Models of Computation

Written Exam on June 21, 2010

(First part: Exercises 1 and 2 Second part: Exercises 3 and 4)

Exercise 1 (8)

Extend **IMP** with the new iterative command **when** b **do** c_0 **exit** c_1 with a semantics similar to **while** b **do** c_0 but such that when the guard is false command c_1 must be executed before exiting.

Give the (i) operational and (ii) denotational semantics of the new command and the proof of their equivalence. Finally, prove using the denotational semantics that **when** b **do** c_0 **exit** c_1 is equivalent to (**while** b **do** c_0) ; c_1. (Hint: prove that $\mathcal{C}[\![c_1]\!]^*(\Gamma^n(\bot)\sigma) = \Gamma'^n(\bot)\sigma$. Then apply continuity.)

Exercise 2 (7)

Equivalence relations on a set $\mathcal{S}$ have the usual properties of reflexivity, symmetry and transitivity. Prove that they form under inclusion a complete partial ordering with bottom. Consider also *partitions* $P = \{S_i\}$ on $\mathcal{S}$, namely sets of nonempty subsets $S_i \subseteq \mathcal{S}$ with the property that they are nonintersecting and that their union returns $\mathcal{S}$. Express the bijective correspondence between equivalence relations and partitions and show what is the ordering between partitions which corresponds to inclusion between equivalence relations.

Exercise 3 (8)

Modify the denotational semantics of HOFL by restricting the use of the *lifting* domain construction only to integers, namely $V_{int} = N_\bot$ but $V_{\tau_1 * \tau_2} = V_{\tau_1} \times V_{\tau_2}$ and similarly for functions. List all the modified clauses of the denotational semantics and prove that $t \to c$ implies $[\![t]\!]\rho = [\![c]\!]\rho$. Finally prove that it is not true that $t \to c$ implies $[\![t]\!]\rho \neq \bot$. (Hint: consider the HOFL program $t\colon int \to int = rec\ f.\lambda x.(f\ x)$.

Exercise 4 (7)

Prove that strong early ground bisimilarity is a congruence for restriction of π-calculus. Distinguish the case of input action. Assume that if R is a bisimulation, also $R' = \{(\sigma(x), \sigma(y)) | (x, y) \in R\}$ is a bisimulation, where σ is a one-to-one renaming.