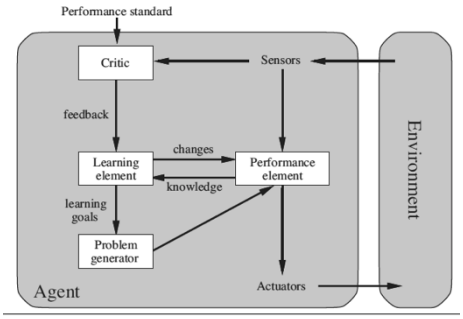
# Concept learning

*Maria Simi, 2011/2012*
Machine Learning, Tom Mitchell
Mc Graw-Hill International Editions, 1997
(Cap 1, 2).

## Introduction to machine learning

- Introduction to machine learning
  - When appropriate and when not appropriate
  - Task definition
  - Learning methodology: design, experiment, evaluation
  - Learning issues: representing hypothesis
  - Learning paradigms
    - Supervised learning
    - Unsupervised learning
    - Reinforcement learning
- Next week …

## AIMA learning architecture




## Machine learning: definition

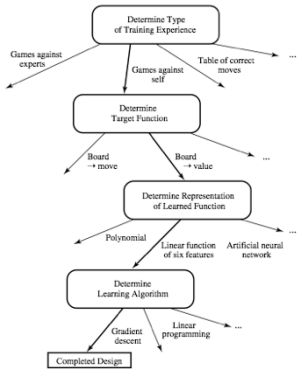
- *A computer program is said to **learn** from experience E with respect to some class of tasks T and performance measure P, if its performance at tasks in T, as measured by P, improves with experience E* [Mitchell]
- Problem definition for a learning agent
  - Task T
  - Performance measure P
  - Experience E

## Designing a learning system

1. Choosing the training experience
   - Examples of best moves, games outcome …
2. Choosing the target function
   - board-move, board-value, …
3. Choosing a representation for the target function
   - linear function with weights (hypothesis space)
4. Choosing a learning algorithm for approximating the target function
   - A method for parameter estimation

## Design of a learning system

Mitchell



## Inductive learning

- Inductive learning
  - Inducing a general function from training examples
  - A supervised paradigm
- Basic schemas that assume a logical representation of the hypothesis
  - Concept learning
  - Decision trees learning
  - Important issues
    - Inductive bias (definition)
    - The problem of overfitting
- Bibliography:
  - Mitchell, cap1,2,3

## Definition of concept learning

- *Task*: learning a category description (*concept*) from a set of positive and negative training examples.
  - Concept may be an event, an object …
- *Target function*: a boolean function $c$: $X \rightarrow \{0, 1\}$
- *Experience*: a set of training instances $D$:$\{\langle x, c(x)\rangle\}$
- A search problem for best fitting hypothesis in a hypotheses space
  - The space is determined by the choice of representation of the hypothesis (all boolean functions or a subset)

## Sport example

- Concept to be learned:
  *Days in which Aldo can enjoy water sport*
  Attributes:
  ***Sky***: *Sunny, Cloudy, Rainy* ***Wind***: *Strong, Weak*
  ***AirTemp***: *Warm, Cold* ***Water***: *Warm, Cool*
  ***Humidity***: *Normal, High* ***Forecast***: *Same, Change*
- Instances in the training set (out of the 96 possible):

| Sky | Temp | Humid | Wind | Water | Forecst | EnjoySpt |
|---|---|---|---|---|---|---|
| Sunny | Warm | Normal | Strong | Warm | Same | Yes |
| Sunny | Warm | High | Strong | Warm | Same | Yes |
| Rainy | Cold | High | Strong | Warm | Change | No |
| Sunny | Warm | High | Strong | Cool | Change | Yes |

## Hypotheses representation

- $h$ is a set of constraints on attributes:
  - a specific value: e.g. *Water = Warm*
  - any value allowed: e.g. *Water = ?*
  - no value allowed: e.g. *Water = Ø*
- Example hypothesis:

  *Sky* *AirTemp* *Humidity* *Wind* *Water* *Forecast*
  ⟨*Sunny*, ?, ?, *Strong*, ?, *Same*⟩
  Corresponding to boolean function:
  Sunny(Sky) ∧ Strong(Wind) ∧ Same(Forecast)
- $H$, hypotheses space, all possible $h$

