for problem solving.
- $\Rightarrow$  Decisions within a competition.

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### THE MOTIVATIONS

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## THE MOTIVATIONS





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## THE MOTIVATIONS



- (1) to analyze the interactions among various types of actors;
- (2) to describe allocation (indivisible), distribution (shareable) and negotiation (redistribution, reallocation) tools that involve such actors;
- (3) to propose methods and models that implement such tools and can be easily and fairly used by the actors.

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## THE MOTIVATIONS



#### The main motivations of the thesis are:

#### (1) to analyze the interactions among various types of actors;

- (2) to describe allocation (indivisible), distribution (shareable) and negotiation (redistribution, reallocation) tools that involve such actors;
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## ACTORS: DECIDERS, STAKEHOLDERS, EXPERTS

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# ACTORS: DECIDERS, STAKEHOLDERS, EXPERTS



### Actors: deciders, stakeholders and experts

- ⇒ The deciders "drive" the procedures and bear the main responsibilities.
- ⇒ The stakeholders are part of the affected reactive environment of the deciders.
- ⇒ The experts are part of the affected reactive environment of the deciders.
- ⇒ Stakeholders and experts form dynamically interacting subsets.
- $\Rightarrow$  Deciders form dynamically interacting subsets.

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- $\Rightarrow$  Deciders form dynamically interacting subsets.

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## DECIDERS: MONOLITHIC VERSUS STRUCTURED

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## DECIDERS: MONOLITHIC VERSUS STRUCTURED



- $\Rightarrow$  Monolithic deciders: as single players, no inner dynamics.
- Monolithic deciders: mainly decisions, competitive approaches.
- ⇒ Structured deciders: inner structure, dynamics, local data and value systems.
- ⇒ Structured deciders: mainly negotiation and deliberation, cooperative approaches.

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## DECIDERS: MONOLITHIC VERSUS STRUCTURED



#### Monolithic deciders versus structured deciders.

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Two levels: within a structured decider and among monolithic deciders.

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### THE ROLES OF AUCTIONS

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- $\Rightarrow$  Autonomous tools for allocation or distribution.
- $\Rightarrow$  Ancillary procedures for the definition of:
  - · initial allocations,
  - · initial distributions,
  - to be followed by negotiation or redistribution as a post auction phase.

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- $\Rightarrow$  Autonomous tools for allocation or distribution.
- $\Rightarrow$  Ancillary procedures for the definition of:
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  - · initial distributions,
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The preliminaries

### **REASONS FOR AUCTIONING**

The preliminaries

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## **REASONS FOR AUCTIONING**



- ⇒ to describe one-to-many relations auctioneer versus bidders;
- $\Rightarrow$  to allocate single bads or chores;
- ⇒ to share benefits and costs, initial distribution proportional to the bids and redistribution (dissatisfaction, post auction);
- ⇒ to define initial endowments made of fractions of benefits and costs (link with barter mechanisms);
- ⇒ to compensate a lack of knowledge from the auctioneer of the capabilities of the bidders.

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### We can use auctions :

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The preliminaries

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The preliminaries

## THE AUCTION MODELS





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### THE AUCTION MODELS



#### Models of auctions.

- $\Rightarrow$  Positive.
  - · The bidders bid for getting the auctioned item.
- $\Rightarrow$  Negative.
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The preliminaries

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### THE AUCTION MODELS



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# **NEGATIVE AUCTIONS**

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# **NEGATIVE AUCTIONS**





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## **NEGATIVE AUCTIONS**



- $\Rightarrow$  bidders bid for not getting an item (bad or chore),
- ⇒ the less offering bidder gets it together with a compensation,
- $\Rightarrow$  the other bidders proportionally compensate him,
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- ⇒ the less offering bidder gets it together with a compensation,
- $\Rightarrow$  the other bidders proportionally compensate him,
- ⇒ weakly dominant strategy: bid =evaluation + $\delta$  with  $\delta \rightarrow 0$  as  $n \rightarrow \infty$ .

