Models of Computation

Written Exam on July 12, 2010

(First part: Exercises 1 and 2 — Second part: Exercises 3 and 4)

Exercise 1 (8)

Let $w = \textbf{while } x \neq 0 \textbf{ do } (x := x - 1\ ;\ y := y + k)$. Prove by computational induction that $\mathcal{C}[\![w]\!]\sigma = \sigma'$ implies $\sigma'(x) = 0, \sigma'(y) = \sigma(y) + k\sigma(x)$.

Exercise 2 (7)

Let us define on the terms t with the binary operation symbol f and the unary symbol c the relation $\sqsubseteq$ as follows:

$$c \sqsubseteq t \qquad \frac{t_1 \sqsubseteq t_2 \quad t_1' \sqsubseteq t_2'}{f(t_1, t_1') \sqsubseteq f(t_2, t_2')}$$

e.g. $f(c, f(c, c)) \sqsubseteq f(f(c, c), f(c, c))$, but $f(c, f(c, c)) \not\sqsubseteq f(f(c, c), c.)$. Prove by structural/rule induction that $\sqsubseteq$ is a partial ordering with $\perp$, but show with a counterexample that it is not complete.

Exercise 3 (7)

The following equations hold in the λ-calculus we use as a metalanguage for the denotational semantics of HOFL:

$$\pi_1(x, y) = x \quad \pi_2(x, y) = y \quad (\pi_1(x), \pi_2(x)) = x.$$

Show if and when the three of them hold also at the language level, e.g. $[\![fst(x, y)]\!]\rho = [\![x]\!]\rho$, etc.

Exercise 4 (8)

Consider the PEPA program B with

$$A = (\alpha, \lambda).B + (\alpha, \lambda).C \quad B = (\alpha, \lambda).A + (\alpha, \lambda).C \quad C = (\alpha, \lambda).B.$$

and derive the corresponding finite state CTMC. What is the probability distribution of staying in B? If $\lambda = 0.1\ sec^{-1}$, what it the probability that the system be still in B after 10 seconds? Are there bisimilar states? Finally, to study the steady state behavior of the system, introduce the self loops, decorated with negative rates which are negated apparent rates, namely the negated sums of all the outgoing rates, show that the system is ergodic and write and solve a system of linear equations similar to the one seen for DTMC: $\vec{s} \times M = \vec{0}$ and $\sum_i s_i = 1$.