Errata to *The Tile Model* draft available on the net

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The first version of "The tile model" appeared on the web at the end of December 1997. We could say that it aged gracefully, since it indeed spawned a lot of subsequent research themes (see the additional references [1–7, 9–13], besides those already present in the paper). Nevertheless, a few changes were necessary. Most of them are just typos, with the exception of some of the results in Section 3.2, on which we gained more insights after the publication of [9]. The note is addressed to those already familiar with the work, and it shows the following structure: We point out the parts of the original paper that have to be modified, adding after each of them the modified version.

The updated version of the *The Tile Model* paper, containing all the corrections pointed out here, is also available on the net.

1 On tile functoriality

When we defined *functoriality* in Section 3.2, we thought of it as a straightforward extension on the standard congruence for algebras. It turns out that it is a more difficult, and intriguing, concept than it looks, as shown in [9]. In particular, while decomposition still implies functoriality, it is not true that what we called the *basic source* property implies decomposition, as it is clarified in the next section. We then decided to add the related notion of *algebraicity*, tightly connected to congruence, simply proving that basic source implies it.

Now, we extend Definition 13, in order to include also the new concept of algebraicity. In the first version it is was stated as follows.

Definition 13 (tile functoriality). Let $\mathcal{R} = \langle \Sigma_\sigma, \Sigma_\tau, N, R\rangle$ be an ARS. A symmetric equivalence relation $\cong_{\mathtt{f}} \subseteq \mathbf{A}(\Sigma_\sigma) \times \mathbf{A}(\Sigma_\sigma)$ is *functorial* for $\mathcal{R}$ if, whenever $s \cong_{\mathtt{f}} t, s' \cong_{\mathtt{f}} t'$ for generic s, s', t, t' elements of $\mathbf{A}(\Sigma_\sigma)$, then $s; s' \cong_{\mathtt{f}} t; t'$ (whenever defined) and $s \otimes s' \cong_{\mathtt{f}} t \otimes t'$. ∎

The definition should be updated as follows.

Definition 13 (tile functoriality and algebraicity). Let $\mathcal{R} = \langle \Sigma_\sigma, \Sigma_\tau, N, R\rangle$ be an ARS. An equivalence relation $\cong_{\mathtt{f}} \subseteq \mathbf{A}(\Sigma_\sigma) \times \mathbf{A}(\Sigma_\sigma)$ is *functorial* for $\mathcal{R}$ if for s, s', t, t' elements of $\mathbf{A}(\Sigma_\sigma)$, whenever $s \cong_{\mathtt{f}} t$ and $s' \cong_{\mathtt{f}} t'$, then $s; s' \cong_{\mathtt{f}} t; t'$ (whenever defined) and $s \otimes s' \cong_{\mathtt{f}} t \otimes t'$. It is *algebraic* for $\mathcal{R}$

if for $s_i, t_i : \underline{0} \to \underline{1}$, $i = 1 \ldots n$, elements of $\mathbf{A}(\Sigma_\sigma)$, whenever $s_i \cong_{\mathtt{f}} t_i$, then $(s_1 \otimes \ldots \otimes s_n); f \cong_{\mathtt{f}} (t_1 \otimes \ldots \otimes t_n); f$ for each n-ary operator $f \in \Sigma_\sigma$. □

The decomposition result still holds, but now the stage is taken by the notion of algebraicity. Theorem 16 in the first version was stated as follows.

Theorem 16 (basic components and decomposition). Let $\mathcal{R}$ be the ARS $\langle \Sigma_\sigma, \Sigma_\tau, N, R\rangle$ such that, for all $\alpha : s \xrightarrow[b]{a} t \in R$, $s \in \Sigma_\sigma$ and $b \in \Sigma_\tau$ (hence, both source and effect are just operators). Then $\mathcal{R}$ is decomposable. □

In the updated version we just slightly strengthen the conditions on the vertical signature, thus recovering that the basic source property implies algebraicity.