The preliminaries

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## **NEGATIVE AUCTIONS**



- $(ph_1)$  **A** auctions  $\zeta$ ;
- $(ph_2)$  the  $b_i$  make their bids  $x_i$  in a sealed bid one shot auction;
- (ph<sub>3</sub>) the bids are revealed;
- $(ph_4)$  the lowest bidding bidder  $b_1$  gets  $\zeta$  and  $x_1$  as a compensation for this allocation;
- $(ph_5)$  each of the other bidders  $b_i$  pays to  $b_1$  a fraction  $c_i$  of  $x_1$  such that:  $\sum_{i \neq 1} c_i = x_1 \quad c_i = x_1 \frac{x_i}{X} \quad X = \sum_{j \neq 1} x_j$

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### **USES OF NEGATIVE AUCTIONS**

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The main body Conclusions

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- ⇒ the auctioned item involves a single bidder though it may benefit not only the auctioneer but also other bidders;
- $\Rightarrow$  the influence (as a damage) on other bidders is negligible;
- ⇒ the influence (as a damage) on actors distinct from the bidders is negligible;
- $\Rightarrow$  all the costs and damages can be summarized with  $m_i$ ;
- ⇒ solid waste disposal plants, hazardous waste disposal plants, incinerators;
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# INTRODUCTION

**1** INTRODUCTION

- The Thesis
- Overview (Thesis) & main themes (presentation)

THE PRELIMINARIES
 The motivations

- The actors
- 3 THE MAIN BODY
  - The auction models
  - The barter models
  - Coalitions for problem solving
  - Deciding within a competition

CONCLUSIONS

The preliminaries

## **REASONS FOR BARTERING**



The preliminaries

### **REASONS FOR BARTERING**





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- ⇒ describing the swap of sets of items (generally benefits and costs), either exogenous or endogenous, initial endowments from auction phases;
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The preliminaries

MODELS FOR BILATERAL BARTERS



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### MODELS FOR BILATERAL BARTERS



- ⇒ involve two actors each with a basket of items (endowments of benefits and costs),
- $\Rightarrow$  explicit barter if each actor reveals his basket,
- $\Rightarrow$  implicit barter if each actor conceals his basket,
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The preliminaries

### BILATERAL EXPLICIT BARTER, MERGE AND SPLIT

The preliminaries

# BILATERAL EXPLICIT BARTER, MERGE AND SPLIT





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#### BILATERAL EXPLICIT BARTER, MERGE AND SPLIT



- (1)  $B^h_{i,j} = bk^h_i \oplus bk^h_j$
- (2) if negotiation $(B_{i,j}^h)$  is successful then
  - i takes  $bk_i^{h+1} \succ_i bk_i^h$
  - j takes  $bk_j^{h+1} \succ_j bk_j^h$

else if negotiation $(B_{i,j}^h)$  fails

- i takes back  $bk_i^h$
- j takes back  $bk_j^h$
- (3) end;

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- (3) end;



The preliminaries

**BILATERAL EXPLICIT BARTER, NEGOTIATION** 

The preliminaries

# **BILATERAL EXPLICIT BARTER, NEGOTIATION**





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## **BILATERAL EXPLICIT BARTER, NEGOTIATION**



- (1) random selection to choose player 1;
- (2) 1 proposes a split of the set  $B_{i,j}^h$  as  $bk_1^{h+1}$ ,  $bk_2^{h+1}$ ;
- (3) if 2 accepts then
  - negotiation successful, go to (5);
- (4) if 2 refuses then
  - (4a) 2 proposes a split of the set  $B_{i,j}^h$  as  $bk_2^{h+1}$ ,  $bk_1^{h+1}$ ;
  - (4b) if 1 accepts then
    - negotiation successful, go to (5); else
    - negotiation fails, go to (5);
- (5) end;

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# MULTILATERAL BARTERS

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The preliminaries

# MULTILATERAL BARTERS





The preliminaries

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### MULTILATERAL BARTERS



- $\Rightarrow$  involve more than two actors each with a basket of items,
- $\Rightarrow$  explicit barter if each actor reveals his basket,
- $\Rightarrow$  implicit barter if each actor conceals his basket,
- $\Rightarrow$  mixed barter if some reveal and the some others conceal.

