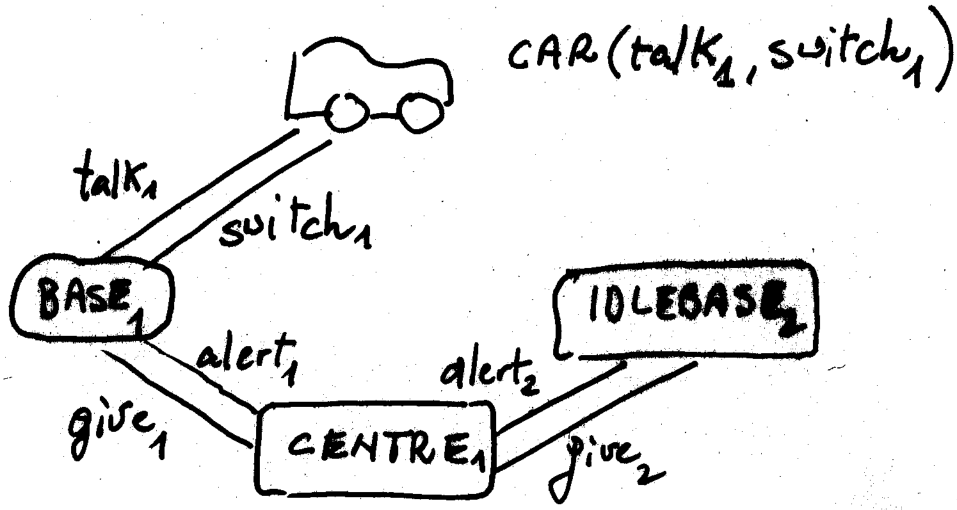
Mobile Telephones in π-calculus



$$SYSTEM_1 \overset{def}{=} \nu\, talk_i, switch_i, give_i, alert_i : i = 1,2$$
$$(CAR(talk_1, switch_1) \mid BASE_1 \mid IDLEBASE_2 \mid CENTRE_1)$$

$$CAR(talk, switch) \overset{def}{=} \overline{talk}.CAR(talk, switch) +$$
$$+\; switch(talk', switch').CAR(talk', switch')$$

$$BASE_i \overset{def}{=} BASE(talk_i, switch_i, give_i, alert_i) \overset{def}{=}$$
$$talk_i.BASE_i + give_i(t's').\overline{switch_i}\, t's'.IDLEBASE_i$$

$$IDLEBASE_i \overset{def}{=} IDLEBASE(talk_i, switch_i, give_i, alert_i) \overset{def}{=}$$
$$alert_i.BASE_i$$

$$CENTRE_1 \overset{def}{=} \overline{give_1}\, talk_2\, switch_2 . \overline{alert_2}.CENTRE_2$$
$$CENTRE_2 \overset{def}{=} \overline{give_2}\, talk_1\, switch_1 . \overline{alert_1}.CENTRE_1$$

$$x(y_1 \cdots y_n) = x(w).w(y_1). \cdots .w(y_n)$$
$$\overline{x}\, y_1 \cdots y_n = (\nu w)\, \overline{x}w.\overline{w}y_1. \cdots .\overline{w}y_n$$

Operational semantics of the π-calculus

$$p ::= nil \mid \alpha p \mid [x=y]p \mid p+p \mid p|p \mid (y)p \mid\ !p$$

$$\alpha ::= \tau \mid x(y) \mid \bar{x}y$$

$$\tau p \xrightarrow{\tau} p \qquad x(y)p \xrightarrow{x(w)} p\{w/y\} \quad w \notin fn((y)p) \qquad \bar{x}y\,p \xrightarrow{\bar{x}y} p$$

$$\frac{p \xrightarrow{\alpha} p'}{[x=x]\,p \xrightarrow{\alpha} p'} \qquad \frac{p \xrightarrow{\alpha} p'}{p+q \xrightarrow{\alpha} p'} \qquad \frac{p \xrightarrow{\alpha} p'}{p|q \xrightarrow{\alpha} p'|q} \quad bn(\alpha) \cap fn(q) = \phi$$

