Models of Computation

Written Exam on June 6, 2012

(First part: Exercises 1 and 2 Second part: Exercises 3 and 4)

Exercise 1 (8)

We modify the definition of memory σ in IMP as a partial function from locations to integers. The update function $_{-}[_{-}]: \Sigma \times \Sigma \to \Sigma$ is computed as follows: $\sigma[\sigma'](x) = \text{if } \sigma'(x)$ is defined, then $\sigma'(x)$, else if $\sigma(x)$ is defined then $\sigma(x)$, else undefined.

The syntax of IMP is modified as follows. A new syntactic category Mass of multiple assignments, with notation m, is introduced, with two constructs, ordinary (single) assignment x := a and multiple assignment composition m|m. Then multiple assignments are commands: $c := \{m\} \mid \ldots$, while single assignments are not directly commands anymore. The denotational semantics is as follows:

$$\mathcal{M}[\![x:=a]\!]\sigma = (\mathcal{A}[\![a]\!]\sigma/x) \qquad \mathcal{M}[\![m_1]\!]\sigma = (\mathcal{M}[\![m_1]\!]\sigma)[\mathcal{M}[\![m_2]\!]\sigma] \qquad \mathcal{C}[\![\{m\}]\!]\sigma = \sigma[\mathcal{M}[\![m]\!]\sigma]$$

where (n/x) is a memory with (n/x)(x) = n and undefined elsewhere.

Prove that: (i) multiple assignment composition is associative, namely

$$\mathcal{M}[\![(m_1|m_2)|m_3]\!]\sigma = \mathcal{M}[\![m_1|(m_2|m_3)]\!]\sigma;$$

(ii) the denotational semantics of a single assignment x:=a in the ordinary version of IMP coincides with $\mathcal{C}[\![\{x:=a\}]\!];$ (iii) $\mathcal{C}[\![\{x:=1\};\{x:=2\}]\!]\sigma\neq\mathcal{C}[\![\{x:=1\};\{x:=2\};\{y:=x\}]\!]\sigma$. Finally, (iv) give the operational semantics of the new constructs and (v) prove their equivalence with the denotational semantics.

Exercise 2 (8)

We consider strings $\alpha, \beta \in V^*$ with $V = \{1, 2, \dots, b\}$, which are intended to represent natural numbers in base b with digits without zeroes. A relation < is defined on them:

$$\frac{|\alpha| < |\beta|}{\alpha < \beta} \qquad \frac{|\alpha| = |\beta| \land n < m}{n\alpha < m\beta} \qquad \frac{|\alpha| = |\beta| \land \alpha < \beta}{n\alpha < n\beta}.$$

Prove that: (i) < is a transitive, well founded relation on V^* ; (ii) $\le = < \cup \{(n,n)\}$ is a non complete partial ordering with bottom; and (iii) it is a *total* ordering. Then, (iv) show that total ordering $(V^*,<)$ is isomorphic to $(\omega,<)$ via monotone function $[\![.]\!]$ defined as $[\![\lambda]\!] = 0$ and $[\![n\alpha]\!] = n + b[\![\alpha]\!]$ (where λ is the string of lenght zero), namely $[\![.]\!]$ is bijective and $\alpha < \beta \to [\![\alpha]\!] < [\![\beta]\!]$.

Exercise 3 (8)