Theorem 16 (basic components and algebraicity). Let $\mathcal{R} = \langle \Sigma_\sigma, \Sigma_\tau, N, R\rangle$ be an ARS. If Σ_τ contains only unary operators, and for all rules $\alpha : s \xrightarrow[b]{a} t \in R$ we have that $s \in \Sigma_\sigma$, then the strong tile bisimulation associated to $\mathcal{R}$ is algebraic. □

A sketch of the proof may be found in the updated version. The interested reader may find more details in Section 3 of this note.

2 A worthy example

We provide now an example of an ARS satisfying the (updated version of the) basic source condition, that does not verify the decomposition property.

Let us consider the signatures $\Sigma_v = \{x, y, z : 1 \to 1\}$ and $\Sigma_h = \{a, b, c : 0 \to 1; f, g : 2 \to 1\}$, and the ARS $\mathcal{R}_F = \langle \Sigma_h, \Sigma_v, N_F, R_F\rangle$, such that

$$R_F = \{d_x : a \xrightarrow[x]{id} b, d_y : a \xrightarrow[y]{id} c, d_f : f \xrightarrow[z]{x \otimes y} g\}$$

where id is shorthand for id_0. For rules d_x and d_y the intuitive meaning is that the element a can be rewritten either to x or to y; for d_f, f can be rewritten to g, producing an effect z, whenever there exists a rewrite with effect $x \otimes y$.

Now, f is obviously not bisimilar to g, and $f(a, a)$ is not bisimilar to $g(a, a)$. At the same time, $f(x, x)$ *is* bisimilar to $g(x, x)$, since the shared occurrence of the variable x forbids the execution of any possible action in $f(x, x)$.

In general, if we have terms u that may evolve with steps having different effects, the naturality of the ∇ operator allows for situation of the following kind

- terms s and t are not bisimilar;
- terms $\nabla; s$ and $\nabla; t$ are bisimilar;
- terms $u; \nabla; s = (u \otimes u); s$ and $u; \nabla; t = (u \otimes u); t$ are not bisimilar.

In more general terms, this is basically related to the *lack of substitutivity*, a problem arising e.g. in connection with non-deterministic algebraic specifications, and often solved introducing a suitable notion of "sharing" (see [8] for a recollection). Then, algebraicity simply asks a *closure* property (with respect to contexts) for the bisimulation equivalence. An alternative proposal would be to require that bisimulation enjoys substitutivity *only with respect to linear contexts*, that is, with respect to elements of $\mathbf{M}(\Sigma_h)$, ruling out the bisimilar, yet not linear terms $f(x,x)$ and $g(x,x)$.

3 A detailed proof

This section is devoted to the proof of Theorem 16. It is split in a series of technical lemma, which appear in a seriously abridged form in the sketch of the same proof in the updated version of the paper. Thus, in the following, we assume an ARS $\mathcal{R} = \langle \Sigma_\sigma, \Sigma_\tau, N, R \rangle$ satisfying the basic components property.

Lemma 1 (arrow decomposition). *The terms of the two theories enjoy the following properties.*

- *Any arrow* $s : \underline{m} \to \underline{n} \in \mathbf{A}(\Sigma_\sigma)$ *can be uniquely factored as* $s_\nabla; (s_1 \otimes \ldots \otimes s_n)$, *for* $s_i : \underline{m}_i \to \underline{1} \in \mathbf{M}(\Sigma_\sigma)$ *and* $s_\nabla : \underline{m} \to \underline{m}_1 \otimes \ldots \otimes \underline{m}_n$ *a* functional *(namely, an arrow obtained composing only identities, duplicators, dischargers and permutations).*
- *The source and target of any arrow* $a \in \mathbf{M}(\Sigma_\tau)$ *coincide, and the arrow can be factored as* $a_1 \otimes \ldots \otimes a_p$ *for* $a_i : \underline{1} \to \underline{1}$ *and* $p \in \mathbb{N}$.