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MULTILATERAL EXPLICIT BARTER, LAST MODIFIER

The preliminaries

# MULTILATERAL EXPLICIT BARTER, LAST MODIFIER



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### MULTILATERAL EXPLICIT BARTER, LAST MODIFIER



- a merge operation is executed so to define B<sup>h</sup><sub>S</sub> = ⊕<sub>i∈S</sub>bk<sup>h</sup><sub>i</sub>;
- (2) one of the players  $i \in S$  is randomly selected;
- the selected player i proposes a basket bk ⊂ B<sup>h</sup><sub>S</sub> and passes it along to the others;
- (4) if nobody modifies it in any way (so that i is conventionally the last modifier) then the basket is assigned to i and becomes bk<sub>i</sub><sup>h+1</sup> so that i exits from S (and so from the game);
- (5) if other players modify it and if j is the last modifier we have the following cases:
  - (5a) if *i* accepts the modified basket he gets it so that it becomes  $bk_i^{h+1}$  and then *i* exits from *S* (and so from the game);
  - (5b) if *i* refuses the modified basket *j* gets it so that it becomes  $bk_j^{h+1}$  and then *j* exits from *S* (and so from the game);
- (6) the items allocated to either *i* or *j* must be removed from  $B_S^h$ ;
- (7) if there are still at least two players go to (2) else end;

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The preliminaries

PARALLEL AND CASCADED BARTERS

The preliminaries

# PARALLEL AND CASCADED BARTERS





The preliminaries

#### PARALLEL AND CASCADED BARTERS



- $\Rightarrow$  executed in parallel,
- $\Rightarrow$  executed in cascade among the same actors,
- ⇒ executed in cascade among at least partially different actors,
- ⇒ this reduces the level of common knowledge among the actors.

The preliminaries

The main body Conclusions

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#### THE EVALUATION CRITERIA, BASIC DEFINITIONS

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The preliminaries

#### THE EVALUATION CRITERIA, BASIC DEFINITIONS

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# We say a barter is fair if the following conditions are satisfied, otherwise it is unfair.

- ⇒ Envy-freeness: nobody would prefer the portion of somebody else to his own.
- ⇒ Proportionality: each of the n players thinks to have received at least 1/n of the total value.
- ⇒ Equitability: each player thinks he has received a portion that is worth the same in one's evaluation as the other's portion in the other's evaluation.
- ⇒ Pareto efficiency: there is no other allocation where one of the players is better off and none of the others is worse off.

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THE EVALUATION CRITERIA, THE PARAMETERS

The preliminaries

#### THE EVALUATION CRITERIA, THE PARAMETERS





#### THE EVALUATION CRITERIA, THE PARAMETERS



## We define the following parameters for player *i*:

- $\Rightarrow$  *a<sub>i</sub>* the value of what *i* gets from the barter,
- $\Rightarrow$  *I<sub>i</sub>* the value of what *i* gives away in the barter,
- $\Rightarrow$   $(a_j)_i$  the value of what j gets from the barter in i's opinion,
- ⇒  $v_i^{h+1}$  and  $v_i^1$  the worths (for *i*) of *i*'s basket after and before the barter,

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- $\Rightarrow$   $v_i^{h+1}$  and  $v_i^1$  the worths (for *i*) of *i*'s basket after and before the barter,
- $\Rightarrow A = \sum_{j \neq i} (a_j)_i$ .

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THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (1)

The preliminaries

The main body Conclusions

#### THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (1)





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# THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (1)



 $v_i^{h+1} = v_i^h - l_i + a_i$  so that player i accepts a proposed barter (since  $v_i^{h+1} \geq v_i^h$ ) if and only if:  $a_i > l_i$ 

In the case of two players a barter is envy-free if we have for player *i*:

$$\frac{a_i}{l_i} \ge 1$$

In the case of more than two players if we consider player i we have that the following relation must hold for all  $j \neq i$ :

$$a_i \ge (a_j)_i$$

In the case of **two players** we want to maintain the equivalence between proportionality and envy-freeness

$$\frac{a_i}{a_i + l_i} \ge \frac{1}{2}$$

In the general case of more than two players

$$\begin{array}{l} envy - freeness \ \Rightarrow \ proportionality \\ proportionality \ \Rightarrow \ envy - freeness \\ \hline \\ \frac{a_i}{a_i+A} \geq \frac{1}{n} \end{array}$$