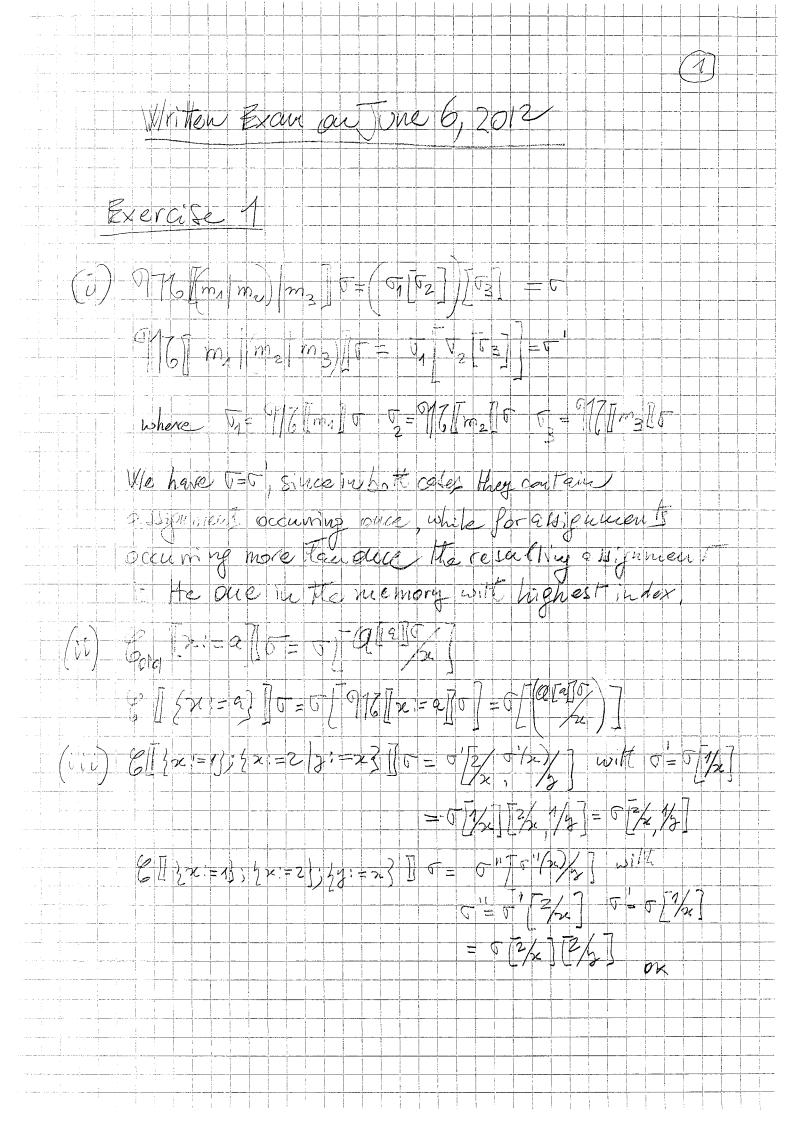
Consider the HOFL program:

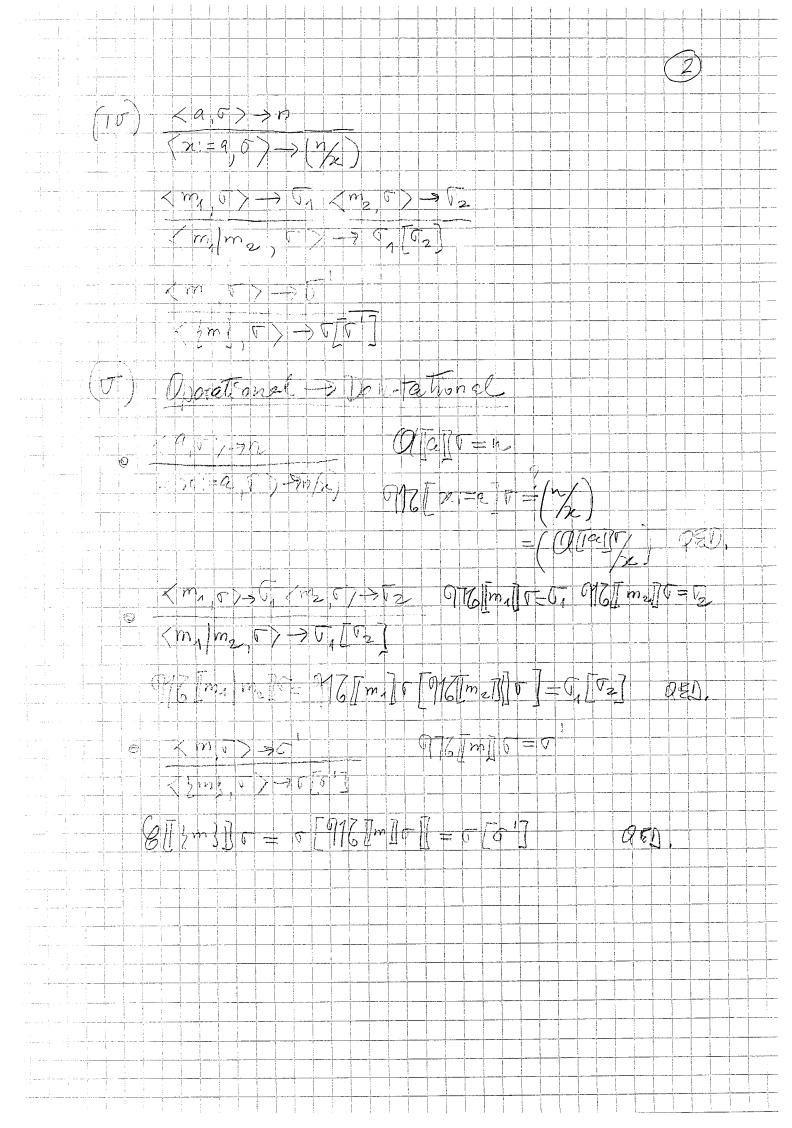
$$t = rec \ f. \lambda n.$$
if n then 0 else if $n-1$ then 1 else $2 \times (f \ n-1) - (f \ n-2),$