Both properties are proved by structural induction. In particular, the second property depends on the fact that Σ_τ contains only unary operators. It implies that, for any sequent $\alpha : s \xrightarrow[b]{a} t$ entailed by $\mathcal{R}$, such that $s : \underline{m} \to \underline{n} \in \mathbf{A}(\Sigma_\sigma)$, then $t : \underline{m} \to \underline{n}$, $a : \underline{m} \to \underline{m}$ and $b : \underline{n} \to \underline{n}$.

Proposition 1 (cell decomposition, I). *Let* α *be a sequent entailed by* $\mathcal{R}$, *with initial configuration* $s : \underline{m} \to \underline{n} \in \mathbf{A}(\Sigma_\sigma)$. *Then, it can be uniquely factored as* $\alpha_\nabla; (\alpha_1 \otimes \ldots \otimes \alpha_n)$ *for* α_∇ *a functional sequent (namely, a sequent obtained composing only identity, duplicator, discharger and permutation cells) and* α_i*'s monoidal sequents (namely, sequents obtained composing only basic and composition rules), with respective initial configurations* $s_i : \underline{m}_i \to \underline{1} \in \mathbf{M}(\Sigma_\sigma)$.

Proof. The proof goes by structural induction. Note that the base case is immediately verified, since the initial configuration of any rule must be an operator.

So, as far as horizontal composition is concerned, let us consider $\alpha * \beta$: They can be factored, by induction hypothesis, as $\alpha = \alpha_\nabla; (\alpha_1 \otimes \ldots \otimes \alpha_p)$ and $\beta = \beta_\nabla; (\beta_1 \otimes \ldots \otimes \beta_n)$. By naturality, $(\alpha_1 \otimes \ldots \otimes \alpha_p) * \beta_\nabla$ can be factored as $\alpha'_\nabla * (\alpha'_1 \otimes \ldots \otimes \alpha'_q)$, such that for each j, $\alpha'_j = \alpha_i$ for some i. Then, the result follows by inductive hypothesis.

Vertical composition is proved similarly. Let us consider $\alpha \cdot \beta$: They can be factored, by induction hypothesis, as $\alpha = \alpha_\nabla; (\alpha_1 \otimes \ldots \otimes \alpha_n)$ and $\beta = \beta_\nabla; (\beta_1 \otimes \ldots \otimes \beta_n)$ with initial configurations factored as $s = s_\nabla; (s_1 \otimes \ldots \otimes s_n)$ and $t = t_\nabla; (t_1 \otimes \ldots \otimes t_n)$, respectively, according to (2) of Lemma 1. The final configuration of α must then be $u = s_\nabla; (u_1 \otimes \ldots \otimes u_n)$, with u_i's not necessarily in $\mathbf{M}(\Sigma_\sigma)$. They can be however factored as $((u_i)_\nabla); u_i'$, for $u_i' \in \mathbf{M}(\Sigma_\sigma)$, in such a way that $u_i' = t_i$, and then the result holds. $\square$

In general, if $\alpha : s \xrightarrow[b]{a} t$ is a monoidal sequent entailed by $\mathcal{R}$, then the basic components property ensures us that $s \in \mathbf{M}(\Sigma_\sigma)$. Moreover, if α is a *coordinating* sequent, namely, a monoidal sequent obtained without using the (*vert*) rule, then $a, b \in G(\Sigma_\tau)$.

Proposition 2 (cell decomposition, II). *Let α be a monoidal sequent entailed by $\mathcal{R}$. Then, it can be factored as $\alpha_1 \cdot \ldots \cdot \alpha_n$, for α_i's coordinating sequents.*

Proof. The proof goes by structural induction. The base case is immediately verified, as well as the parallel and vertical composition.

