Summary so far

- Classes P and NP.
- $P \subseteq NP$ and it is assumed that $P \neq NP$.
- Problems in P are tractable.
- Problems in $NP \setminus P$ are untractable.
- HPP is untractable.
  - What else is (un)tractable?
  - Is “my problem” tractable or untractable?

Proving tractability

- "My problem" is tractable if...

  ... I exhibit a polynomial algorithm that solves it.
- What's next?
  - I try to conceive the most efficient algorithm (see the rest of this course).
  - Is the best algorithm efficient enough?
    - If yes, then it's great.
    - If not, I try heuristics (see later).

Proving untractability: the idea

- It is hard to prove that “I cannot do this”.
- Easier to prove that “if I can do this, then I can also do that (and show how)”.

- Assume I know a problem Pr that is untractable: if I show that the polynomial solution of my problem would also solve Pr with some extra work then...

Polynomial Reduction $\leq_p$ /1

- A problem P1 can be polynomially reduced to a problem P2 (P1 $\leq_p$ P2) iff:
  - An instance i1 of P1 can be transformed in polynomial time in an instance i2 of P2 such that:
  - A solution of i2 corresponds to a solution of i1.

Somehow, it means that P1 is not harder to solve than P2

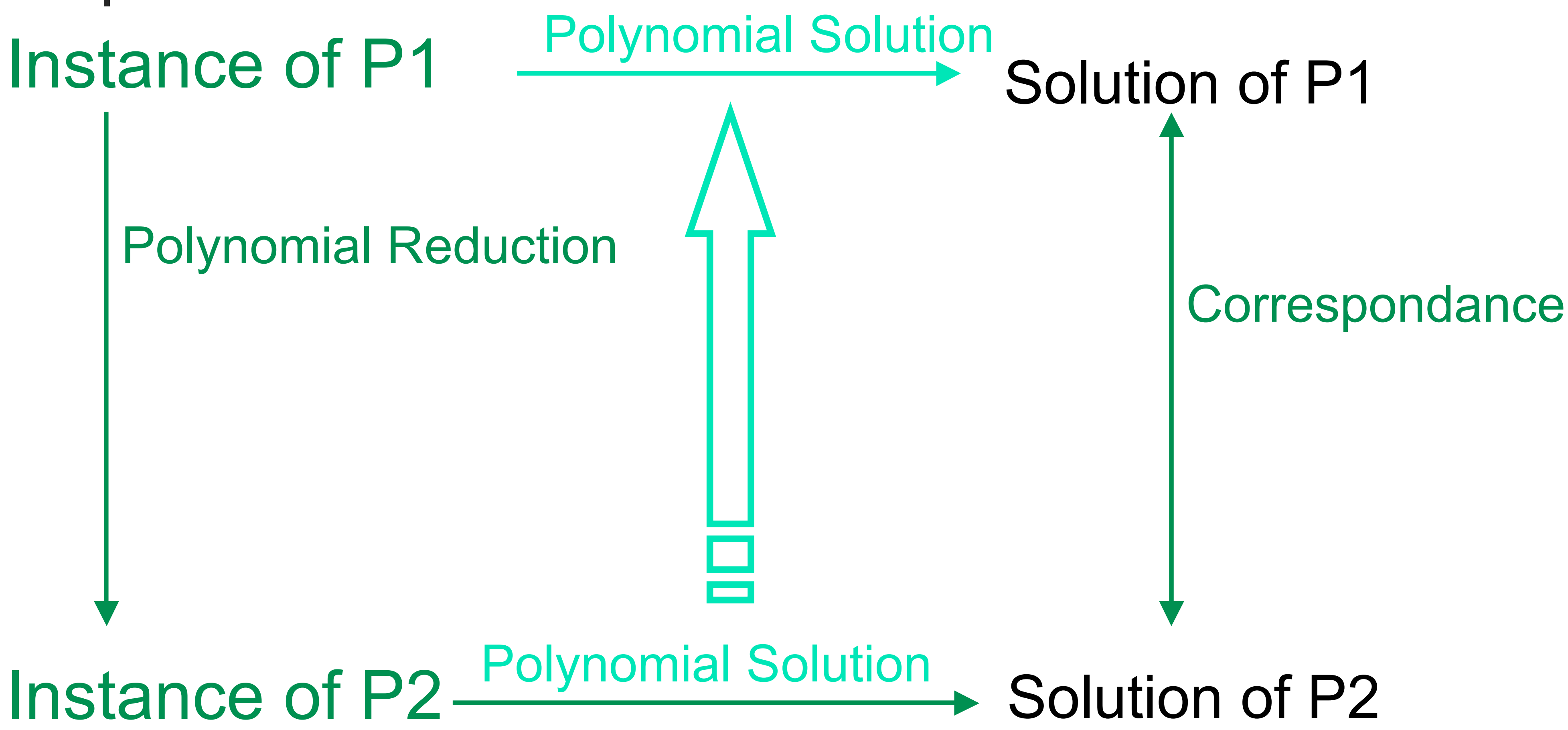
Another way to see (and use) it:

- A “new” problem P2 can be polynomially reduced FROM a “known” problem P1 ($P1 \leq_p P2$) if ...

Then P2 is at least as difficult as P1.

In particular, if P1 is untractable, then also P2 is.

P1 $\leq_p$ P2 when P1 is untractable



NP-complete Problems

- A problem P is NP-complete ($P \in NPC$) iff:
  - It is in NP and
  - $P' \leq_p P$ for each $P' \in NP$.

- Somehow, they are *the most difficult* problems in NP: if I can efficiently solve one of them, then I can efficiently solve all of them, and in that case P = NP.

NP-complete Problems

- A problem P is NP-complete ($P \in NPC$) iff:
  - It is in NP and
  - $P' \leq_p P$ for each $P' \in NP$.

- If $Pr \in NPC$ is polynomially solvable, since any problem $P' \in NP$ can be reduced to it, then P' can also be solved in polynomial time.

NP-complete Problems

- The existence of NP-complete problems enforces the conjecture that P $\neq$ NP:
  - Disproving the conjecture would mean that there exist polynomial solutions for a huge class of problems...
  - ... for which nobody has found a polynomial solution so far!

P, NP, and NPC