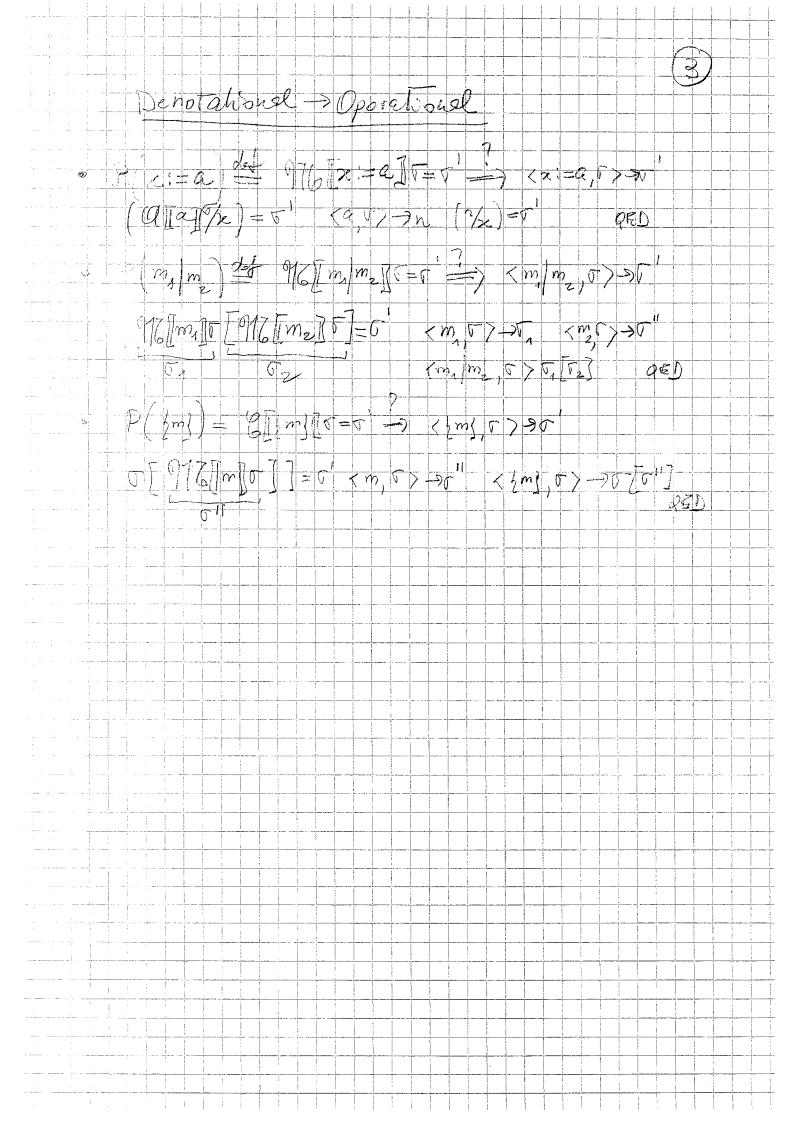
Compute the corresponding $\Gamma: [N_{\perp} \to N_{\perp}]_{\perp} \to [N_{\perp} \to N_{\perp}]_{\perp}$ and prove that $\lfloor f^* \rfloor : [N_{\perp} \to N_{\perp}]_{\perp}$ - where $f(n) = \lfloor n \rfloor$ - is a fixpoint of Γ . Observing that t is in fact a definition by well-founded recursion, conclude that $\lfloor f^* \rfloor$ is the unique fixpoint.