## Hypothesis satisfaction

- An instance $x$ *satisfies* an hypothesis $h$ iff all the constraints expressed by *h* are satisfied by the attribute values in $x$.
- Example 1:
  $x_1$: ⟨*Sunny, Warm, Normal, Strong, Warm, Same*⟩
  $h_1$: ⟨*Sunny*, ?, ?, *Strong*, ?, *Same*⟩ Satisfies? Yes
- Example 2:
  $x_2$: ⟨*Sunny, Warm, Normal, Strong, Warm, Same*⟩
  $h_2$: ⟨*Sunny*, ?, ?, Ø, ?, *Same*⟩ Satisfies? No

## Formal task description

- Given:
  - $X$ all possible days, as described by the attributes
  - A set of hypothesis $H$, a conjunction of constraints on the attributes, representing a function $h$: $X \rightarrow \{0, 1\}$
    [$h(x) = 1$ if $x$ satisfies $h$; $h(x) = 0$ if $x$ does not satisfy $h$]
  - A target concept: $c$: $X \rightarrow \{0, 1\}$ where
    $c(x) = 1$ iff *EnjoySport = Yes*;
    $c(x) = 0$ iff *EnjoySport = No*;
  - A training set of possible instances $D$: $\{\langle x, c(x)\rangle\}$
- Goal: find a hypothesis $h$ in $H$ such that
  $h(x) = c(x)$ for all $x$ in $D$
  Hopefully $h$ will be able to predict outside $D$…

## The inductive learning assumption

- We can at best guarantee that the output hypothesis fits the target concept over the training data
- *Assumption*: an hypothesis that approximates well the training data will also approximate the target function over unobserved examples
- i.e. given a significant training set, the output hypothesis is able to make predictions

## Concept learning as search

- Concept learning is a task of searching an hypotheses space
- The representation chosen for hypotheses determines the search space
- In the example we have:
  - 3 x $2^5$ = 96 possible instances (6 attributes)
  - 1 + 4 x $3^5$= 973 possible hypothesis

    considering that all the hypothesis with some ∅ are semantically equivalent, i.e. inconsistent
- Structuring the search space may help in searching more efficiently

## General to specific ordering

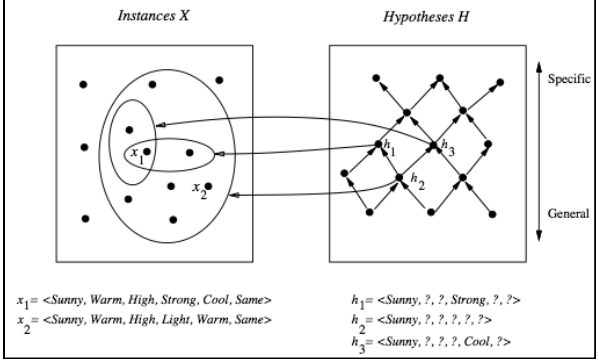
- Consider:

  $h_1 = \langle Sunny, ?, ?, Strong, ?, ? \rangle$

  $h_2 = \langle Sunny, ?, ?, ?, ?, ? \rangle$
- Any instance classified positive by $h_1$ will also be classified positive by $h_2$
- $h_2$ is more general than $h_1$
- Definition: $h_j \geq_g h_k$ *iff* $(\forall x \in X)\ [(h_k = 1) \rightarrow (h_j = 1)]$

  $\geq_g$ *more general or equal;* $>_g$ *strictly more general*
- Most general hypothesis: $\langle ?, ?, ?, ?, ?, ? \rangle$
- Most specific hypothesis: $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$

## General to specific ordering: induced structure




$x_1$ = <Sunny, Warm, High, Strong, Cool, Same>
$x_2$ = <Sunny, Warm, High, Light, Warm, Same>

$h_1$ = <Sunny, ?, ?, Strong, ?, ?>
$h_2$ = <Sunny, ?, ?, ?, ?, ?>
$h_3$ = <Sunny, ?, ?, ?, Cool, ?>

## *Find-S*: finding the most specific hypothesis

- Exploiting the structure we have alternatives to enumeration …

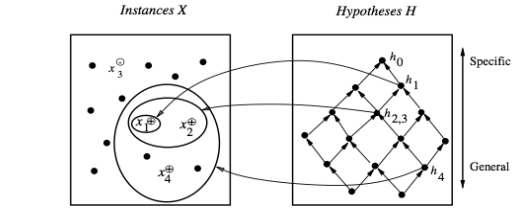
1. Initialize $h$ to the most specific hypothesis in $H$
2. For each positive training instance:

   *for each* attribute constraint $a$ in $h$:

   *If* the constraint $a$ is satisfied by $x$ *then do nothing*

   *else* replace $a$ in $h$ by the next more general constraint satified by $x$ (move towards a more general hp)
3. Output hypothesis $h$

## Find-S in action




$x_1$ = <Sunny Warm Normal Strong Warm Same>, +
$x_2$ = <Sunny Warm High Strong Warm Same>, +
$x_3$ = <Rainy Cold High Strong Warm Change>, -
$x_4$ = <Sunny Warm High Strong Cool Change>, +

$h_0$ = <∅, ∅, ∅, ∅, ∅, ∅>
$h_1$ = <Sunny Warm Normal Strong Warm San
$h_2$ = <Sunny Warm ? Strong Warm Same>
$h_3$ = <Sunny Warm ? Strong Warm Same>
$h_4$ = <Sunny Warm ? Strong ? ? >

## Properties of *Find-S*

- *Find-S* is guaranteed to output the *most specific hypothesis* within $H$ that is consistent with the *positive* training examples
- The final hypothesis will also be consistent with the negative examples
- Problems:
  - There can be more than one "most specific hypotheses"
  - We cannot say if the learner converged to the correct target
  - Why choose the most specific?
  - If the training examples are inconsistent, the algorithm can be mislead: no tolerance to rumor.
  - Negative example are not considered

## Candidate elimination algorithm: the idea

- *The idea*: output a description of the set of *all hypotheses consistent* with the training examples (correctly classify training examples).
- *Version space*: a representation of the set of hypotheses which are *consistent* with $D$
  1. an explicit list of hypotheses (List-Than-Eliminate)
  2. a compact representation of hypotheses which exploits the *more_general_than* partial ordering (Candidate-Elimination)

## Version space

- The version space $VS_{H,D}$ is the subset of the hypothesis from $H$ *consistent* with the training example in $D$

  $VS_{H,D} \equiv \{h \in H \mid Consistent(h, D)\}$
- An hypothesis $h$ is consistent with a set of training examples $D$ iff $h(x) = c(x)$ for each example in $D$

  $Consistent(h, D) \equiv (\forall \langle x, c(x)\rangle \in D)\ h(x) = c(x))$

  Note: "$x$ *satisfies* $h$" ($h(x)=1$) different from "$h$ *consistent with* $x$"

  In particular when an hypothesis $h$ is consistent with a negative example $d = \langle x, c(x)=No\rangle$, then $x$ must not satisfy $h$

## The *List-Then-Eliminate* algorithm

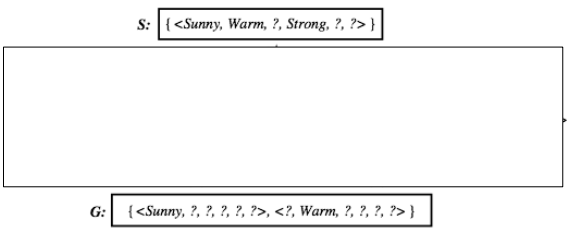
Version space as list of hypotheses

1. $VersionSpace \leftarrow$ a list containing every hypothesis in $H$
2. For each training example, $\langle x, c(x)\rangle$

   Remove from $VersionSpace$ any hypothesis $h$ for which $h(x) \neq c(x)$
3. Output the list of hypotheses in $VersionSpace$

- Problems
  - The hypothesis space must be finite
  - Enumeration of all the hypothesis, rather inefficient

## A compact representation for *Version Space*




*Note*: The output of *Find-S* is just $\langle Sunny, Warm, ?, Strong, ?, ?\rangle$

- Version space represented by its most general members G and its most specific members S *(boundaries)*

## General and specific boundaries

- The *Specific boundary*, S, of version space $VS_{H,D}$ is the set of its minimally general (most specific) members

  $S \equiv \{s \in H \mid Consistent(s, D) \wedge (\neg \exists s' \in H)[(s >_g s') \wedge Consistent(s', D)]\}$

  Note: any member of S is satisfied by all positive examples, but more specific hypotheses fail to capture some
- The *General boundary*, G, of version space $VS_{H,D}$ is the set of its maximally general members

  $G \equiv \{g \in H \mid Consistent(g, D) \wedge (\neg \exists g' \in H)[(g' >_g g) \wedge Consistent(g', D)]\}$

  Note: any member of G is satisfied by no negative example but more general hypothesis cover some negative example

## Version Space representation theorem

- G and S completely define the Version Space
- *Theorem*: Every member of the version space ($h$ consistent with $D$) is in S or G or lies between these boundaries

$$VS_{H,D}=\{h \in H \mid (\exists s \in S)\ (\exists g \in G)\ (g \geq_g h \geq_g s)\}$$

where $x \geq_g y$ means $x$ is more general or equal to $y$

*Sketch of proof*:

⇐ If $g \geq_g h \geq_g s$, since $s$ is in S and $h \geq_g s$, $h$ is satisfied by all positive examples in $D$; $g$ is in G and $g \geq_g h$, then $h$ is satisfied by no negative examples in $D$; therefore $h$ belongs to $VS_{H,D}$

⇒ It can be proved by assuming a consistent $h$ that does not satisfy the right-hand side and by showing that this would lead to a contradiction

## Candidate elimination algorithm-1

$S \leftarrow$ minimally general hypotheses in $H$,

$G \leftarrow$ maximally general hypotheses in $H$

Initially any hypothesis is still possible

$S_0 = \langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing, \varnothing \rangle$ $G_0 = \langle ?, ?, ?, ?, ?, ? \rangle$

For each training example $d$, do:

If $d$ is a *positive* example:

1. Remove from $G$ any $h$ inconsistent with $d$
2. *Generalize*($S$, $d$)

If $d$ is a *negative* example:

1. Remove from $S$ any $h$ inconsistent with $d$
2. *Specialize*($G$, $d$)

Note: when $d = \langle x, \text{No} \rangle$ is a negative example, an hypothesis $h$ is inconsistent with $d$ iff $h$ *satisfies* $x$

## Candidate elimination algorithm-2

*Generalize*($S$, $d$): $d$ is positive

For each hypothesis $s$ in $S$ not consistent with $d$:

1. Remove $s$ from $S$
2. Add to $S$ all minimal generalizations of $s$ consistent with $d$ and having a generalization in $G$
3. Remove from $S$ any hypothesis with a more specific $h$ in $S$

*Specialize*($G$, $d$): $d$ is negative

For each hypothesis $g$ in $G$ not consistent with $d$: i.e. $g$ *satisfies* $d$, but $d$ *is negative*

1. Remove $g$ from $G$
2. Add to $G$ all minimal specializations of $g$ consistent with $d$ and having a specialization in $S$
3. Remove from $G$ any hypothesis having a more general hypothesis in $G$

## Example: initially

$S_0$: $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing. \varnothing \rangle$

$G_0$ $\langle ?, ?, ?, ?, ?, ? \rangle$

## Example:

after seing $\langle$*Sunny,Warm, Normal, Strong, Warm, Same* $\rangle$ +

$S_0$: $\langle \varnothing, \varnothing, \varnothing, \varnothing, \varnothing. \varnothing \rangle$

$S_1$: $\langle$*Sunny,Warm, Normal, Strong, Warm, Same*$\rangle$

$G_0$, $G_1$ $\langle ?, ?, ?, ?, ?, ? \rangle$

## Example:

after seing $\langle$*Sunny,Warm, High, Strong, Warm, Same*$\rangle$ +

$S_1$: $\langle$*Sunny,Warm, Normal, Strong, Warm, Same*$\rangle$

$S_2$: $\langle$*Sunny,Warm, ?, Strong, Warm, Same*$\rangle$

$G_1$, $G_2$ $\langle ?, ?, ?, ?, ?, ? \rangle$

## Example:

after seing ⟨*Rainy, Cold, High, Strong, Warm, Change* ⟩-

$S_2, S_3$: ⟨*Sunny, Warm, ?, Strong, Warm, Same*⟩

$G_3$: ⟨*Sunny, ?, ?, ?, ?, ?*⟩ ⟨*?, Warm, ?, ?, ?, ?*⟩ ⟨*?, ?, ?, ?, ?, Same*⟩

$G_2$: ⟨*?, ?, ?, ?, ?, ?*⟩

## Example:

after seing ⟨*Sunny, Warm, High, Strong, Cool Change* ⟩ +

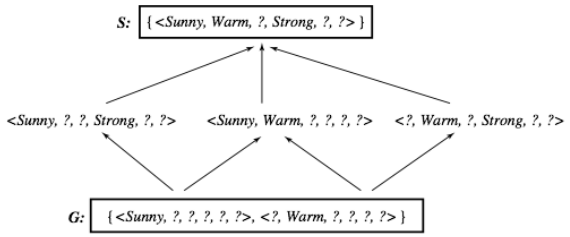
$S_3$: ⟨*Sunny, Warm, ?, Strong, Warm, Same*⟩