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 $v_i^{h+1} = v_i^h - l_i + a_i$  so that player i accepts a proposed barter (since  $v_i^{h+1} \geq v_i^h$ ) if and only if:  $a_i > l_i$ 

In the case of two players a barter is **envy-free** if we have for player *i*:

$$\frac{a_i}{l_i} \ge 1$$

In the case of more than two players if we consider player i we have that the following relation must hold for all  $j \neq i$ :

$$a_i \ge (a_j)_i$$

In the case of **two players** we want to maintain the equivalence between proportionality and envy-freeness

$$\frac{a_i}{a_i + l_i} \ge \frac{1}{2}$$

In the general case of more than two players

$$\begin{array}{l} envy - freeness \ \Rightarrow \ proportionality \\ proportionality \ \Rightarrow \ envy - freeness \\ \hline \\ \frac{a_i}{a_i + A} \geq \frac{1}{n} \end{array}$$



THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (2)

The preliminaries

#### THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (2)



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# THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (2)



$$\frac{a_i}{v_i^{h+1}} \geq \frac{l_i}{v_i^h} \qquad \frac{a_j}{v_j^{h+1}} \geq \frac{l_j}{v_j^h}$$

If both relations hold we say that the barter is **equitable**.

$$\begin{split} v_i^{h+1} &= v_i^h + a_i - l_i & \bar{v} = v_i^{h+1} - a_i = v_i^h - l_i \\ v_i^{h+1} &= \bar{v} + a_i & v_i^h = \bar{v} + l_i \\ &\frac{a_i}{\bar{v} + a_i} \geq \frac{l_i}{\bar{v} + l_i} & \text{we can easily derive } a_i \geq l_i \end{split}$$

from equitability we derive envy-freeness envy-freeness can be expressed as  $a_i \ge l_i$  (and  $v_i^{h+1} \ge v_i^h$ )

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In this way we get that, in the case of two players, envy-freeness necessarily implies equitability and vice versa.

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# THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (2)



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THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (3)

The preliminaries

### THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (3)





#### THE EVALUATION CRITERIA, MODIFIED DEFINITIONS (3)



 $a_{ij}$  the value of what i gets from j

 $l_{ij}$  the value of what *i* gives to *j* 

bilaterally equitable if for a pair *i*, *j*:

$$\frac{a_{ij}}{v_i^{h+1}} \geq \frac{l_{ij}}{v_i^{h}}$$

If such relations (that scale easily to the two players case) are satisfied for every i and for every  $j \neq i$  we say that the barter satisfies **bilateral equitability**.

If, for a given *i*, we sum all the relations over all the  $j \neq i$  we get:

$$\begin{split} \frac{a_i}{v_i^{h+1}} &\geq \frac{l_i}{v_i^h} \quad a_i = \sum_{j \neq i} a_{ij} \quad l_i = \sum_{j \neq i} l_{ij} \quad \text{an hypothesis of additivity} \\ \frac{a_i}{\bar{v} + a_i} &\geq \frac{l_i}{\bar{v} + l_i} \quad \text{or:} \quad a_i \geq l_i \quad \text{and:} \quad v_i^{h+1} \geq v_i^h \end{split}$$

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THE EVALUATION CRITERIA, SATISFACTION (1)

The preliminaries

# THE EVALUATION CRITERIA, SATISFACTION (1)





## THE EVALUATION CRITERIA, SATISFACTION (1)



- $\Rightarrow$  occurrence of the barter,
- $\Rightarrow$  envy-freeness,
- $\Rightarrow$  proportionality,
- $\Rightarrow$  equitability.



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## THE EVALUATION CRITERIA, SATISFACTION (1)



For the models of bilateral barter the following conditions are equivalent:

- $\Rightarrow$  occurrence of the barter,
- $\Rightarrow$  envy-freeness,
- $\Rightarrow$  proportionality,
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Efficiency must be verified case by case on ex-post conditions.

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THE EVALUATION CRITERIA, SATISFACTION (2)

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The preliminaries

# THE EVALUATION CRITERIA, SATISFACTION (2)



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The preliminaries

# THE EVALUATION CRITERIA, SATISFACTION (2)



- $\Rightarrow$  envy-freeness,
- $\Rightarrow$  proportionality,
- $\Rightarrow$  bilateral equitability,
- $\Rightarrow$  efficiency,



The preliminaries

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The preliminaries

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# THE EVALUATION CRITERIA, SATISFACTION (2)



## The multilateral barter models in general satisfy:

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only as ex-post conditions to be verified case by case.