+simm

$$\frac{p \xrightarrow{\bar{x}y} p' \quad q \xrightarrow{x(z)} q'}{p|q \xrightarrow{\tau} p'|q'\{y/z\}} \qquad \frac{p \xrightarrow{\alpha} p'}{(y)p \xrightarrow{\alpha} (y)p'} \quad y \notin n(\alpha)$$

$$\frac{p \xrightarrow{\bar{x}y} p' \quad y \neq x}{(y)p \xrightarrow{\bar{x}(w)} p'\{w/y\}} \qquad \frac{p \xrightarrow{\bar{x}(w)} p' \quad q \xrightarrow{x(w)} q'}{p|q \xrightarrow{\tau} (w)(p'|q')}$$

$$\frac{p\,|\,!p \xrightarrow{\alpha} p'}{!p \xrightarrow{\alpha} p'}$$

open close

Example

$$\cfrac{\cfrac{\cfrac{\bar{x}y.p \xrightarrow{\bar{x}y} p}{(y)\bar{x}y.p \xrightarrow{\bar{x}(v)} p\{v/y\}} \quad v \notin fn((y)p)}{((y)\bar{x}y.p)|q \xrightarrow{\bar{x}(v)} p\{v/y\}|q} \quad v \notin fn(q) \qquad \cfrac{v \notin fn((z)r)}{x(z).r \xrightarrow{x(v)} r\{v/z\}}}{(((y)\,\bar{x}y.p)|q)|x(z).r \xrightarrow{\tau} (v)((p\{v/y\}|q)|r\{v/z\})}$$

Early bisimulations for the π-calculus

symmetric relation S is a strong early ground bisimulation iff pSq implies

if $p \xrightarrow{\alpha} p'$ with $\alpha \neq x(y)$ and $bn(\alpha) \notin fn(p,q)$ then for some q', $q \xrightarrow{\alpha} q'$ and $p'Sq'$

if $p \xrightarrow{x(y)} p'$ with $y \notin fn(p,q)$ then for all w there exists q' such that $q \xrightarrow{x(y)} q'$ and $p'\{w/y\} \, S \, q'\{w/y\}$

$p \dot{\sim}_E q$ iff pSq for some segb S

Symmetric relation S is a weak early ground bisimul. iff pSq implies

if $p \xrightarrow{\alpha} p'$ with $\alpha \neq x(y)$ and $bn(\alpha) \notin fn(p,q)$ then for some q', $q \stackrel{\hat{\alpha}}{\Longrightarrow} q'$ and $p'Sq'$

if $p \xrightarrow{x(y)} p'$ with $y \notin fn(p,q)$ then for all w there exists q' such that $q \Rightarrow \xrightarrow{x(y)} \Rightarrow q'$ and $p'\{w/y\} \, S \, q'\{w/y\}$

$p \dot{\approx}_E q$ iff pSq for some wegb S

$p \dot{\simeq}_E q$ iff $p \dot{\approx} q$ and $p \xrightarrow{\tau} p'$ implies $q \stackrel{\tau}{\Longrightarrow} q'$ and $p' \dot{\approx} q'$

- $p \sim_E q$ strong early bisimilar iff $\forall \sigma$, $p\sigma \dot{\sim}_E q\sigma$

$p \simeq_E q$ weak late bisimilar iff $\forall \sigma$, $p\sigma \dot{\simeq}_E q\sigma$

remaining $\sim_E$ and $\dot{\simeq}_E$ are congruences

Directly with additional transitions $p \xrightarrow{[x/y]} p[x/y]$

Late

Bisimulations for the π-calculus

Symmetric relation S is a strong late ground bisimulation iff pSq implies

if $p \xrightarrow{\alpha} p'$ with $\alpha \neq x(y)$ and $bn(\alpha) \notin fn(p,q)$ then
for some q', $q \xrightarrow{\alpha} q'$ and $p' S q'$

if $p \xrightarrow{x(y)} p'$ with $y \notin fn(p,q)$, then
for some q', $q \xrightarrow{x(y)} q'$ and for all w $p'\{w/y\} S q'\{w/y\}$