So, as far as horizontal composition is concerned, let us consider $\alpha * \beta$, with initial configurations factored as $s = s_1 \otimes \ldots \otimes s_p$ and $t = t_1 \otimes \ldots \otimes t_n$, respectively, according to (2) of Lemma 1. By Proposition 1, they can be factored as $\alpha_1 \otimes \ldots \otimes \alpha_p$ and $\beta_1 \otimes \ldots \otimes \beta_n$, for α_i's and β_i's monoidal sequents. By induction hypothesis, they can also be factored as $\alpha_i^1 \cdot \ldots \cdot \alpha_i^h$ and $\beta_j^1 \cdot \ldots \cdot \beta_j^k$. So, let us consider the initial levels $\alpha_1^1 \otimes \ldots \otimes \alpha_p^1$ and $\beta_1^1 \otimes \ldots \otimes \beta_n^1$, with trigger a_j^1's and effect b_i^1's, respectively. The factorisations for the sequents implies that each b_i^1 can be factored as $a_j^1 \otimes \ldots \otimes a_{j+l}^1$ for some j, k, and then the result holds. The only exception to such a factorisation for b_i^1 is that for some l, a_l^i is the identity, while the component needed by b_i^1 is actually at a deeper level. It is then enough to "shift upward" the α_l, adding an identity sequent to all the other components of α. $\square$

Note that the proposition does not hold, if α is not monoidal. Moreover, note also that the ruling out of identity $id_{\underline{1}}$ as the possible effect of a rule, as required in the original version of the basic decomposition property, is actually not necessary. Let us consider once more the signatures $\Sigma_h = \{a, b, c : 0 \to 1; f, g : 2 \to 1\}$ and $\Sigma_v = \{x, y, z : 1 \to 1\}$, and the ARS $\mathcal{R}_A = \langle \Sigma_h, \Sigma_v, N_F \cup N_A, R_F \cup R_A \rangle$, for

$$R_A = \{d_i : c \xrightarrow[id]{id} a\},$$

where id is shorthand for both $id_{\underline{0}}$ and $id_{\underline{1}}$. Then, the sequent $((d_x \otimes d_i) \cdot (id^b \otimes d_y)) * d_f : (a \otimes c); f \xrightarrow[z]{id} (b \otimes b); f$ can be factored, as sketched in the previous proof, as $((id^a \otimes d_i) * id^f) \cdot ((d_x \otimes d_y) * d_f) \cdot ((id^b \otimes id^b)) * id^g$.

Proposition 3 (cell decomposition, III). *Let α be a coordinating sequent entailed by $\mathcal{R}$, with initial configuration $s : \underline{m} \to \underline{n} \in \mathbf{M}(\Sigma_\sigma)$. Then, for any factorisation $(s_1 \otimes \ldots \otimes s_l); s'$ of s, such that $s' \in \mathbf{M}(\Sigma_\sigma)$, α can be decomposed as $(\alpha_1 \otimes \ldots \otimes \alpha_l) * \alpha'$, for α_i's and α' coordinating sequents.*

Proof. First, note that, since α is a coordinating sequent, then all s_i's are elements of $\mathbf{M}(\Sigma_\sigma)$. The proof goes by structural induction. The base case is immediately verified, since rules satisfy the basic components property. Also parallel composition is immediate, thanks to the factorisation of Proposition 1.

So, as far as horizontal composition is concerned, let us consider $\alpha * \beta$, with initial configurations factored as $s = s_1 \otimes \ldots \otimes s_p$ and $t = t_1 \otimes \ldots \otimes t_n$, respectively. Let us then consider a factorisation $s; t = (u_1 \otimes \ldots \otimes u_m); v$, such that $v \in \mathbf{M}(\Sigma_\sigma)$: It must be that $v = v_1 \otimes \ldots \otimes v_n$.