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The preliminaries

The main body Conclusions 

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# INTRODUCTION

- The Thesis
- Overview (Thesis) & main themes (presentation)

- The motivations
- The actors
- - THE MAIN BODY
  - The auction models
  - The barter models
  - Coalitions for problem solving

The preliminaries

The main body Conclusions

# INTRODUCTORY REMARKS

The preliminaries

The main body Conclusions

# **INTRODUCTORY REMARKS**





The main body Conclusions

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## INTRODUCTORY REMARKS



- $\Rightarrow$  dynamic setting: the sets of deciders *N*, issues *I* and criteria *C* are defined from seminal sets;
- $\Rightarrow$  stability conditions: fixed point conditions on such sets;
- ⇒ static setting: issue selection according to the agreed on criteria from the admitted deciders;
- ⇒ conditions of failure: inability to choose, reopening of the dynamic setting with possibly new seminal sets.

The preliminaries

The main body Conclusions

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The preliminaries

# THE BASIC FRAMEWORK

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The preliminaries

# THE BASIC FRAMEWORK



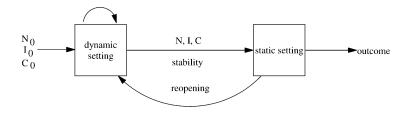


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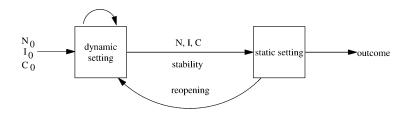




The preliminaries

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# THE STATIC SETTING

The preliminaries

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The preliminaries

The main body Conclusions

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# THE STATIC SETTING



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- $\Rightarrow$  the deciders of N can:
- ⇒ proceed cooperatively (early merge) so to produce a single decision matrix to be used for a collective ranking;
- ⇒ proceed competitively (late merge) so that each produces a private ranking;
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The preliminaries

The main body Conclusions

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The preliminaries

## LATE MERGE

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The preliminaries

## LATE MERGE



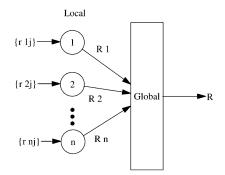


The preliminaries

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## LATE MERGE

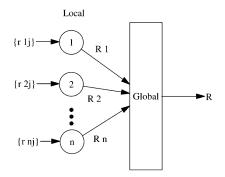




The preliminaries

## LATE MERGE



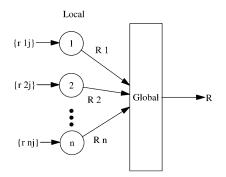


 $r_{ij}$  are local rankings of each decider *i* for each criterion *j* whereas the  $R_i$  are the "overall" ranking for each decider

The preliminaries

# LATE MERGE





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The preliminaries

LATE MERGE OF RANKINGS WITH TIES (1)

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The preliminaries

### LATE MERGE OF RANKINGS WITH TIES (1)





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- ⇒ strict preferences or indifferences with traditional properties,
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LATE MERGE OF RANKINGS WITH TIES (2)

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## LATE MERGE OF RANKINGS WITH TIES (2)



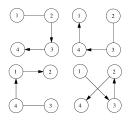
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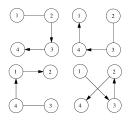


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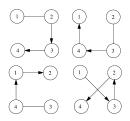
The preliminaries

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### LATE MERGE OF RANKINGS WITH TIES (2)



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The preliminaries

LATE MERGE OF RANKINGS WITH TIES (3)

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The preliminaries

### LATE MERGE OF RANKINGS WITH TIES (3)



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The preliminaries

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### LATE MERGE OF RANKINGS WITH TIES (3)



Two more deciders, [same four] issues, [same four] criteria, the resulting multigraph.

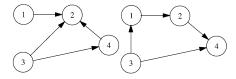
The preliminaries

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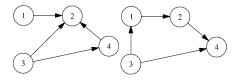


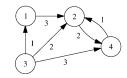
The preliminaries

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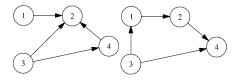
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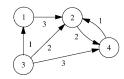
The preliminaries

### LATE MERGE OF RANKINGS WITH TIES (3)



Two more deciders, [same four] issues, [same four] criteria, the resulting multigraph.







