Models of Computation

Written Exam on June 24, 2011

(First part: Exercises 1 and 2, 90 minutes   Second part: Exercises 3, 4 and 5, 90 minutes)

(Previous TSD students: Exercises 1, 2 and 3, 135 minutes)

Exercise 1 (8)

Let us replace the **while** command of IMP with the new command **until** x **LE** n **step** k **do** c, with k positive natural number and $x \notin FV(c)$. The **until** command keeps executing c increasing every time x by k until it becomes larger than integer n. Its operational semantics is as follows:

$$\frac{\sigma(x) > n}{< \textbf{until}\ x\ \textbf{LE}\ n\ \textbf{step}\ k\ \textbf{do}\ c, \sigma > \rightarrow \sigma}$$

$$\frac{\sigma(x) \leq n \quad < c;\ x := x + k, \sigma > \rightarrow \sigma'' \quad < \textbf{until}\ x\ \textbf{LE}\ n\ \textbf{step}\ k\ \textbf{do}\ c, \sigma'' > \rightarrow \sigma'}{< \textbf{until}\ x\ \textbf{LE}\ n\ \textbf{step}\ k\ \textbf{do}\ c, \sigma > \rightarrow \sigma'.}$$

Define the denotational semantics of the new construct and prove the equivalence of the two semantics. Finally, assuming that $< c, \sigma > \rightarrow \sigma' \;\Rightarrow\; \sigma(x) = \sigma'(x)$ prove that in the new version of IMP all commands do terminate. (Hint: Prove the property by structural induction on commands. Consider only the **until** construct. Observe that in a goal oriented computation the value of $n - \sigma(x)$ strictly decreases, and when it becomes smaller that 0 the execution terminates.)

Exercise 2 (7)

Let us consider the set $(\{0, 1, \ldots, N-1\} \times \omega) \cup \{(N, 0)\}$ of pairs of natural numbers with the lexicographical relation $\sqsubseteq$ defined as $(n_1, m_1) \sqsubseteq (n_2, m_2)$ if $n_1 < n_2$ or if $n_1 = n_2$ and $m_1 \leq m_2$. Prove that $\sqsubseteq$ is a complete partial ordering with bottom, which is also a total ordering, namely for every pair (n_1, m_1), (n_2, m_2), either $(n_1, m_1) \sqsubseteq (n_2, m_2)$ or $(n_2, m_2) \sqsubseteq (n_1, m_1)$, or both. Finally, given the function $f : (\{0, 1, \ldots, N-1\} \times \omega) \cup \{(N, 0)\} \rightarrow \omega \cup \{\infty\}$ (where $\omega \cup \{\infty\}$ is ordered with $n \leq n + k, x \leq \infty$) defined as $f(n, m) = n + m$, show that it is not monotone, while $g : \{0, 1, \ldots, n-1\} \times \omega \rightarrow \omega \cup \{\infty\}$, defined as $g(n, m) = n$, is monotone but not continuous.

Exercise 3 (6)

Show which are the processes reachable from $p = ((rec\ x.(\alpha.x|\beta.nil)) \mid (rec\ x.\overline{\alpha}.x) \mid (rec\ x.(\alpha.x|\gamma.nil)))\backslash\alpha$ and from $p' = rec\ x.(\beta.x + \gamma.x)$ and prove that p and p' are not strong bisimilar but that they are weak bisimilar. Structural axioms $p|q = q|p$, $(p|q)|r = p|(q|r)$, $p|nil = p$ and $rec\ x.p = p[rec\ x.p/x]$ can be assumed.

Exercise 4 (5)

Consider the new LTS $\mapsto$ for the π-calculus defined as

$$\frac{p \stackrel{\alpha}{\rightarrow} q \quad \alpha \neq x(y)}{p \stackrel{\alpha}{\mapsto} q} \qquad \frac{p \stackrel{x(y)}{\rightarrow} q}{p \stackrel{x}{\mapsto} \lambda y.q \stackrel{w}{\mapsto} q[w/y]}$$

Prove that on the new LTS the strong late ground bisimulations can be defined without considering input a special case, namely with only the first of the two clauses seen during the course. (Hint: given an Alice-Bob strategy for a LTS, derive a strategy for the other LTS, and viceversa.)

Exercise 5(4)

A probabilistic system has three possible states, *idle*, *working* and *failure*. From *idle*, the system can move with 50% probability to *working*, or otherwise it can remain in the same state. From *working*, the system returns idle with 90% probability and otherwise moves to *failure*. Finally, from *failure* the system moves with 50% probabiity to *working*, or otherwise remains in the same state. Define the DTMC corresponding to the description above, observe that the chain is ergodic, and compute the stationary probabilities.