

Es. 1)

$S \rightarrow AB$

$A \rightarrow abb \mid aAbb \mid Ab$

$B \rightarrow bc \mid bBc \mid bB$

oppure

$S \rightarrow ABC$

$A \rightarrow abb \mid aAbb$

$C \rightarrow bc \mid bcc$

$B \rightarrow \varepsilon \mid bB$

Es. 2)

$$i) L = \{ a^m b^m \mid m > 0, m > 1 \}$$

$$ii) S = \{a\} S \{b\} \cup \{a\} A \{b\}$$

$$A = \{a\} A \cup B$$

$$B = \{b\} B \cup \{b\}$$

iii)

$$S^0 = \{ \}$$

$$A^0 = \{ \}$$

$$B^0 = \{ \}$$

$$S^1 = \{ \}$$

$$A^1 = \{ \}$$

$$B^1 = \{ b \}$$

$$S^2 = \{ \}$$

$$A^2 = \{ b \}$$

$$B^2 = \{ b, bb \}$$

Es. 3)

$$i) \text{ bool} \rightarrow (\text{bool} \rightarrow \text{bool}) \rightarrow \text{bool}$$

$$ii) ('a \rightarrow 'b) \rightarrow 'a \rightarrow (('a \rightarrow 'b) \rightarrow 'b \text{ list}) \rightarrow 'b \text{ list}$$

$$iii) 'a \text{ list} \rightarrow 'a \rightarrow ('a \rightarrow 'a \text{ list}) \rightarrow 'a \text{ list}$$

Es. 4)

Relazione di precedenza

$$\left(\forall m, m', m'', m''' \in \mathbb{N}. (m, m') < (m'', m''') \right)$$

$$\equiv$$

$$\left(m' < 10 \wedge m = m' + 1 \wedge m'' = m' \right) \vee$$

$$\left(m > 10 \wedge m'' = m + 1 \wedge m' = m + 1 \right)$$

$$f(m, m) = \begin{cases} 10 + 2m + 1 & \text{re } m = 10 \\ f(m+1, m) + X & \text{re } m < 10 \\ f(m-1, m+1) + Y & \text{re } m > 10 \end{cases}$$

$$X = 1$$

$$Y = 1$$

Caso base

$$f(10, m) = 10 + 2m + 1$$

$$f(10, m) \\ \approx \left\{ \text{def } f, 10^{\text{ caso}} \right\}$$

$$10 + 2m + 1$$

Caso induttivo: $v_0 = 1$, $m < 10$

$$f(m+1, m) = m+1 + 2m+1$$

$$\Rightarrow f(m, m) = m + 2m + 1$$

$$f(m, m) = \{ \text{def } f, m < 10, 2^{\circ} \text{ caso} \}$$

$$f(m+1, m) + X = \{ (m+1, m) < (m, m) \}$$

ip. ind.

$$m+1 + 2m+1 + X = \{ \text{calcolo, } X = -1 \}$$

$$m + 2m + 1$$

Caso moltiplo vs 2, $m > 10$

$$f(m-1, m+1) = (m-1) + 2(m+1) + 1$$

$$\Rightarrow f(m, m) = m + 2m + 1$$

$$f(m, m)$$

$$= \{ \text{def } f, m > 0, 3^{\circ} \text{ caso} \}$$

$$f(m-1, m+1) + \gamma$$

$$\stackrel{N}{=} \{ (m-1, m+1) < (m, m) \}$$

ip. moltiplo va

$$(m-1) + 2(m+1) + 1 + \gamma$$

$$= \{ \text{calcolo}, \gamma = -1 \}$$

$$m + 2m + 1$$

Es. 6)

let rec foo l m x = match (m, l) with

(0, l) → l

| (m, []) when m > 0 → []

| (m, w :: ws) when m > 0 & w = x
 → foo ws (m-1) x

| (m, w :: ws) when m > 0 & w <> x
 → w :: foo ws m x ;;