

CAML

let sum x y = x + y;;

↑ ↑ ↑ ↓
nome 1° arg 2° arg exp

SUM: 'a → 'b → 'c

tipo x tipo y res.

sum : int → int → int

INFERENZA DI TIPO

FUNZIONI DI ORDINE SUPERIORE

Sono funzioni che ricevono come argomenti altre funzioni e/o restituiscono come risultato una funzione

sum 3 j;

- i int \rightarrow int = <fun>

let sum x y = x + y j;

sum; int \rightarrow int \rightarrow int

let inc3 = sum 3 j;

inc3 : int \rightarrow int = <fun>

inc3 8 j;

- i int = 11

inc3 25 j;

- i int = 28

let gt x y = x > y ;; greater than

gt : int → int → bool = <fun>

let neg = gt 0 ;;

neg : int → bool = <fun>

let h = apply neg ;;

h : int → bool = <fun>

apply : ('a → 'b) → 'a → 'b
neg : int → bool

INFERENZA DI TIPO

let $f \ x_1 \ x_2 \ \dots \ x_m = \text{Exp } j$

① STRUTTURA DEL TIPO DI f

$f : 'a_1 \rightarrow 'a_2 \rightarrow \dots \rightarrow 'a_n \rightarrow 'r$

② Si ragiona sull'uso di x_1, \dots, x_m
in Exp per ISTANZIARE (CONCRETIZZARE)
' $a_1 \dots 'a_n$ (se possibile)

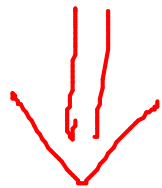
In questo ci si può accorgere che alcune
variabili di tipo DEVONO COINCIDERE