Exercise 4 (6)

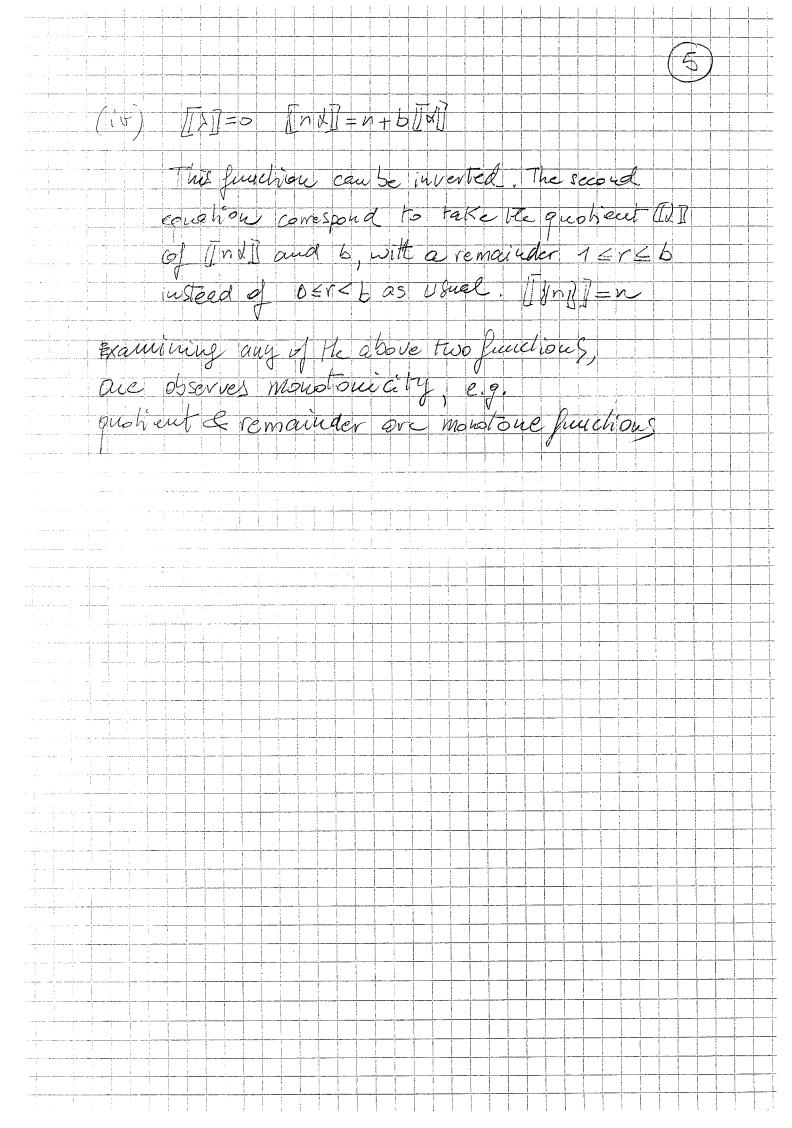
Determine the processes reachable from $p = ((rec \ x.\alpha x)|(rec \ x.(\overline{\alpha}.x|\beta.nil)))\setminus \alpha$ and from $q = rec \ x.\beta x$ and prove that $p \in q$ are not strong bisimilar, but that they are weak bisimilar. For simplicity, assume the structural axioms p|q = q|p, (p|q)|r = p|(q|r) e p|nil = p.