The preliminaries

## THE DYNAMIC SETTING

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The preliminaries

# THE DYNAMIC SETTING

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The main body Conclusions

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# THE DYNAMIC SETTING



- $\Rightarrow$  from the seminal sets  $N_0$ ,  $I_0$ ,  $C_0$ ,
- ⇒ new deciders are admitted so to define  $N_i$  with i > 1 from  $N_{i-1}$  (direct involvement, sponsoring),
- $\Rightarrow$  new criteria are merged with the existing criteria so to define  $C_i$  in similar ways,
- $\Rightarrow$  new issues are merged with the existing issues so to define  $l_i$  in similar ways.

The main body Conclusions

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# THE DYNAMIC SETTING



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- ⇒ new issues are merged with the existing issues so to define *I<sub>i</sub>* in similar ways.

The main body Conclusions

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# THE DYNAMIC SETTING



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The main body Conclusions

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# THE DYNAMIC SETTING



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The main body Conclusions

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# THE DYNAMIC SETTING



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# THE DYNAMIC SETTING



In the dynamic setting:

- $\Rightarrow$  from the seminal sets  $N_0$ ,  $I_0$ ,  $C_0$ ,
- ⇒ new deciders are admitted so to define  $N_i$  with i > 1 from  $N_{i-1}$  (direct involvement, sponsoring),
- $\Rightarrow$  new criteria are merged with the existing criteria so to define  $C_i$  in similar ways,
- $\Rightarrow$  new issues are merged with the existing issues so to define  $I_i$  in similar ways.

The updating of the sets occurs through quick and dirty procedures since the real refinement occurs in the static setting. We have fixed point conditions to end the dynamic setting (such as  $N_{i+1} = N_i$ ).

The main body Conclusions

### THE DYNAMIC SETTING



In the dynamic setting:

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The preliminaries

The main body Conclusions

# INTRODUCTION

1 INTRODUCTION

- The Thesis
- Overview (Thesis) & main themes (presentation)

2 THE PRELIMINARIES

- The motivations
- The actors
- 3

#### THE MAIN BODY

- The auction models
- The barter models
- Coalitions for problem solving
- Deciding within a competition



The main body Conclusions

**DECIDING WITHIN A COMPETITION** 



The preliminaries

The main body Conclusions

#### **DECIDING WITHIN A COMPETITION**





The preliminaries

The main body Conclusions

#### DECIDING WITHIN A COMPETITION



# We disregard the process of formation of a common knowledge.



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### **ONE PROJECT, THE SETS**

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The preliminaries

The main body Conclusions

### **ONE PROJECT, THE SETS**



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## **ONE PROJECT, THE SETS**



$$C_{a} = \{C_{a_{i}} \mid i \in I\}$$

$$B_{a} = \{B_{a_{j}} \mid j \in J\}$$

$$C_{1} = \{\alpha_{i}C_{a_{i}} \mid i \in I\} \quad B_{1} = \{\beta_{j}B_{a_{j}} \mid j \in J\}$$

$$C_{2} = \{(1 - \alpha_{i})C_{a_{i}} \mid i \in I\} \quad B_{2} = \{(1 - \beta_{j})B_{a_{j}} \mid j \in J\}$$

The main body Conclusions

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#### ONE PROJECT, THE SETS



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- $\Rightarrow$  a key role is played by the sets { $\alpha_i$ } and { $\beta_i$ } (how).



### **ONE PROJECT, THE STRUCTURE**

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The preliminaries

The main body Conclusions

## ONE PROJECT, THE STRUCTURE

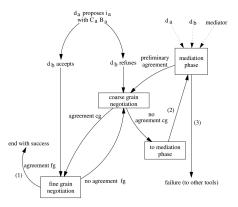




The main body Conclusions

#### **ONE PROJECT, THE STRUCTURE**

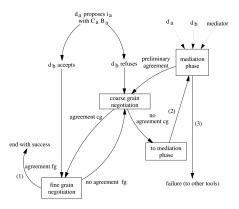




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#### **ONE PROJECT, THE STRUCTURE**







