$$S \to ab \mid aSb$$

$$T_S(X) = \ \dots$$

trasformazione da
insieme di stringhe
in insieme di stringhe

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$$S \to ab \mid aSb$$

$$\Downarrow$$

$$T(S) = \{a\}\{b\} \cup \{a\}S\{b\}$$

$$X = T(X)$$

$$\boxed{X = \{a\}\{b\} \cup \{a\}X\{b\}}$$

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2013_10_15

$$AB = \{\underline{\underline{\alpha\beta}} \mid \alpha \in A \text{ e } \beta \in B\}$$

$$A, B \subseteq \Lambda^*$$

$$A\{\} = \{\alpha\beta \mid \alpha \in A \text{ e } \beta \in \{\}\} = \{\}$$

$$A\{\varepsilon\} = A$$

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$$X = \{a\}\{b\} \cup \{a\} X \{b\} \leftarrow$$

$$T(X) = \{a\}\{b\} \cup \{a\} X \{b\}$$

$$T^0(\{\}) = \{\}$$

$$T^1(\{\}) = \underline{\underline{\{a\}\{b\}}} \cup \underline{\underline{\{a\}\{\}\{b\}}} = \{ab\}$$

$$T^2(\{\}) = T(T^1(\{\})) = T(\{ab\})$$

$$= \{a\}\{b\} \cup \{a\}\{ab\}\{b\} =$$

$$\{ab, aabb\}$$

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$$X = \{a\}\{b\} \cup \{a\} X \{b\} \leftarrow$$

$$T(X) = \{a\}\{b\} \cup \{a\} X \{b\}$$

$$T^3(\{\}) = T(T^2(\{\})) =$$

$$T(\{ab, aabb\}) =$$

$$\{a\}\{b\} \cup \{a\}\{ab, aabb\}\{b\} =$$

$$\{ab, aabb, aaabbb\} \ldots$$

$$X = \{a\}\{b\} \cup \{a\} X \{b\} \leftarrow$$

$$T(X) = \{a\}\{b\} \cup \{a\} X \{b\}$$

$$\bigcup_{i \geq 0} T^i(\{\}) =$$

$$\{a^n b^n \mid n \geq 0\}$$

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$$AB = \{\alpha\beta \mid \alpha \in A ,\ \beta \in B\}$$

$$A \cup B = \{\alpha \mid \alpha \in A \text{ oppure } \alpha \in B\}$$

$$A = \{ab, cd\}.$$

$$B = \{ab, aab\}$$

$$AB = \{abab, abaab, cdab, cdaab\}$$

$$A \cup B = \{ab, cd, aab\}$$

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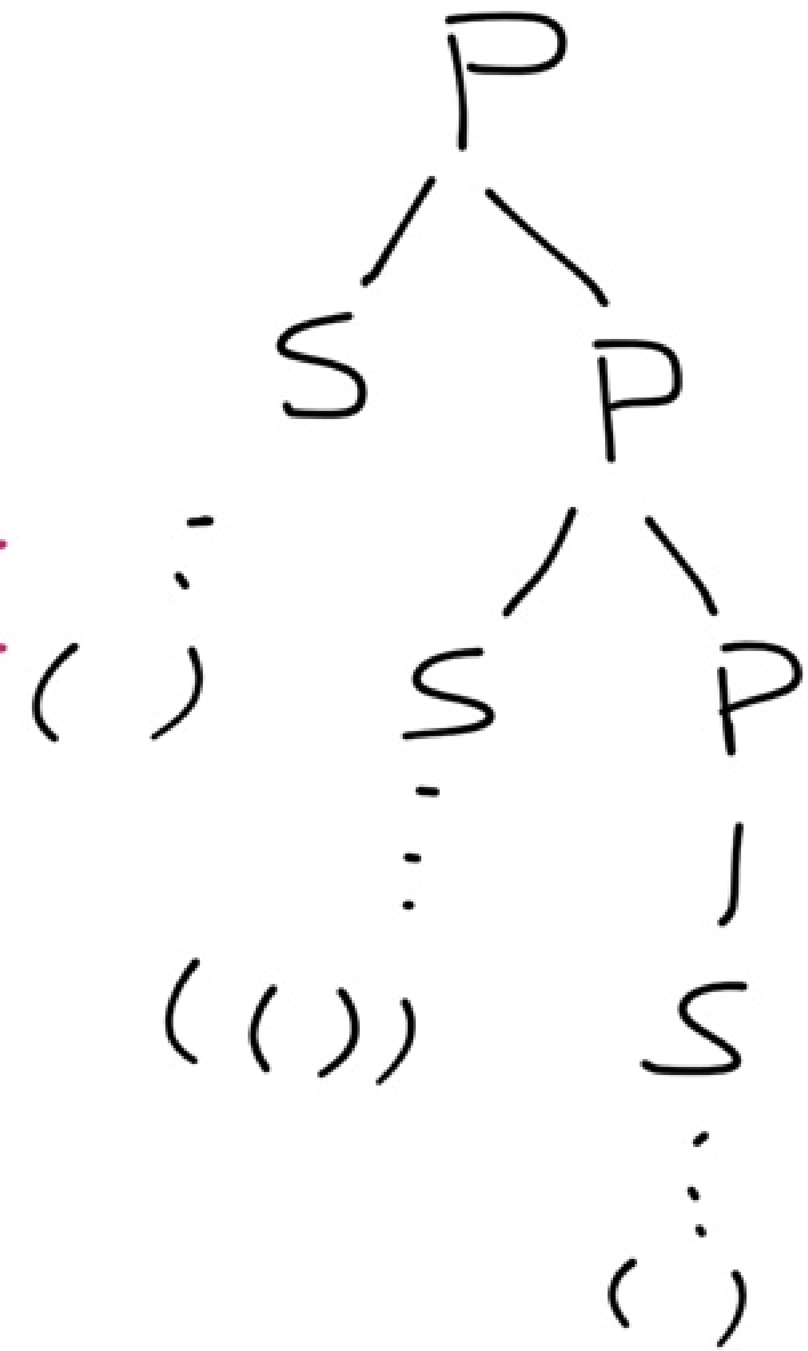
$$P \to SP \mid S$$
$$S \to () \mid (P)$$