Now, note the two factorisations must be equated by the functoriality and/or the identity axioms. This implies that either $v_i = (s''_j \otimes \ldots \otimes s''_{j+k}; t_i$ and for each $l, l = j, \ldots, j+k, u_l = s'_l$ and $s_l = s'_l; s''_l$; or similarly, that $t_i = (u''_j \otimes \ldots \otimes u''_{j+k}; v_i$ and for each $l, l = j, \ldots, j+k$, $s_l = u'_l$ and $u_l = u'_l; u''_l$. By induction hypothesis, $(s'_1 \otimes \ldots s'_p) * (s''_1 \otimes \ldots s''_p)$ (and similarly for t) can be factored *via* suitable α'_i's and α''_i', and then the result holds (since each sequent with initial configuration $id_{\underline{n}}$ for some n, can be proved equivalent to the identity) by a suitable reshuffling of the components. ∎

Note that the previous result does not hold, if the sequent α is monoidal but eventually not coordinating. This is due to the fact that the basic components property allows the final configuration of a rule to contain a functional part.

We state now a stronger property than algebraicity.

Definition 1 (contextuality). *Let $\mathcal{R} = \langle \Sigma_\sigma, \Sigma_\tau, N, R \rangle$ be an* ARS. *An equivalence relation $\cong_{\mathtt{f}} \subseteq \mathbf{A}(\Sigma_\sigma) \times \mathbf{A}(\Sigma_\sigma)$ is* contextual *for $\mathcal{R}$ if for $s_i, t_i : \underline{0} \to \underline{1}$, $i = 1 \ldots n$, elements of $\mathbf{M}(\Sigma_\sigma)$, whenever $s_i \cong_{\mathtt{f}} t_i$, then $(s_1 \otimes \ldots \otimes s_n); t \cong_{\mathtt{f}} (t_1 \otimes \ldots \otimes t_n); t$ for each $t \in \mathbf{A}(\Sigma_\sigma)$.*

Of course, contextuality implies algebraicity. We can then state and prove the following, stronger version of Theorem 16.

Theorem 16bis (basic components and contextuality). Let $\mathcal{R} = \langle \Sigma_\sigma, \Sigma_\tau, N, R \rangle$ be an ARS. If Σ_τ contains only unary operators, and for all rules $\alpha : s \xrightarrow[b]{a} t \in R$ we have that $s \in \Sigma_\sigma$, then the strong tile bisimulation associated to $\mathcal{R}$ is contextual.

Proof. It suffices to prove that $\equiv_c = \{\langle (s_1 \otimes \ldots \otimes s_n); u, (t_1 \otimes \ldots \otimes t_n); u \rangle \mid s_i \equiv_b t_i\}$ is a bisimulation. The case where t is just a functional is immediate, thanks to the factorisation of Proposition 1. Then, we can safely assume that $t \in \mathbf{M}(\Sigma_\sigma)$.

Then, let α be a sequent entailed by $\mathcal{R}$, with initial configuration $(s_1 \otimes \ldots s_n); t$. Proposition 1 implies that α is a monoidal sequent, and by Proposition 2 it can be factored into the vertical composition of coordinating sequents. We now proceed by induction on the length of such a sequence.

So, for the base case, let α be a coordinating sequent. By Proposition 3 it can be factored as $(\alpha_1 \otimes \ldots \otimes \alpha_n) * \alpha'$, for α_i's and α' coordinating sequents, with final configuration s'_i, t'. By hypothesis, there exists $\beta_1, \ldots, \beta_n$ with initial configuration t_i and final configuration t'_i, bisimilar to s'_i. We then just need to prove that $(s'_1 \otimes \ldots s'_n); t'$ is bisimilar to $(t'_1 \otimes \ldots t'_n); t'$. But this is immediately verified by coinductive hypothesis (even if t' is not necessarily an element of $\mathbf{M}(\Sigma_\sigma)$). Moreover, if $t' = t'_\nabla; t''$, then $(s'_1 \otimes \ldots s'_n); t' = (s''_1 \otimes \ldots s''_p); t''$ such that for each j, $s''_j = s'_i$ for some i, and the the result holds for any possible sequence of vertical components of α. $\square$

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