FUNZIONE UNCURRIED

Caso particolare in cui abbiamo 1 solo argomento

$$\underline{\text{let}} \ f(y_1, \dots, y_n) = \text{Exp}$$

$$f : 'a \rightarrow 'b$$

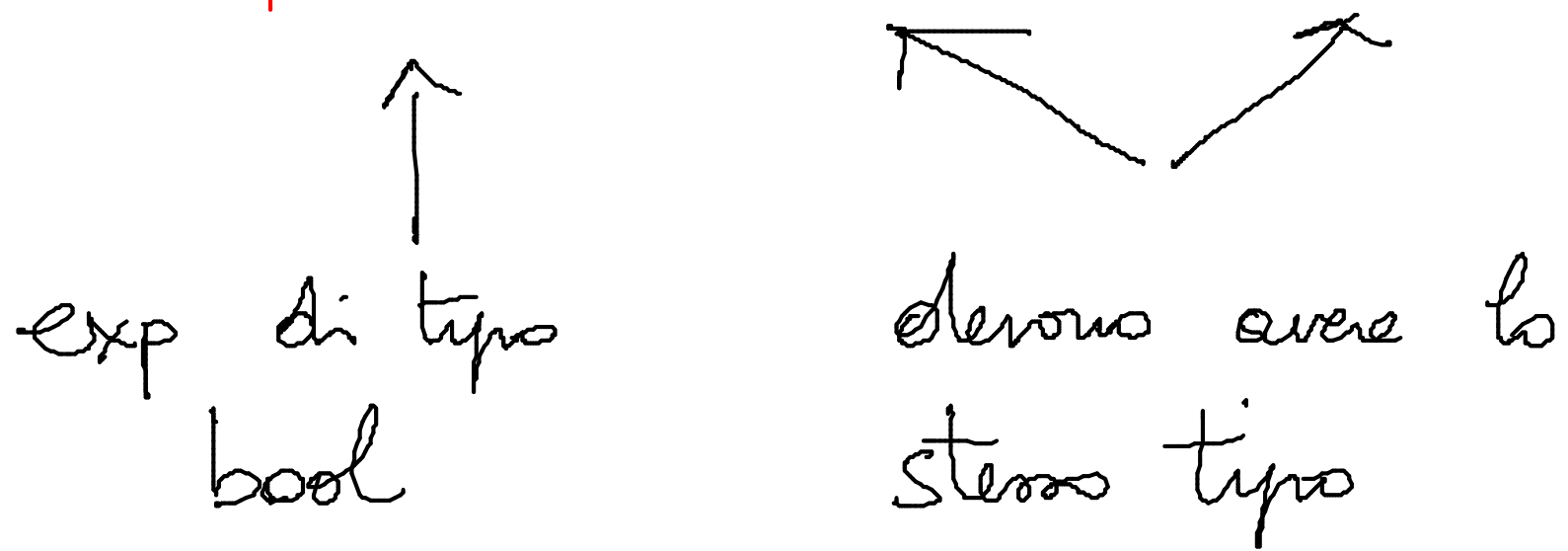


$$f : 'a_1 * 'a_2 * \dots * 'a_n \rightarrow 'b$$

CONDIZIONALE : espressioni

let abs $x =$ if $x > 0$ then x
else $-x$ جز
abs: int \rightarrow int = <fun>

if C then E1 else E2



let rec pot (x, y) = if y = 0 then 1 else x * pot (x, y-1);

pot : (int * int) → int = <fun>

pot (x, y) = $\begin{cases} 1 & \text{se } y=0 \\ x * \text{pot}(x, y-1) & \text{altrimenti} \end{cases}$

$$\text{pot}(3, 2)$$

$$= \{ \text{def. di pot} \}$$

$$3 * \text{pot}(3, \underline{1})$$

$$= \{ \text{def. di pot} \}$$

$$3 * 3 * \text{pot}(3, \underline{0})$$

$$= \{ \text{def. di pot} \}$$

$$3 * 3 * 1$$

"

9

$$\text{pot}(3, -1)$$

$$= \{ \text{def. di pot} \}$$

$$3 * \text{pot}(3, -2)$$

=

$$3 * 3 * \text{pot}(3, -3)$$

⋮

$$\text{pot}(x, y) = \text{if } y = 0$$

then 1

else

$$x * \text{pot}(x, y-1)$$

$$\text{pot} : \mathbb{Z} * \mathbb{Z} \rightarrow \mathbb{Z}$$

Relazione involotta \prec_{pot}

$m \in \mathbb{Z}^+$

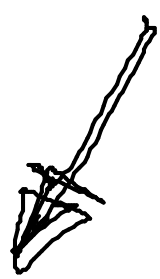
(m, m)

$(m, m-1)$

⋮

$(m, 1)$

(m, \emptyset)



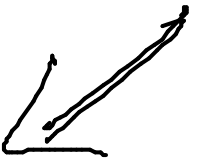
(n, n)

$(n, n-1)$

$(n, n-2)$

⋮

$m \in \mathbb{Z}^-$



$$\forall (x, y) \in (\mathbb{Z}^+ * \mathbb{Z}^+ \text{ (N * N)}). \text{pot}(x, y) = x^y$$

• pot \bar{e} TOTALE su $\mathbb{Z} * \mathbb{N}$

• pot \bar{e} PARZIALE (non TOTALE) su $\mathbb{Z} * \mathbb{Z}$

Caso base

$$\text{pot}(x, \emptyset)$$

$$= \{ \text{def. di pot, nono then} \}$$

1

$$= x^\emptyset$$

Caso induttivo (x, y) con $y > \emptyset$

$$\text{pot}(x, y)$$

$$= \{ \text{def. di pot, nono else} \}$$

$$x * \text{pot}(x, y-1)$$

$$= \{ \text{Ip. ind. } (x, y-1) \prec_{\text{pot}} (x, y) \}$$

$$x * x^{y-1}$$

$$= x^{y-1+1}$$

c.v.d.

DIMOSTRAZIONI PER INDUZIONE BEN FONDATA

let rec pot x y = if y=0 then 1 else x * pot x y + 1;



dimostrazione per ind. b.f. la faccenda
come nella versione uncurried.