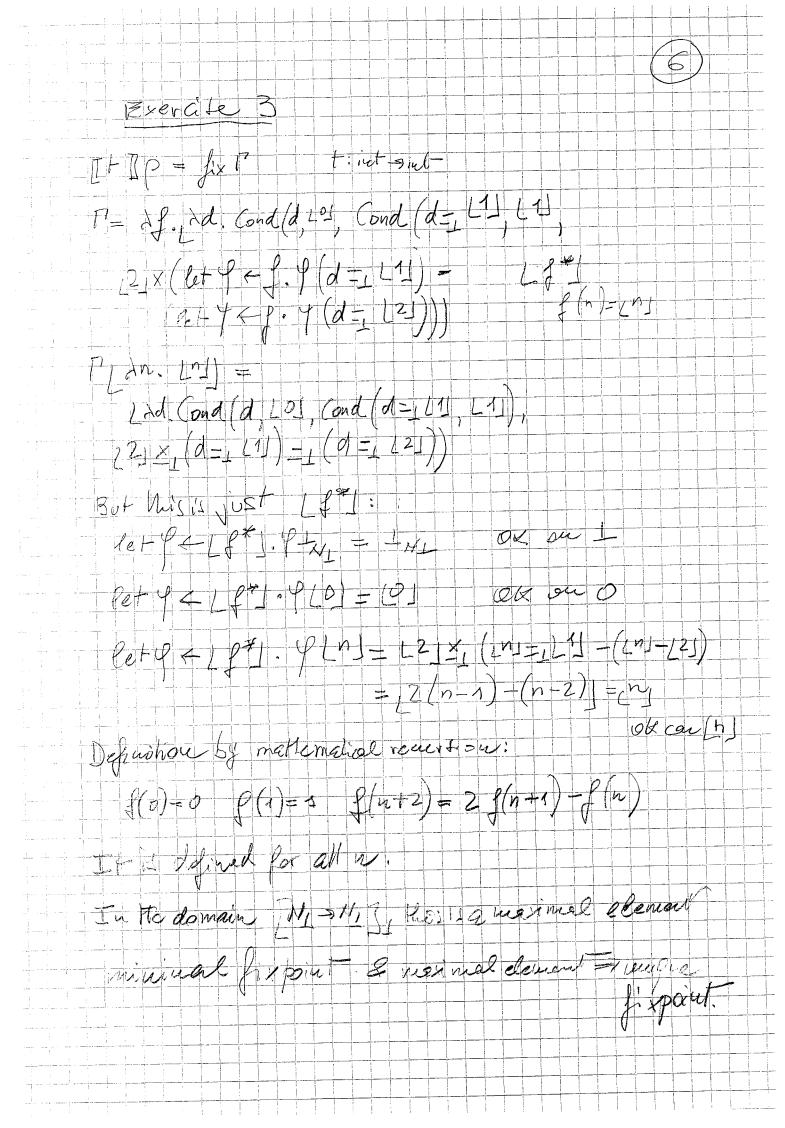






Exercise 2 1 C 13 have the 1 12 port Is efter D<B or p<8 Her also of KO # PS = P = O Hew He hypotheris in importable IP W= 1B = 1 = K we assure transhrity helds 10,0 = 102/5 = 100/ = K+1 and 9,00, or 92 × 93 Han transhirity holds IP lad = a B = /27/= FM Hen translivity hold for the male, inducation hypotheti L'is well sunded it is acyclic since it is transhive it is enough to notice XXX devict be proved, no we applied If |d|= + +0- sings Bwith (0< x) can be at most 26h. it we already proved transhirity and autisymmetry acidiants). Reflexivity is aded by hanny Human of meet 12 to on pts of his. 1 < 11 × 111 mis ache a with no led 1 4 is attotal ordering. For I B only Liceportony are postable: either is /> or It or 19 th. It is possible to be It by applying the interessee rules symbolise by a good en exted way. Only one of Here applies





Exercise 4 ccz, Zz Bul=p" rec x, xx = p Bull Bull on Buil = rn P=(P/P") \ P/\.P)\\
\[\bar{B} \] (P') P/(1) X & (P') ZP" (F1) X $(P'|P''/r_n)$ $P_n = (F'|P''|r_n) \times P_n = (P'/\nabla P''/r_n)$ n=0,100 $P_{n} \xrightarrow{P} P_{n} \xrightarrow{P_{n}} P_{n}$ $P_{n} \xrightarrow{P} P_{n+1} \xrightarrow{P} P_{n}$ $P_{n+1} \xrightarrow{P} P_{n} \xrightarrow{P} P_{n}$ 999 P + 9 in fact P = (P/P"/") and 97 P ≈ 9 ju fact from every state we have