$$P \to () \mid (P) \mid PP$$

$()(())()$



mutuamente ricorsive

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$$P \to SP \mid S$$
$$S \to () \mid (P)$$

$$T_P(P,S) = SP \cup S$$

$$T_S(P,S) = \{()\} \cup \{(\} P \{)\}$$

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$$P = SP \cup S$$

$$S = \{()\} \cup \{(P)\}$$

$$P = T_P(P, S)$$

$$S = T_S(P, S)$$

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$$P = SP \cup S$$

$$S = \underline{\{(\ )\} \cup \{(\}P\{)\}}$$

$$T_P^0(\{\},\{\}) = \{\}$$

$$T_S^0(\{\},\{\}) = \{\}$$

$$T_P^1(\{\},\{\}) = T_P(\{\},\{\}) = \{\}\{\} \cup \{\} = \{\}$$

$$T_S^1(\{\},\{\}) = T_S(\{\},\{\}) = \{(\ )\} \cup \{(\}\{\}\{)\}$$

$$= \{(\ )\}$$

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$$T_p^2(\{\},\{\}) = T_p\left(T_p^1(\{\},\{\}), T_s^1(\{\},\{\})\right)$$

$$= T_p(\{\},\{()\}) = \{()\}\{\} \cup \{()\}$$

$$= \{()\}$$

$$T_s^2(\{\},\{\}) = T_s\left(T_p^1(\{\},\{\}), T_s^1(\{\},\{\})\right)$$

$$= T_s(\{\},\{()\}) =$$

$$\{()\} \cup \{(\}\{\}\{)\} = \{()\}$$

$$T_p^3(\{\},\{\}) = T_p(T_p^2(\{\},\{\}), T_s^2(\{\},\{\}))$$

$$= T_p(\{()\},\{()\}) =$$

$$\{()\}\{()\} \cup \{()\} = \{(),()()\}$$

$$T_s^3(\{\},\{\}) = T_s(\{()\},\{()\}) =$$

$$\{()\} \cup \{(\{()\})\} =$$

$$\{(),(())\}$$

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$$\bigcup_{i\geq 0} T_p^{i}(\{\},\{\}) =$$

linguaggio infinito delle parentesi bilanciate

$$\bigcup_{i\geq 0} T_s^{i}(\{\},\{\}) =$$

linguaggio infinito delle parentesi bilanciate racchiuse tra parentesi.

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Applicazione del Teorema di ricorsione alle definizioni di linguaggi

come:

Trasformando le produzioni delle grammatiche in def. di trasformazioni da insiemi di stringhe in insiemi di stringhe

$$S \to 0A1 \mid 0A$$

$$A \to 01 \mid 10 \mid 12$$

$$T_S(S,A) = \underline{\{0\}A\{1\}} \cup \underline{\{0\}A}$$

$$T_A(S,A) = \{01\} \cup \{10\} \cup \{12\}$$

$$T_S^0(\{\},\{\}) = \{\} \qquad T_S^1(\{\},\{\}) = \{\}$$

$$T_A^0(\{\},\{\}) = \{\} \qquad T_A^1(\{\},\{\}) = \{01, 10, 12\}$$

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$$T_S(\{\}, \{01, 10, 12\}) = T_S^2(\{\}, \{\}) =$$

$$\{0\}\{01, 10, 12\}\{1\} \cup \{0\}\{01, 10, 12\}$$

$$= \{0011, 0101, 0121\} \cup \{001, 010, 012\}$$

$$= \{ \quad '' \quad , \quad '' \quad \}$$

$$T_A(\{\}, \{01, 10, 12\}) = \{01, 10, 12\}$$

$$= T_A^2(\{\}; \{\})$$

$$T_A^3(\{\}, \{\}) = T_A^2(\{\}, \{\})$$

$$T_S^3(\{\}, \{\}) = T_S^2(\{\}, \{\})$$

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$$f(x) = \begin{cases} \emptyset & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

posso applicare il Teo di Ric? NO

$$f : \mathbb{N} \to \mathbb{N}$$

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GRAFICO DI UNA FUNZIONE

Il grafico di una funzione $f : \mathbb{N} \to \mathbb{N}$ è l'insieme delle coppie

$$\{ \langle n, m \rangle \mid n, m \in \mathbb{N} \text{ e } m = f(n) \}$$

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Es. $f : \mathbb{N} \to \mathbb{N}$

$$f(x) = x+1$$

$$F = \{\langle 0,1\rangle, \langle 1,2\rangle, \langle 2,3\rangle, \dots\}$$

$$\langle n, m\rangle$$

tale che $m = f(n)$,

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$$f(x) = \begin{cases} 0 & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

$$F = \{\langle 0, 0 \rangle\} \cup$$

$$\{\langle x, 2+m \rangle \mid \langle x-1, m \rangle \in F\}$$

$$f : \mathbb{N} \to \mathbb{N}$$

$$F \subseteq \mathbb{N} \times \mathbb{N}$$

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$$f(x) = \begin{cases} 0 & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

$$F = \{\langle 0, 0 \rangle\} \cup \{\langle n, n+2 \rangle \mid \langle n-1, n \rangle \in F\}$$

$$\Downarrow$$

$$T_F : \mathcal{P}_{\mathbb{N}\times\mathbb{N}} \to \mathcal{P}_{\mathbb{N}\times\mathbb{N}}$$

$$F = T_F(F)$$

$$T_F(X) = \{\langle 0, 0 \rangle\} \cup \{\langle n, n+2 \rangle \mid \langle n-1, n \rangle \in X\}$$

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$$\begin{cases} F = T_F(F) \\ F = \{\langle 0,0\rangle\} \cup \{\langle n, m+2\rangle \mid \langle n-1, m\rangle \in F\} \end{cases}$$

$$T_F^0(\{\}) = \{\}$$

$$T_F^1(\{\}) = \{\langle 0,0\rangle\} \cup \{\langle n, m+2\rangle \mid \langle n-1, m\rangle \in \{\}\} = \{\langle 0,0\rangle\}$$

$$T_F^2(\{\}) = T_F(\{\langle 0,0\rangle\}) =$$

$$\{\langle 0,0\rangle\} \cup \{\langle n, m+2\rangle \mid \langle n-1, m\rangle \in \{\langle 0,0\rangle\}\}$$

$$= \{\langle 0,0\rangle, \langle 1,2\rangle\}$$

$$\left.\begin{matrix} n-1=0 \\ m=0 \end{matrix}\right\} \Rightarrow n=1,\ m+2=2$$

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$$T_F^3(\{1\}) = T_F(\{\langle 0,0\rangle, \langle 1,2\rangle\}) =$$

$$\{\langle 0,0\rangle\} \cup \{\langle m, m+2\rangle \mid \langle m-1, m\rangle \in \{\langle 0,0\rangle, \langle 1,2\rangle\}\}$$

$$= \{\langle 0,0\rangle\} \cup \{\langle 1,2\rangle, \langle 2,4\rangle\}$$

$$\langle m-1, m\rangle \in \{\langle 0,0\rangle, \langle 1,2\rangle\}$$

$\langle m-1,m\rangle = \langle 0,0\rangle$	$\langle m-1,m\rangle = \langle 1,2\rangle$	
$m = 1$	$m-1 = 1$	$m = 2$
$m+2 = 2$	$m = 2$	$m+2 = 4$

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$$F = \{ \langle 0,0 \rangle, \langle 1,2 \rangle, \langle 2,4 \rangle, \dots \}$$

$$f(x) = \begin{cases} 0 & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

$$f(x) = 2 \cdot x$$

$$F = \{ (n, m) \mid n, m \in \mathbb{N} \text{ e } m = 2 \cdot n \}$$

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