PATTERN MATCHING

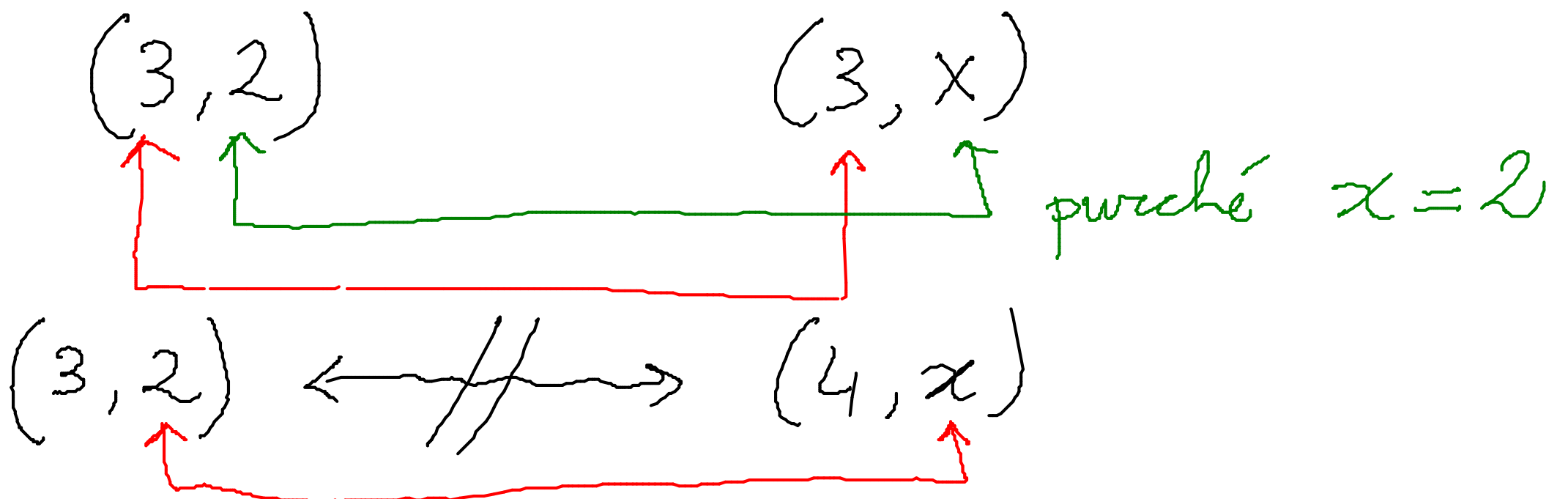
$(3, 2)$

$(3, x)$

oggetto 1

oggetto 2

fare pattern matching su questi due oggetti
significa domandarsi: possono COINCIDERE?



let rec pot (x,y) = if y=0 then 1 else x * pot (x,y-1);;