$S_4$: ⟨*Sunny, Warm, ?, Strong, ?, ?*⟩

$G_4$: ⟨*Sunny, ?, ?, ?, ?, ?*⟩ ⟨*?, Warm, ?, ?, ?, ?*⟩

$G_3$: ⟨*Sunny, ?, ?, ?, ?, ?*⟩ ⟨*?, Warm, ?, ?, ?, ?*⟩ ⟨*?, ?, ?, ?, ?, Same*⟩

## Learned Version Space



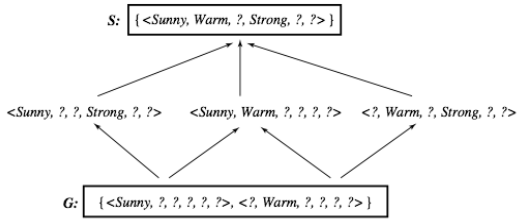

## Observations

- The learned Version Space correctly describes the target concept, provided:
  1. There are no errors in the training examples
  2. There is some hypothesis that correctly describes the target concept
- If *S* and *G* converge to a single hypothesis the concept is exactly learned
- In case of errors in the training, useful hypothesis are discarded, no recovery possible
- An empty version space means no hypothesis in $H$ is consistent with training examples

## Ordering on training examples

- The learned version space does not change with different orderings of training examples
- Efficiency does
- Optimal strategy (if you are allowed to choose)
  - Generate instances that satisfy half the hypotheses in the current version space. For example:
    ⟨*Sunny, Warm, Normal, Light, Warm, Same*⟩ satisfies 3/6 hyp.
  - Ideally the $VS$ can be reduced by half at each experiment
  - Correct target found in $\lceil log_2 |VS| \rceil$ experiments

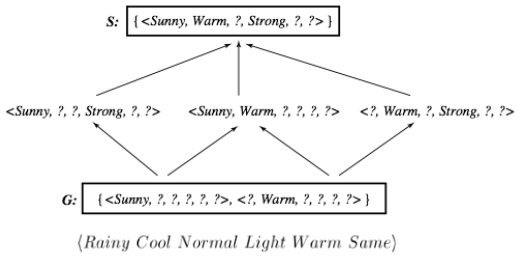
## Use of partially learned concepts




⟨*Sunny Warm Normal Strong Cool Change*⟩

Classified as *positive* by all hypothesis, since satisfies any hypothesis in S

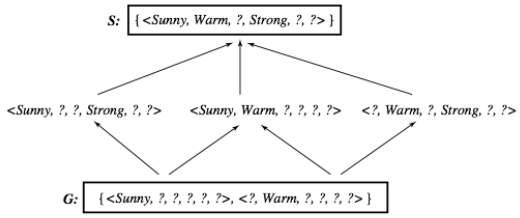
## Classifying new examples




⟨*Rainy Cool Normal Light Warm Same*⟩

Classified as *negative* by all hypothesis, since does not satisfy any hypothesis in G

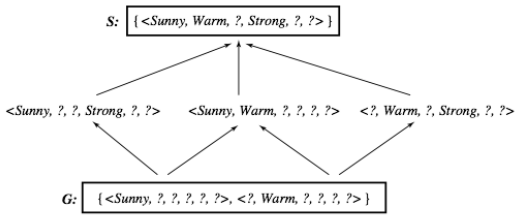
## Classifying new examples




⟨*Sunny Warm Normal Light Warm Same*⟩

Uncertain classification: half hypothesis are consistent, half are not consistent

## Classifying new examples




⟨*Sunny, Cold, Normal, Strong, Warm, Same*⟩

4 hypothesis not satisfied; 2 satisfied
Probably a negative instance. Majority vote?

## Hypothesis space and bias

- What if H does not contain the target concept?
- Can we improve the situation by extending the hypothesis space?
- Will this influence the ability to generalize?
- These are general questions for inductive inference, addressed in the context of Candidate-Elimination
- Suppose we include in H every possible hypothesis … including the ability to represent disjunctive concepts

## Extending the hypothesis space

| | *Sky* | *AirTemp* | *Humidity* | *Wind* | *Water* | *Forecast* | *EnjoyS* |
|---|---|---|---|---|---|---|---|
| 1 | *Sunny* | *Warm* | *Normal* | *Strong* | *Cool* | *Change* | *YES* |
| 2 | *Cloudy* | *Warm* | *Normal* | *Strong* | *Cool* | *Change* | *YES* |
| 3 | *Rainy* | *Warm* | *Normal* | *Strong* | *Cool* | *Change* | *NO* |

