Models of Computation

Written Exam on February 2, 2011

AA031: 6 credits, Exercises 1-3, 2h:20
375AA: 9 credits, Exercises 1-4, 3h

Esercizio 1 (9)

Si dimostri che, se x non appare nel comando c, allora

$$< c, \sigma > \rightarrow \sigma' \quad \Rightarrow \quad < c, \sigma[n/x] > \rightarrow \sigma'[n/x].$$

Si assuma che $x \notin VAR(a) \;\Rightarrow\; (< a, \sigma > \rightarrow m \;\Rightarrow < a, \sigma[n/x] > \rightarrow m)$. Similmente per le espressioni booleane.

Exercise 2 (8)

Si modifichi la semantica di HOFL assumendo per il condizionale, solo nel caso il dominio delle alternative e del risultato sia piatto, la seguente semantica operazionale:

$$\frac{t_0 \rightarrow 0 \quad t_1 \rightarrow c_1}{\textbf{if } t_0 \textbf{ then } t_1 \textbf{ else } t_2 \rightarrow \; c_1} \quad \frac{t_0 \rightarrow n \quad n \neq 0 \; t_2 \rightarrow c_2}{\textbf{if } t_0 \textbf{ then } t_1 \textbf{ else } t_2 \rightarrow \; c_2} \quad \frac{t_1 \rightarrow c \quad t_2 \rightarrow c}{\textbf{if } t_0 \textbf{ then } t_1 \textbf{ else } t_2 \rightarrow \; c}.$$

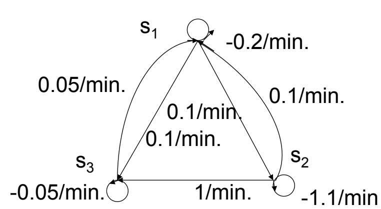
Si fornisca la corrispondente semantica denotazionale e si dimostri che anche per la semantica modificata $t \rightarrow c$ implica $[\![t]\!] = [\![c]\!]$. Si controlli infine che la semantica operazionale e quella denotazionale di **if** $rec\ x.x$ **then** 0 **else** 0 coincidono, mentre quelle di **if** $rec\ x.x$ **then** $(0,0)$ **else** $(0, 0+0)$ sono diverse.

Exercise 3 (5)

Si dimostri che ogni relazione R tra agenti CCS per cui valga $\alpha.\tau.\beta.nil \; R \; \alpha.\beta.nil$, se è una bisimulazione weak, allora non e' una congruenza. (Cenno: si consideri la mossa di Alice $\alpha.\tau.\beta.nil \xrightarrow{\alpha} \tau.\beta.nil$.)

Exercise 4 (8)

Consider the taxi driver example represented as a CTMC, but this time with the option of going back to state s_1 (parking) from state s_2 (moving slowly looking for costumers): see the Figure below.



Assuming a unique label l for all the transitions, and disregarding self loops, define a PEPA agent for the system, and show that all states are different in terms of bisimilarity. Finally, to study the steady state behavior of the system, introduce the self loops, decorated with negative rates which are negated apparent rates, namely the negated sums of all the outgoing rates, and write and solve a system of linear equations similar to the one seen for DTMC: $\vec{s} \times M = \vec{0}$ and $\sum_i s_i = 1$. The equations express the fact that, for every state i, the probability flow from the other states to state i is the same as the probability flow from state i to the other states.