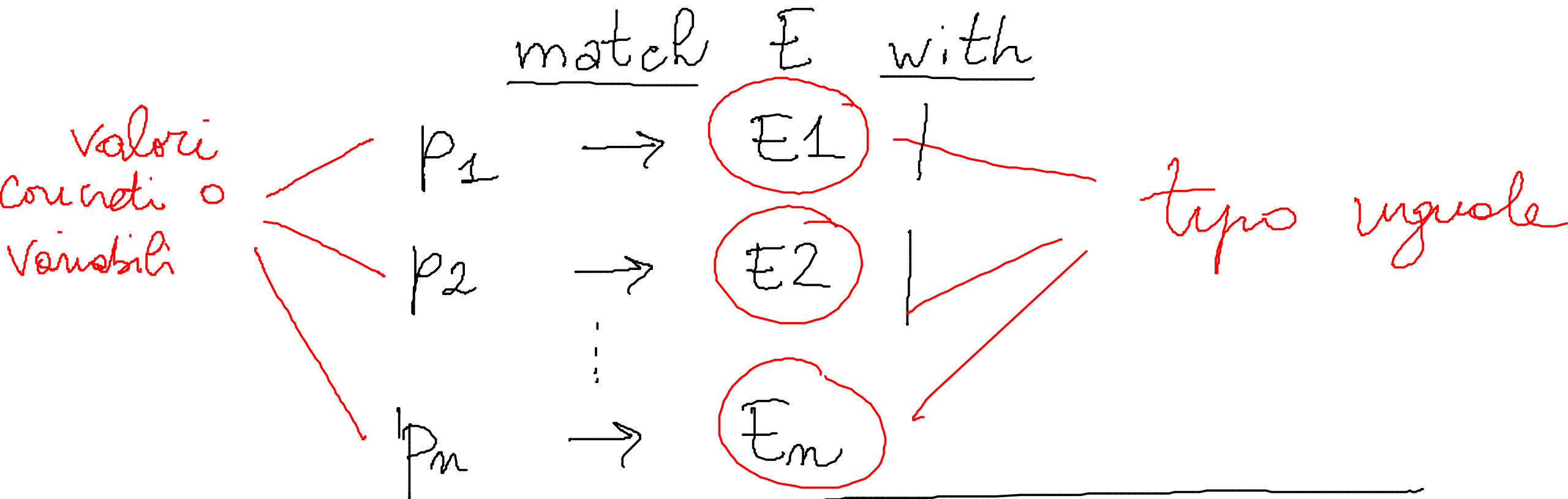
$$\text{pot}(x,y) = \begin{cases} 1 & 1^{\circ} \text{ caso } y=0 \\ x * \text{pot}(x,y-1) & 2^{\circ} \text{ caso} \end{cases}$$

let rec pot (x,y) = match y with

→ \emptyset → 1 |

→ $-$ → $x * \text{pot}(x,y-1)$;;

PATTERN MATCHING



if ($E = \underline{P_1}$) then E_1 else

if ($E = \underline{P_2}$) then E_2 else ...

... if ($E = \underline{P_n}$) then E_n else?

~~$x * 3$~~ match $y + 1$ with

\uparrow
numero (costante intera)

\uparrow
variable (nome, sta per il pattern "generico")
_ (underscore " ")

\rightarrow (in tutti gli altri casi)

let $f(x) = \text{match } x \text{ with}$

$\emptyset \rightarrow 1 \mid$

$1 \rightarrow 25 \mid$

$_ \rightarrow 45 \mid _ _$

$$f(n) = \begin{cases} 1 \\ 25 \\ 45 \end{cases}$$

se $x = 0$

se $x = 1$

ALTRIMENTI

let $f \ x = \text{match } \textcircled{x}$ with

$\textcircled{\emptyset} \rightarrow \textcircled{1} \quad |$
 $\textcircled{1} \rightarrow \textcircled{10} \quad | \textcircled{j}$

$f : \text{int} \rightarrow \underline{\text{int}} = \langle \text{fun} \rangle$

Warning: il p.m. non è ESAUSTIVO
non copre tutti i possibili
valori

let f $x =$ match x with

~~0~~ \rightarrow 1 |

1 \rightarrow 10 |

$_$ \rightarrow 45 | |

f : $\text{int} \rightarrow \text{int} = \langle \text{fun} \rangle$

$$g(m) = \begin{cases} 1 & \text{se } m = 0 \\ 10 & \text{se } m > 0 \\ 25 & \text{se } m < 0 \end{cases}$$

PATTERN MATCHING

match m with

$\emptyset \rightarrow 1$ /
 \textcircled{x} when $x > 0 \rightarrow 10$

\textcircled{x} when $x < 0 \rightarrow 25$

let $g\ n =$ match n with

$\emptyset \rightarrow 1$ |

x when $x > 0$ $\rightarrow 10$ |

x when $x < 0 \rightarrow 25$;;