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#### ONE PROJECT, THE COARSE GRAIN NEGOTIATION

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#### ONE PROJECT, THE COARSE GRAIN NEGOTIATION

- d<sub>b</sub> [refuses the sets B<sub>a</sub> and C<sub>a</sub> and] presents the sets B'<sub>a</sub> and C'<sub>a</sub>;
- d<sub>a</sub> presents the sets B<sup>"</sup><sub>a</sub> and C<sup>"</sup><sub>a</sub>;
- (3) we have the following cases:
  - (3.a) if  $B''_a = B'_a$  and  $C''_a = C'_a$  then go to (4);
  - (3.b) if  $B''_a \neq B'_a$  or  $C''_a \neq C'_a$  then:
    - (a) with a random device we define an ordering between the two players;
    - (b) the player who comes first in the ordering gets a token only if he declares he wishes to rethink about his proposal otherwise the token assignment procedure is repeated with the other player;
    - (c) if  $d_a$  has the token then:
      - (i) d<sub>a</sub> presents the modified sets B<sup>"</sup><sub>a</sub> and C<sup>"</sup><sub>a</sub>;
      - (ii) go to (3);
    - (d) if d<sub>b</sub> has the token then:
      - (i)  $d_b$  presents the modified sets  $B'_a$  and  $C'_a$ ;
      - (ii) go to (3);
    - (e) if none of them has the token then go to (5);
- (4) go to the fine grain negotiation algorithm;
- d<sub>a</sub> and d<sub>b</sub> may either agree to go on or to give up;
- (6) if they agree to go on then go to (1);
- (7) if they agree to give up then call mediator procedure;

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#### ONE PROJECT, THE COARSE GRAIN NEGOTIATION

- d<sub>b</sub> [refuses the sets B<sub>a</sub> and C<sub>a</sub> and] presents the sets B'<sub>a</sub> and C'<sub>a</sub>;
- d<sub>a</sub> presents the sets B<sup>"</sup><sub>a</sub> and C<sup>"</sup><sub>a</sub>;
- (3) we have the following cases:
  - (3.a) if  $B''_a = B'_a$  and  $C''_a = C'_a$  then go to (4);
  - (3.b) if  $B''_a \neq B'_a$  or  $C''_a \neq C'_a$  then:
    - (a) with a random device we define an ordering between the two players;
    - (b) the player who comes first in the ordering gets a token only if he declares he wishes to rethink about his proposal otherwise the token assignment procedure is repeated with the other player;
    - (c) if  $d_a$  has the token then:
      - (i) d<sub>a</sub> presents the modified sets B<sup>"</sup><sub>a</sub> and C<sup>"</sup><sub>a</sub>;
      - (ii) go to (3);
    - (d) if  $d_b$  has the token then:
      - (i)  $d_b$  presents the modified sets  $B'_a$  and  $C'_a$ ;
      - (ii) go to (3);
    - (e) if none of them has the token then go to (5);
- go to the fine grain negotiation algorithm;
- d<sub>a</sub> and d<sub>b</sub> may either agree to go on or to give up;
- (6) if they agree to go on then go to (1);
- (7) if they agree to give up then call mediator procedure;

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#### ONE PROJECT, THE FINE GRAIN NEGOTIATION



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#### ONE PROJECT, THE FINE GRAIN NEGOTIATION

- (1)  $d_a$  presents the sets  $\alpha$  and  $\beta$ ;
- (2)  $d_b$  presents the sets  $\alpha'$  and  $\beta'$ ;
- (3) we have the following cases:
  - (3.a) if  $\alpha = \alpha'$  and  $\beta = \beta'$  then go to (4);
  - (3.b) if  $\alpha \neq \alpha'$  or  $\beta \neq \beta'$  then:
    - (a) with a random device we define on ordering between the two players;
    - (b) the player who comes first in the ordering gets a token only if he declares he wishes to rethink about his proposal otherwise the token assignment procedure is repeated with the other player;
    - (c) if  $d_a$  has the token then:
      - (i)  $d_a$  presents the modified sets  $\alpha$  and  $\beta$ ;
      - (ii) go to (3);
    - (d) if  $d_b$  has the token then:
      - (i)  $d_b$  presents the modified sets  $\alpha'$  and  $\beta'$ ;
      - (ii) go to (3);
    - (e) if none of them has the token then go to (5);
- (4) end;
- (5) go back to the coarse grain negotiation procedure;