- No hypothesis consistent with the three examples with the assumption that the target is a conjunction of constraints
  ⟨?, *Warm, Normal, Strong, Cool, Change*⟩ is too general
- Target concept exists in a different space $H'$, including disjunction and in particular the hypothesis
  *Sky=Sunny* or *Sky=Cloudy*

## An unbiased learner

- Every possible subset of $X$ is a possible target
  $|H'| = 2^{|X|}$, or $2^{96}$ (vs $|H| = 973$, a strong bias)
- This amounts to allowing conjunction, disjunction and negation
  ⟨*Sunny, ?, ?, ?, ?, ?*⟩ V <*Cloudy, ?, ?, ?, ?, ?*⟩
  Sunny(Sky) V Cloudy(Sky)
- We are guaranteed that the target concept exists
- No generalization is however possible!!!
  Let's see why …

## No generalization without bias!

- $VS$ after presenting three positive instances $x_1$, $x_2$, $x_3$, and two negative instances $x_4$, $x_5$
  - $S = \{(x_1 \vee x_2 \vee x_3)\}$
  - $G = \{\neg(x_4 \vee x_5)\}$
  - … all subsets including $x_1 x_2 x_3$ and not including $x_4 x_5$
- We can only classify precisely examples already seen!
- Take a majority vote?
  - Unseen instances, e.g. $x$, are classified positive (and negative) by half of the hypothesis
  - For any hypothesis $h$ that classifies $x$ as positive, there is a complementary hypothesis $\neg h$ that classifies $x$ as negative

## No inductive inference without a bias

- *A learner that makes no a priori assumptions regarding the identity of the target concept, has no rational basis for classifying unseen instances*
- The *inductive bias* of a learner are the assumptions that justify its inductive conclusions or the policy adopted for generalization
- Different learners can be characterized by their bias
- See next for a more formal definition of *inductive bias* …

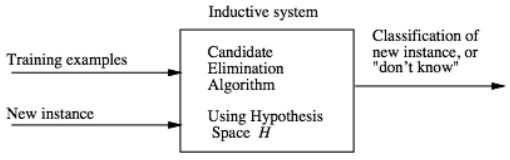
## Inductive bias: definition

- Given:
  - a concept learning algorithm *L* for a set of instances *X*
  - a concept c defined over *X*
  - a set of training examples for c: $D_c = \{\langle x, c(x)\rangle\}$
  - $L(x_i, D_c)$ outcome of classification of $x_i$ after learning
- Inductive inference ( $\succ$ ):
  $D_c \wedge x_i \succ L(x_i, D_c)$
- The *inductive bias* is defined as a minimal set of assumptions ***B***, such that ( $\vdash$ for deduction)
  $\forall (x_i \in X)\ [\ (B \wedge D_c \wedge x_i) \vdash L(x_i, D_c)\ ]$

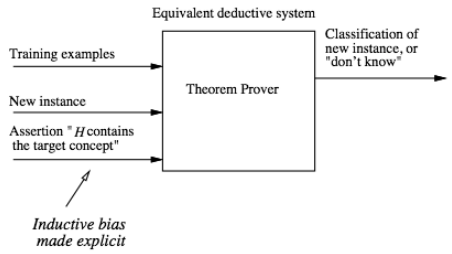
## Inductive bias of Candidate-Elimination

- Assume *L* is defined as follows:
  - compute $VS_{H,D}$
  - classify new instance by complete agreement of all the hypotheses in $VS_{H,D}$
- Then the inductive bias of Candidate-Elimination is simply
  $B \equiv (c \in H)$
- In fact by assuming $c \in H$:
  1. $c \in VS_{H,D}$, in fact $VS_{H,D}$ includes all hypotheses in *H* consistent with D
  2. $L(x_i, D_c)$ outputs a classification "by complete agreement", hence any hypothesis, including c, outputs $L(x_i, D_c)$

## Inductive system




## Equivalent deductive system



## Each learner has an inductive bias

- Three learner with three different inductive bias:
  1. *Rote learner*: no inductive bias, just stores examples and is able to classify only previously observed examples
  2. C*andidateElimination*: the concept is a conjunction of constraints.
  3. *Find-S*: the concept is in *H* (a conjunction of constraints) plus "all instances are negative unless seen as positive examples" (stronger bias)
  - The stronger the bias, greater the ability to generalize and classify new instances (greater inductive leaps).

## Bibliography


- Machine Learning, Tom Mitchell, Mc Graw-Hill International Editions, 1997 (Cap 2).