$p \dot\sim_L q$ iff pSq for some slgb S

Symmetric relation S is a weak late ground bisimulation iff pSq implies

if $p \xrightarrow{\alpha} p'$ with $\alpha \neq x(y)$ and $bn(\alpha) \notin fn(p,q)$ then
for some q' $q \stackrel{\hat\alpha}{\Longrightarrow} q'$ and $p' S q'$

if $p \xrightarrow{x(y)} p'$ with $y \notin fn(p,q)$, then
for some q' $q \Longrightarrow \xrightarrow{x(y)} q'$ and for all w $p'\{w/y\} S q'\{w/y\}$

$p \dot\approx_L q$ iff pSq for some wlgb S

$p \dot\simeq_L q$ iff $p \dot\approx q$ and $p \xrightarrow{\tau} p'$ implies $q \stackrel{\tau}{\Longrightarrow} q'$ and $p' \dot\approx q'$

- $p \sim_L q$ strong late bisimilar iff $\forall \sigma.\ p\sigma \dot\sim_L q\sigma$

$p \simeq_L q$ weak late bisimilar iff $\forall \sigma.\ p\sigma \dot\simeq_L q\sigma$

Relations $\sim_L$ and $\simeq_L$ are congruences

Example (ground bisimulations are not congruences)

$$[x=y]\,\bar{x}x.\,nil \quad \begin{matrix} \dot{\sim}_E & \dot{\sim}_L \\ \dot{\approx}_E & \dot{\approx}_L \\ \dot{\simeq}_E & \dot{\simeq}_L \end{matrix} \quad nil$$

$\bar{x}x.\,nil \mid x(y).\,nil$ $\qquad$ $\bar{x}x.\,x'(y).\,nil + x'(y).\,\bar{x}x\,nil$

but

$$x(y).\,[x=y]\,\bar{x}x.\,nil \quad \begin{matrix} \not\dot{\sim}_E & \not\dot{\sim}_L \\ \not\dot{\approx}_E & \not\dot{\approx}_L \\ \not\dot{\simeq}_E & \not\dot{\simeq}_L \end{matrix} \quad x(y).\,nil$$

matching not essential.

Example

$$x(y).\tau.\,nil + x(y)\,nil = P$$

$$\dot{\sim}_E \text{ but } \not\dot{\sim}_L$$

$$P + x(y).\,[y=z].\tau.\,nil$$

Theorem:

Late bisimulations are strictly stronger than early bisim.

Categorical Semantics of π-Calculus

CCS : algebra + coalgebra $\Rightarrow$ bialgebra

π-calculus : names/name generation in addition

Some problems

$$\frac{p \xrightarrow{\alpha} p'}{p|q \xrightarrow{\alpha} p'|q} \quad bn(\alpha) \cap fn(q) = \phi \quad \text{is not Definmove}$$

$p \, S \, q$ implies $p \xrightarrow{\alpha} p'$ with $bn(\alpha) \cap fn(q) = \phi$ ...
this does not corresponds to a coalgebra

States with names

FM sets sets equipped with actions as
Fraenkel-Mostowski permutations on natural numbers

FM sets are algebras with finite-kernel permutations
as operations

$Autf = \{\pi : \omega \to \omega\}$ $\quad \{x \in \omega \mid \pi(x) \neq x\}$ the kernel

monadic operations $\quad id(x) = x$
with axioms $\quad \pi_1(\pi_2(x)) = (\pi_1 \circ \pi_2)(x)$ $\quad \int \pi_\pi$

