# Models of Computation 

## Written Exam on May 24, 2010

## Exercise 1 (8 punti)

Modify the ordinary HOFL semantics by defining the denotational semantics of the conditional construct as follows

$$
\begin{aligned}
\llbracket \text { if } t_{0} \text { then } t_{1} \text { else } t_{2} \rrbracket \rho= & \operatorname{Condd}\left(\llbracket t_{0} \rrbracket \rho, \llbracket t_{1} \rrbracket \rho, \llbracket t_{2} \rrbracket \rho\right) \text { where } \\
\operatorname{Condd}\left(z_{0}, z_{1}, z_{2}\right)= & z_{1} \text { if } z_{0}=\lfloor 0\rfloor \text { oppure } z_{1}=z_{2} \\
& z_{2} \text { if } z_{0}=\lfloor n\rfloor, n \neq 0 \\
& \perp \text { otherwise. }
\end{aligned}
$$

Assume that $t_{1}, t_{2}$ : int. Prove that Condd is a monotonic, continuous function, show a HOFL term with a different semantics than the ordinary, and explain how the relation between operational and denotational semantics of HOFL is actually changed.

## Exercise 2 (6 punti)

Prove with a counterexample that the $\tau$-law $\quad \mu \cdot \tau \cdot p=\mu \cdot p$, which holds for CCS weak observational congruence, does not hold for dynamic bisimilarity.

## Exercise 3 (8 punti)

Consider the $\pi$-calculus structural axiom

$$
(x)(p \mid q) \equiv p \mid(x) q \text { if } x \notin f n(p)
$$

and show that the left side $L=(x)(p \mid q)$ and the right side $R=p \mid(x) q$ bisimulate. Namely, given the moves $p \xrightarrow{\mu^{\prime}} p^{\prime}$ and $q \xrightarrow{\mu^{\prime \prime}} q^{\prime}$, prove that for every transition $L \xrightarrow{\mu} L^{\prime}$ there exist a transition $R \xrightarrow{\mu} R^{\prime}$ with $L^{\prime} \equiv R^{\prime}$ according to the axiom, and viceversa. (Hint: proceed goal-oriented in all possible ways from $(x)(p \mid q) \xrightarrow{\mu} r$ and $p \mid(x) q \xrightarrow{\mu} r^{\prime}$ and match the proofs).

## Exercise 4 (8 punti)

In the notes, the CTMC notion of bisimilarity has been defined for unlabeled TSs, while PEPA TS is labeled. Extend the definition of bisimilarity to the labeled version.


Define two PEPA processes for the two TS in the figure above (assume all the transitions as decorated by the same label and disregard the self loop on $E_{31}$ : why?) and compute iteratively the bisimilarity relation as the fixpoint of function $\Phi$.