#### ONE PROJECT, THE FINE GRAIN NEGOTIATION

- (1)  $d_a$  presents the sets  $\alpha$  and  $\beta$ ;
- (2)  $d_b$  presents the sets  $\alpha'$  and  $\beta'$ ;
- (3) we have the following cases:
  - (3.a) if  $\alpha = \alpha'$  and  $\beta = \beta'$  then go to (4);
  - (3.b) if  $\alpha \neq \alpha'$  or  $\beta \neq \beta'$  then:
    - (a) with a random device we define on ordering between the two players;
    - (b) the player who comes first in the ordering gets a token only if he declares he wishes to rethink about his proposal otherwise the token assignment procedure is repeated with the other player;
    - (c) if  $d_a$  has the token then:
      - (i) d<sub>a</sub> presents the modified sets α and β;
      - (ii) go to (3);
    - (d) if  $d_b$  has the token then:
      - (i)  $d_b$  presents the modified sets  $\alpha'$  and  $\beta'$ ;
      - (ii) go to (3);
    - (e) if none of them has the token then go to (5);
- (4) end;
- (5) go back to the coarse grain negotiation procedure;



The main body Conclusions

#### TWO PROJECTS, THE STRUCTURE

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#### TWO PROJECTS, THE STRUCTURE

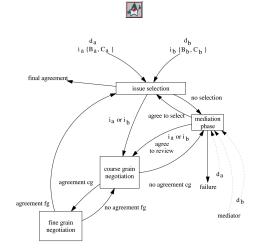




The preliminaries

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# TWO PROJECTS, THE STRUCTURE

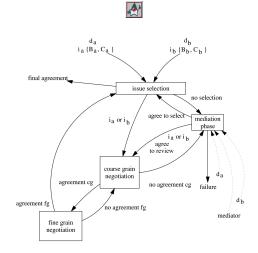


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# TWO PROJECTS, THE STRUCTURE





The preliminaries

The main body Conclusions

MANY PROJECTS & MANY DECIDERS

The preliminaries

The main body Conclusions

# MANY PROJECTS & MANY DECIDERS





The preliminaries

The main body Conclusions

# MANY PROJECTS & MANY DECIDERS



- $\Rightarrow$  one project and more than two deciders,
- $\Rightarrow$  two projects and more than two deciders,
- $\Rightarrow$  more than two projects and more than two deciders,
- $\Rightarrow$  more than two projects and two deciders.

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# MANY PROJECTS & MANY DECIDERS



- $\Rightarrow$  one project and more than two deciders,
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# MANY PROJECTS & MANY DECIDERS



- $\Rightarrow$  one project and more than two deciders,
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# MANY PROJECTS & MANY DECIDERS



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# MANY PROJECTS & MANY DECIDERS



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# MANY PROJECTS & MANY DECIDERS



- $\Rightarrow$  one project and more than two deciders,
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# THE POSSIBLE EXTENSIONS

The preliminaries

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# THE POSSIBLE EXTENSIONS





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# THE POSSIBLE EXTENSIONS



- ⇒ the structured deciders (inner dynamics, two levels of negotiation and decision),
- ⇒ the presence of the stakeholders (pressures, reactive environment),
- ⇒ the presence of the experts (lack of autonomy, reactive environment),
- $\Rightarrow$  the dynamic setting (variable sets).

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# THE POSSIBLE EXTENSIONS



- ⇒ the structured deciders (inner dynamics, two levels of negotiation and decision),
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The preliminaries

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- ⇒ the presence of the experts (lack of autonomy, reactive environment),
- $\Rightarrow$  the dynamic setting (variable sets).

The preliminaries

The main body Conclusions

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# THE POSSIBLE EXTENSIONS



- ⇒ the structured deciders (inner dynamics, two levels of negotiation and decision),
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# CONCLUSIONS

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- $\Rightarrow$  Open issues: "practical" applications to real cases.
- $\Rightarrow$  Open issues: deeper and more complete formalization.

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## AT THE END WE WERE BACK TO SEE AGAIN THE STARS...

