Models of Computation

Written Exam on May 31, 2010

(First part: Exercises 1 and 2 Second part: Exercises 3 and 4)

Exercise 1 (7)

Extend **IMP** in two ways: (i) introduce statements for typed expressions and commands of the form $a\colon X$, $b\colon X$, and $c\colon X$, where X is a set of locations, which, for well-typed expressions and commands, represents the locations appearing in them; (ii) introduce a new *parallel* construct $c ::= c_0|c_1$, where c is well typed only if the locations of c_0 and c_1 are disjoint, and where c_0 and c_1 can be executed in any order.

Give: (a) the typing rules for all the constructs; (b) the operational and (c) the denotational semantics of the new construct. Finally prove the operational $\leftrightarrow$ denotation equivalence for it.

Exercise 2 (8)

Given a cpo $\mathcal{D} = (D, \sqsubseteq_D)$, let $\mathcal{R} = \{(A, R) \mid A \subseteq D, R \subseteq \sqsubseteq_D, R \subseteq A \times A \text{ and } \exists M \in A.\forall a \in A.aRM\}$ be the set of the relations smaller or equal than $\mathcal{D}$, with a maximal element, ordered by inclusion componentwise. Prove that (i) $\mathcal{R}$ is a partial order; and show by counterexamples that: (ii) it may have no bottom; and (iii) it may be not complete. (Hint: take $\omega \cup \{\infty\}$ as D, and an infinite chain of finite sets.)

Exercise 3 (8)

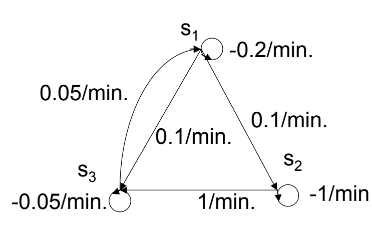
Consider the HOFL program:

$$t = rec\ f.\lambda n.\textbf{if}\ n\ \textbf{then}\ 0\ \textbf{else if}\ n-1\ \textbf{then}\ 1\ \textbf{else}\ (f\ n-2) + 2 \times (f\ n-1),$$

Compute the corresponding $\Gamma : [N_\perp \to N_\perp]_\perp \to [N_\perp \to N_\perp]_\perp$ and prove that $\lfloor f^* \rfloor : [N_\perp \to N_\perp]_\perp$ - where $f(n) = \lfloor \frac{(1+\sqrt{2})^n}{2\sqrt{2})} - \frac{(1-\sqrt{2})^n}{2\sqrt{2})} \rfloor$ restricted on the natural numbers - is a fixpoint of Γ. Observing that t is in fact a definition by well-founded recursion, conclude that $\lfloor f^* \rfloor$ is the minimal fixpoint. Finally, prove that it is the unique fixpoint.

Exercise 4 (7)

Consider the taxi driver example, but this time represented as a CTMC: see the Figure below, where rates are defined in 1/minutes, e.g. costumers show up every 10 minutes and rides last 20 minutes.



Assuming a unique label l for all the transitions, and disregarding self loops, define a PEPA agent for the system, and show that all states are different in terms of bisimilarity. Finally, to study the steady state behavior of the system, introduce the self loops, decorated with negative rates which are negated apparent rates, namely the negated sums of all the outgoing rates, and write and solve a system of linear equations similar to the one seen for DTMC: $\vec{s} \times M = \vec{0}$ and $\sum_i s_i = 1$. The equations express the fact that, for every state i, the probability flow from the other states to state i is the same as the probability flow from state i to the other states.