$g : \text{int} \rightarrow \text{int} = \langle \text{fun} \rangle$

$g(4)$

pattern matching $4 \langle \rightsquigarrow \rangle \emptyset$ NO

$4 \langle \rightsquigarrow \rangle x$ $x=4$ SI

$g(-3)$

$-3 \langle \rightsquigarrow \rangle \emptyset$ NO

$-3 \langle \rightsquigarrow \rangle x$ $x=-3$ ma $-3 \neq 0$ NO

$-3 \langle \rightsquigarrow \rangle x$ $x=-3$ $-3 < 0$ SI

10

25

Sia un nome x che $_$ sono pattern
"generici" che "matchano" qualunque valore

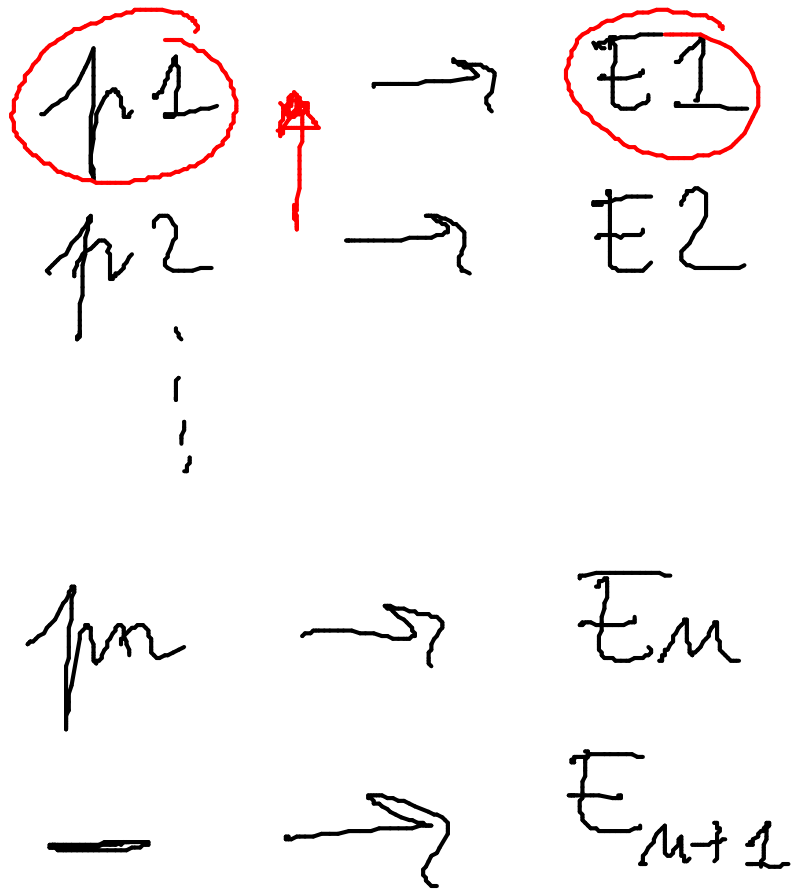
Quando si usa $_$ e quando si usa x ??

match (x, y) with

$(-, 3)$ when ~~$z > 0$~~ $\rightarrow y + 5$

$(z, 3)$ when $z > 0 \rightarrow y + 5$

match E with



alternanti

match n with

$$\emptyset \rightarrow n+1 \quad 1 \quad |$$

$$\rightarrow x \text{ when } x > 0 \rightarrow \cancel{x+2} \quad |$$

$n+2$

$$\rightarrow \boxed{x \rightarrow x+3 \quad ; ;}$$

? case \bar{x} ??
x unbounded

$$_ \rightarrow n+3 \quad ; ;$$

da qui in poi
(fino alla |)
 n e x
sono SINONIMI

PATTERN MATCHING : COPPIE

• let $f(x, y) = x + y$;;

• let $f(z) = \text{match } z \text{ with}$
 $(x, y) \rightarrow x + y$;;

LISTE : sequenze di oggetti

let $\text{pat}(x, y) = \text{match } y \text{ with}$

$\emptyset \rightarrow 1$ |
when $n > 0$ $\rightarrow x * \text{pat}(x, y-1)$ | //

$\ominus \rightarrow -1$; ;

$\text{pat} : \text{int} * \text{int} \rightarrow \text{int} = \langle \text{fun} \rangle$

$f(x, y) = \begin{cases} x^y & \text{se } y \geq 0 \\ -1 & \text{altrimenti} \end{cases}$