Models of Computation

Midterm Exam on April 18, 2011

Exercise 1 (10)

Extend IMP with the command **test** b **do** c having the following denotational semantics:

$$\mathcal{C}[\![\textbf{test } b \textbf{ do } c]\!] = \mathit{fix}\ \Gamma \qquad \Gamma\varphi\sigma = \mathcal{B}[\![b]\!]\sigma \ \rightarrow \ \varphi\sigma, \mathcal{C}[\![c]\!]\sigma.$$

Define the operational semantics of the new command and prove its equivalence with the denotational semantics above. Finally, show that $\mathcal{C}[\![\textbf{test } b \textbf{ do } c]\!] = \mathcal{C}[\![\textbf{if } b \textbf{ then while } \mathit{true} \textbf{ do skip else } c]\!]$.

Exercise 2 (10)

Consider the logic inference system R corresponding to the rules of the grammar:

$$S ::= aB|bA \qquad A ::= a|aS|bAA \qquad B ::= b|bS|aBB$$

where the well formed formulas are of the form $x \in L$, where L is either S or A or B and where x is a string on $\{a, b\}$. Write down explicitly the rules in R.

Prove by rule induction - in one direction - and by mathematical induction on the length of the strings - in the other direction - that the strings generated by S are all the nonempty strings with the same number of a's and b's (formally $P(x \in S) \stackrel{def}{=} x|_a = x|_b \neq 0$), while A generates all the strings with an additional a (formally $P(x \in A) \stackrel{def}{=} x|_a = x|_b + 1$) and B with an additional b.

Finally prove by induction on derivations that $P(d/(x \in L)) \stackrel{def}{=} |d| \leq |x|$, i.e. the depth of any derivation d is smaller or equal that the length of the string x generated by it.

Exercise 3 (10)

Consider the HOFL term:

$$t = \textbf{rec } f.\, \lambda x.\, \textbf{if } \mathit{fst}(x) \times \mathit{snd}(x) \textbf{ then } x \textbf{ else } (f\ (f\ x)).$$

Derive the type, the canonical form and the denotational semantics of t. Finally show another term with the same denotational semantics as t but with different canonical form.