Permutation algebras $\quad \langle A, \{\pi_A : A \to A \mid \pi \in Autf\} \rangle$

$$orb_A(a) = \{\pi_A(a) \mid \pi \in Autf\}$$

(2)

<u>Symmetry of an element of a Perm. Algebra</u>

$G_A(a) = \{\pi \in \mathrm{Aut}\, f \mid \pi_A(a) = a\}$

identity group of $X \subseteq \omega$

$$fix(X) = \{\pi \in \mathrm{Aut}\, f \mid \pi|_X = id_X\}$$

X supports a iff $fix(X) \subseteq G_A(a)$

$supp_A(a)$ is the minimal such X

$\underline{Alg}^{\pi}$ $\underline{FSAlg}^{\pi}$ are categories of algebras

There is a theory morphism (injective!) $-^{+1} : \Gamma \to \Gamma$

$$\pi^{+1}(i) = \begin{cases} 0 & i = 0 \\ \pi(i)+1 & \text{otherwise} \end{cases}$$ right shift operator

$\delta : Alg^{\pi} \to Alg^{\pi}$ is the forgetful functor | name abstraction

associated to the right shift morphism

<u>Syntax of λ calculus : De Bruijn indexes</u>

$$\lambda x.\lambda y.(x\,y) \equiv \lambda.\lambda.(1\,0)$$

$L ::= \lambda x.L \mid LL \mid x$

$$\begin{tikzcd} X \\ T(X) \arrow[u] \end{tikzcd}$$

$$T(X) = \delta(X) + X \times X + \omega$$

Fixpoint $\bar{X} \simeq T(\bar{X})$ is the initial algebra

Similarly for the π-calculus

$$TX = \underbrace{K}_{\text{inaction}} + \underbrace{w\times w\times X}_{\text{output}} + \underbrace{w\times \delta X}_{\text{input}} + \underbrace{X\times X}_{\text{parallel}} + \underbrace{\delta X}_{\text{restriction}} + \cdots$$

Initial algebra Z_π

Now, coalgebras! Where?

First option: coalgebras in $\underline{Alg}^{\pi}$!

$$G(X) = \mathcal{P}_f\left(L\times X + L'\times \delta X\right)$$

transitions may allocate fresh names. LTS in Set can be lifted

Second option: coalgebras in Alg_π

Bisim is not a congruence $\Rightarrow$ no lifting possible

Fix #1: Eliminate prefix

$\bar{x}y.P + z(w).Q$ is a constant, with De Simone axioms
combines well with recursion.

Fix #2: Move from Alg_π to Alg_{fusion}
Additional transitions $p \xrightarrow{[x/y]} p[x/y]$
or equivalent: $p[x/y] \xrightarrow{\mu} q \Rightarrow p \xrightarrow{[x/y],\mu} q$

History-Dependent Automata

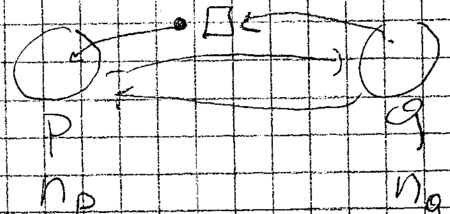
$$\frac{p \xrightarrow{\overline{x}y} q}{(y)\, p \xrightarrow{\overline{x}(z)} q[z/y]}$$

infinite branching
chose one for finite state.
different than all the others.

$p \xrightarrow{\overline{x}(y)}$
$q \xrightarrow{\overline{x}(y)}$ ⇒ choose the same z !

OK on the fly, does not work for minimization.

HD automata local names



injection $n_q \to n_p \cup \{*\}$

new name

Bisimulation : set of states with partial bijection
Group of symmetries : bisimilar to itself
with a p. bijection different from id.

Coalgebraic theory based on Named Sets

$H = \langle Q_N, S_N \rangle$

set : $Q_N \to$ Symset ← cat of groups

Named sets are compact representations of orbits.

Coalgebraic category in named set ≅ in Alg^{π}
but smaller objects